Wiener-Hopf analysis of the field scattered by the edge of a truncated upper Conductor, Parallel Plane waveguide loaded with magnetized plasma

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ABSTRACT

The final scope of this work is the study of microstrip lines an patch antennas printed on magnetized plasma and particularly a solid state one. This is of particular interest since it offers the same features as the magnetized ferrite substrate, but with a dual field behavior. The present effort is concentrated on the study of the scattering by the edge of a semi infinite printed conductor, when the dominant extra-ordinary TEM wave propagating in the parallel plane region is incident normally on this edge. The Wiener-Hopf technique is employed for this purpose and this paper represents our first analytical approach.

INTRODUCTION

When ferrite or plasma materials are subject to constant magnetic field they exhibit anisotropic permeability ($\overline{\overline{\mu}}_r$) and permittivity ($\overline{\overline{\varepsilon}}_r$) respectively. These tensor constitutive parameters depend on both the biasing magnetic field and the operating frequency. This dependence enables their dynamic control through the DC current of an electromagnet, which generates the biasing constant magnetic field. These exceptional features offered by ferrites are extensively used in microwave waveguide, stripline and microstrip devices from long ago. Recently, the exploitation of these features in printed antenna applications received a considerable research effort. These include wideband electronic frequency tuning, beam steering and possible surface wave and RCS reduction. In contrary there was only a limited use of magnetized plasmas in microwave devices. In antenna applications there was a considerable research effort mainly because these antennas were embedded in a magnetized plasma in their operating environment, as for example in satellite communications and nuclear fusion. However, there was only a minor effort in the direction of exploiting the magnetized plasma features in printed antennas. This lack may due to the difficulties in generating and controlling ionized gas plasma. However, the evolution in the electronic solid state plasma technology may enable its application in printed microwave devices and antennas. Some recently published studies [1-3] based on numerical techniques are directed toward these applications, but without making this clear. The present effort aims at the analysis of a rectangular patch antenna printed/placed on a magnetized solid state plasma, based on an analytical approach. In the framework of this study the first step is the solution of a canonical problem of a TEM wave propagating in a parallel-plane waveguide with a semi-infinite upper conductor loaded with a magnetized plasma and normally incident on the edge defined by the truncated upper conductor (Fig.1). The Wiener-Hopf technique is employed for the estimation of the scattered field at the edge and consequently the reflected TEM wave propagating back, in the parallel plane region. The established reflection coefficient can be used in the study of a patch antenna based on the transverse resonance technique, according to our previous work [4]. This approach is based on the approximation that a TEM wave (possibly emanating from a probe feed) propagating zig-zag in the parallel plane region below a rectangular patch radiator incidents almost normally on its radiating edge. This was actually observed for any rectangular patch radiator operating at its dominant mode.

However, independently from the above described specific application the solution of the canonical problem (scattering from the edge of the truncated conductor) is by itself a significant contribution. A lot of interesting phenomena regarding the excitation of surface waves in the grounded plasma region and the radiated space wave can be explained through the scattered field expressions. The dependence of the turn-on/off conditions from the plasma parameters and especially the magnetizing DC field are of particular importance. One possible question is “why do you bother to analyze magnetized plasma since magnetized ferrites offer the same features”? This is not exactly true, because the magnetized plasma ($\overline{\overline{\varepsilon}}_r$) behaves as the dual of magnetized ferrites ($\overline{\overline{\mu}}_r$). So plasma may be more suitable for some applications than ferrite material. For example, in a parallel plane loaded with ferrite the dominant TEM mode is the ordinary one, since the extraordinary TEM mode has an electric field component parallel to the two metallic planes. In contrary, in the magnetized plasma case this occurs vice-versa and the extra-ordinary TEM mode becomes dominant. From this observation one may conclude that is more convenient to exploit the dynamic $\bar{\bar{\varepsilon}}_r$ control in patch antennas printed-placed on magnetized (solid state) plasma. This property was exploited by Johansen [5] as well as herein.

