WAVELET-BASED ELECTROMAGNETIC ANALYSIS OF NONLINEAR OPTICAL STRUCTURES

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ABSTRACT

We have incorporated the instantaneous Kerr nonlinearity and the linear Lorentz dispersion in the wavelet-collocation time domain analysis method. Efficient numerical analysis has been demonstrated for nonlinear optical phenomena of self-focusing effects and temporal and spatial soliton formation. The computational expenditure has been significantly reduced in the proposed method compared to the standard FDTD analysis.

INTRODUCTION

Numerical techniques have been of great importance in modeling electromagnetic and optical signal propagation. However, precise full-wave analysis of optical waveguides has been very difficult due to the electrical largeness of the problems. If the waveguides have any discontinuities, or if they are longitudinally-varying or strongly-guiding structures, the methods based on the paraxial approximation of the nonlinear Shrödinger equation, i.e., the split-step Fourier or the beam propagation method, is no longer applicable, and rigorous full-wave analysis is required. The finite-difference time-domain (FDTD) formulation of Maxwell's equations is one of the best candidates for the rigorous analysis [1], [2]. However, unfortunately, three-dimensional optical waveguide problems are often electrically too extensive to analyze with the FDTD.

Recently, we have shown that the application of wavelets yields an effective large-stencil finite-difference scheme to solve Maxwell's equations [3]. The approach is called "the wavelet-collocation method"; with the interpolation basis functions, the proposed scheme outperforms the standard FDTD by allowing two to four times coarser cells per dimension, significantly reducing the spatial discretization, and thus the computational effort.

The interpolet is not a true wavelet because it has no vanishing moment. However, in the context of the time-domain formulation of the wavelet-collocation technique, it plays the equivalent roll as the other wavelet basis functions. The advantage of applying the interpolet is that it has formal biorthogonality and it significantly simplifies the numerical algorithm compared to the formulation based on the other type of wavelets. The method is particularly suitable for the analysis of large-scaled dielectric waveguides, where the size of the waveguides is much larger than the wavelength of the signal, which is the typical property of the waveguides for optical communication systems.

In this paper, we will demonstrate the incorporation of the instantaneous Kerr-nonlinearity and the linear Lorentz dispersion in the proposed wavelet-based approach. The Kerr nonlinearity is the dominant property that causes interesting phenomena such as self-focusing effects [4], [5] and the soliton propagation in optical media, and has been implemented in the FDTD method [6], [7], [8]. By virtue of the interpolation property of the basis, where the expansion coefficients for the scaling functions are the direct physical sampled values, the nonlinearity is effectively treated even in the variable expansion method without serious computational overhead. We actually simulate in two-dimensional setting the temporal and spatial soliton propagation in nonlinear optical media with reduced computational effort, which can be handled even with personal computers while maintaining the accuracy.

We have already reported that the proposed wavelet analysis technique reduces the computational effort in electrically-large problems compared to the standard FDTD method [9], [3]; the reduction is typically by an order of magnitude both for the memory requirement and for the CPU time. Since the program structure is similar to that of FDTD, the code optimization techniques such as parallelization, vectorization and effective memory management techniques can also be used in the proposed method.

NUMERICAL IMPLEMENTATION

We consider a two-dimensional TE polarized wave propagating in the xz-plane of the Cartesian coordinates represented by Maxwell's equations

$$\begin{aligned} \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= \frac{\sigma}{\epsilon_0} D_y + \frac{\partial D_y}{\partial t} , \\ -\mu \frac{\partial H_x}{\partial t} = -\frac{\partial E_y}{\partial z} , &\quad -\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} , \end{aligned} \tag{1}$$

where E_y denotes the electric field, D_y the electric flux densicy, and H_x and H_z are the magnetic fields, ϵ_0 is the dielectric constant of free space, σ is the electric conductance, and μ is the megnetic permeability of the media.

Ampere's law in (1) is discretized using the semi-implicit scheme for the right hand side of the equation, yielding

$$D_{y,i,k}^{n+1} = \frac{2\epsilon_0 - \sigma_z \Delta t}{2\epsilon_0 + \sigma_z \Delta t} D_{y,i,k}^{n} + \frac{2\epsilon_0 \Delta t}{2\epsilon_0 + \sigma_z \Delta t} \mathcal{H}^y , \tag{2}$$

where $\mathcal{H}^y$ is the space differencing, e.g.

$$\mathcal{H}^y = \frac{H_{x,i,k+1/2}^{n+1/2} - H_{x,i,k-1/2}^{n+1/2}}{\Delta z} - \frac{H_{z,i+1/2,k}^{n+1/2} - H_{z,i-1/2,k}^{n+1/2}}{\Delta x} , \tag{3}$$

for FDTD, which can be replaced by the high-order scheme or the wavelet-based schemes [10], [3].

Considering next the constitutive relation between the flux density D_y and the electric field E_y in the medium of frequency dependent relative dielectric constant $\epsilon_r(\omega)$, the instantaneous Kerr nonlinearity and the linear Lorentz dispersion properties are implemented by the auxiliary differential equation technique [11], [12], [13], nevertheless, with a slight modification to fit in our PML formulation which is described in our succeeding paper. We write in the phaser-domain as in [2],

$$\begin{aligned} \tilde{D}_y(\omega) &= \epsilon_0 \epsilon_r(\omega) \tilde{E}_y(\omega) \\ &= \epsilon_0 \epsilon_\infty \tilde{E}_y(\omega) + \tilde{P}_L(\omega) + \tilde{P}_{NL}(\omega) , \end{aligned} \tag{4}$$

where $\tilde{P}_L$ and $\tilde{P}_{NL}$ denote the linear and the nonlinear polarization. By applying the inverse Fourier transform, the expression in the time domain is obtained as

$$D_y(t) = \epsilon_0 \epsilon_\infty E_y(t) + P_L(t) + P_{NL}(t) . \tag{5}$$

The polarization for the linear Lorentz dispersive medium is given by

$$\begin{aligned} \tilde{P}_L(\omega) &= \epsilon_0 \chi_p(\omega) \tilde{E}_y(\omega) \\ &= \frac{\epsilon_0 \, \Delta\epsilon_p \, \omega_p^2}{\omega_p^2 + 2j\omega\delta_p - \omega^2} \tilde{E}