FORMULATION

The geometry to be studied is shown in Fig.1. Basically it consists of a parallel plane waveguide loaded with magnetized plasma, where the lower conductors (ground plane) and the plasma substrate are assumed extending to infinity while a semi-infinite (truncated z<0) upper conductor is considered. The biasing constant magnetic field ($\overline{H}_{dc}$) is assumed parallel to the two planes and parallel to the edge ($\hat{y}$ -axis, $\overline{H}_{dc} = H_O\hat{y}$). The extra-ordinary TEM wave (possibly emanating from a probe feed) propagating along the $\hat{z}$ -axis (transverse to $\overline{H}_{dc}$) is considered to be incident normally on the edge defined by the truncated upper conductor. Time harmonic fields ($e^{j\omega t}$) and a λ-space spectrum Fourier transform pair in the z-direction are considered (in the form $e^{jko\lambda z}$).

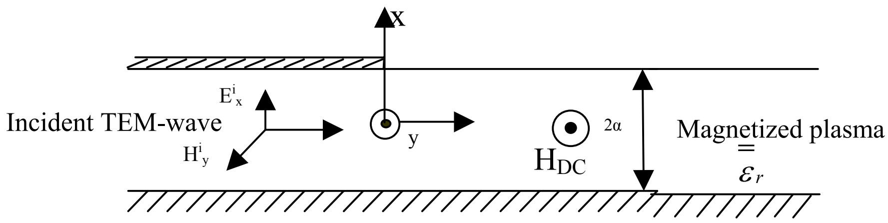

Fig.1 A TEM wave incident upon the edge defined by the truncate upper conductor of a parallel plane waveguide loaded with magnetic plasma.

This assumption results in a simplification of the wave equation by substituting $\partial/\partial y = -jk_0\lambda$. Since the excited extra-ordinary TEM wave propagating in the parallel plane region (z<0) is assumed to be incident normally on the edge z=0, there will be no variation of the scattered field in the also infinitely extending y-direction. This in turn results to a further simplification of $\partial/\partial y$ =0. Moreover, the magnetized plasma ($\overline{H}_{DC} = H_o\hat{y}$) relative permeability tensor is given in [5] us:

$$\bar{\bar{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{r1} & 0 & j\varepsilon_{r2} \\ 0 & \varepsilon_{r3} & 0 \\ -j\varepsilon_{r2} & 0 & \varepsilon_{r1} \end{bmatrix} \tag{1}$$

Where $$\varepsilon_{r1} = \frac{\Omega^2 - R^2 - 1}{\Omega^2 - R^2},\ \varepsilon_{r2} = \frac{R}{\Omega(\Omega^2 - R^2)},\ \varepsilon_{r3} = 1 - \frac{1}{\Omega^2}$$

and $$\Omega=\omega/\omega_p,\ R=\omega_c/\omega_p,\ \omega_p^2=Ne^2/m\varepsilon_o,\ \omega_c=-e\mu_oH_o/m.$$

The plasma frequency is symbolized as ω_p and its gyromagnetic frequency as ω_c. Also, e and m are the charge and the mass of an electron and ε_o, μ_o are the free space permittivity and permeability. With the above considerations, the wave equation for the scattered magnetic field in the λ-domain ($\tilde{H}^s_y$) for the plasma region can be written as:

$$\{\frac{\vartheta^2}{\vartheta x^2} + k^2{}_o(\frac{\varepsilon_{rq}}{\varepsilon_{r1}} - \lambda^2)\}\tilde{H}^s_y = 0 \tag{2}$$

Where $k_o = \omega\sqrt{\mu_o\varepsilon_o}$ the free space wavenumber and $\varepsilon_{rq}=\varepsilon_{r1}^2-\varepsilon_{r2}^2$.

The general solution of (2) in the plasma region $-a \le x \le a$ takes the form:

$$\tilde{H}_y^s = B_p(\lambda)\cosh(k_o u_p x) + C_p(\lambda)\sinh(k_o u_p x) \tag{3}$$

where $u_p = \sqrt{\lambda^2 - \varepsilon_{rq}/\varepsilon_{r1}}$ and $\mathrm{Re}(u_p) \geq 0$.

The transverse field components can also be expressed in the transformed domain as:

$$\tilde{E}_x^s = \frac{\zeta_o}{\varepsilon_{rq}}\{\lambda\varepsilon_{r1}\tilde{H}_y^s - \frac{\varepsilon_{r2}}{k_o}\cdot\frac{\partial\tilde{H}_y^s}{\partial x}\} \qquad \tilde{E}_z^s = j\frac{\zeta_o}{\varepsilon_{rq}}\{\lambda\varepsilon_{r2}\tilde{H}_y^s - \frac{\varepsilon_{r1}}{k_o}\cdot\frac{\partial\tilde{H}_y^s}{\partial x}\} \tag{4}$$

where $\zeta_o = \sqrt{\mu_o/\varepsilon_o} = (120\pi)\Omega$ the free space characteristic impedance.

The remaining field components vanish ($\tilde{E}_y^s = \tilde{H}_x^s = \tilde{H}_z^s = 0$) due to the assumption $\partial/\partial y = 0$. The general solution for the air region can be obtained by substituting in equations (2)-(4) its characteristics ($\varepsilon_{r1} = \varepsilon_{r3} = 1, \varepsilon_{r2} = 0$). However, this solution must also obey the radiation condition at infinity, so for the air region $x \geq a$, we have:

$$\tilde{H}_z^s = A_p(\lambda)\cdot \mathrm{e}^{-k_o u_o (x-a)} \text{ with } u_o = \sqrt{\lambda^2 - 1} \text{ and } \mathrm{Re}(u_o) \geq 0 \tag{5a}$$

and

$$\tilde{E}_x^s = \zeta_o\lambda\cdot\tilde{H}_y^s \qquad \tilde{E}_x^s = \frac{1}{j\omega\varepsilon_o}\cdot\frac{\partial\tilde{H}_y^s}{\partial x} \tag{5b}$$

The quantities $A_p(\lambda)$, $B_p(\lambda)$ *and* $C_p(\lambda)$ involved in the above equations are arbitrary spectral functions to be estimated from the application of the boundary conditions. The incident extra-ordinary TEM wave propagating in the parallel-plane region toward the positive z-direction is given by Johansen,[5] as:

$$H_y^i = \exp\{k_o\varepsilon_{r1}x/\sqrt{\varepsilon_{r1}} - jk_o\sqrt{\varepsilon_{r1}}z\} \quad \text{and} \quad E_x^i = (\zeta_o/\sqrt{\varepsilon_{r1}})\cdot H_y^i \tag{6}$$

where a unit amplitude is assumed for H_y^i just for convenience. The scattering on the edge will excite a reflected TEM wave along with higher order modes, which will in turn vanish at a relatively small distance from the edge, provided that the plasma-substrate thickness is small enough (in order for these modes to be below cut-of). The reflected TEM wave propagating in the negative z-direction can be expressed as:

$$H_y^r = \Gamma_{TEM}\cdot\exp\{-k_o\varepsilon_{r2}\cdot x/\sqrt{\varepsilon_{r1}} + jk_o\sqrt{\varepsilon_{r1}}z\} \quad \text{and} \quad E_x^r = (-\zeta_o/\sqrt{\varepsilon_{r1}})\cdot H_y^r \tag{7}$$

Where Γ_{TEM} is the complex reflection coefficient to be sought from the study of the scattering at the edge, using Wiener-Hopf technique. The Jone's method is employed for convenience reasons,[6-7], namely the wave equation solution and the application of the boundary conditions are carried out in the transformed λ-domain. Specifically, the scattered tangential electric field $\tilde{E}_z^s$ must vanish on the metallic ground plane ($x=-a$) and preserve its continuity at the plasma-air interface at $x = a$. Note that the corresponding tangential component of the incident field is identically zero $\left(E_z^i = E_z^r = 0\right)$. Also the tangential electric field must vanish on the truncated upper conductor $\left(x = a, z < 0\right)$:

$$E_z^s\left(x = a^+, z < 0\right) = f_+\left(z\right) = j\zeta_o\int_{C_+} u_o A_p\left(\lambda\right)e^{-jk_o\lambda z} = 0 \tag{8}$$

In order to satisfy (8), we are looking for a function which must be identically zero for z<0 and with α-value $f_+\left(z\right) \neq 0$ to be defined for $z > 0$. This may result from the inverse Fourier transform of a "positive" spectral function $R_+\left(\lambda\right) = u_o A_p\left(\lambda\right)$ analytic in the upper λ half-plane. Using the two previously described boundary conditions the two spectral functions $B_p\left(\lambda\right)$ and $C_p\left(\lambda\right)$ are also expressed in terms of $R_+\left(\lambda\right)$. The dependence of the scattered field from the incident extra-ordinary TEM wave (excitation) can be obtained either by applying the continuity of the total normal electric flux density $D_{x,tot}$, or the continuity of the total tangential magnetic field $H_{y,tot}$ at the plasma-air interface $\left(x = a, z > 0\right)$. At this point we assume that the incident field propagates unattenuated in the region $z > 0$, or that it exists in the area $\left(|x| \le a, -\infty < z < +\infty \right)$. This is a usual assumption of the Wiener-Hopf technique, e.g. [6, p.126], since it simplifies the problem while its contribution can be evaluated and subtracted latter from the corresponding residue of the propagating field. Imposing in turn the latter boundary condition for $\tilde{H}_{y,tot} = \tilde{H}_y^s + \tilde{H}_y^i$, yields:

$$\tilde{L}_-(\lambda) = \tilde{H}_y^s\left(x = a^+, \lambda\right) - \tilde{H}_y^s\left(x = a^-, \lambda\right) - \tilde{H}_y^i\left(x = a^-, \lambda\right) = 0 \qquad \text{valid} \quad \text{for} \quad z>0 \tag{9}$$

Taking the inverse Fourier transform and requiring the integral to be identically zero for $z > 0$ and to have a non-zero value for $z < 0$, we may define the spectral function $\tilde{L}_-(\lambda)$ as a "negative" one. Namely, a function analytic in the lower λ half-plane. The combination of the above expressions yields a Wiener-Hopf equation of the form:

$$Q(\lambda)\tilde{R}_+(\lambda) = \tilde{L}_-(\lambda) - \tilde{j}_+(\lambda) \tag{10}$$

Where

$$Q(\lambda) = \frac{1}{u_o} + \frac{\lambda\varepsilon_{r2} + \varepsilon_{r1} \cdot u_p \cdot coth\left(k_o u_p a\right)}{\left(\lambda^2 - \varepsilon_{r1}\right)} \tag{11}$$

and

$$\tilde{j}_+(\lambda) = -j\, exp\left(k_o \varepsilon_{r2} a / \sqrt{\varepsilon_{r1}}\right) / \left\{2\pi\left(\lambda - \sqrt{\varepsilon_{r1}}\right)\right\} \tag{12}$$

The spectral function $\tilde{j}_+(\lambda)$ represents the Fourier transform of a fictitious current density that could be induced on $\left(x = a, z > 0\right)$ if a conductor would be placed there. The remaining of the analysis involves the factorization of the kernel $Q(\lambda)$ into a product of "positive" and "negative" function $Q(\lambda) = Q_+(\lambda) \cdot Q_-(\lambda)$ and in turn the evaluation of $\tilde{R}_+(\lambda)$. This enables the evaluation of the scattered field, the TEM wave reflection coefficient and in extension the radiated sky-wave field. More details along with the further process will be presented during the symposium. It must also be noted that following a similar approach and exploiting the remaining boundary conditions a Wiener-Hopf equation involving the induced change density, can be obtained.

CONCLUSIONS

A geometry of a parallel plane waveguide, loaded with a magnetized plasma with a semi-infinite upper conductor is considered. The scattering of the dominant extra-ordinary TEM-wave normally incident upon the edge defined by the truncated upper conductor is treated analytically. The effort arriving up to the Wiener-Hopf equation is currently described, while this continues toward its final solution.

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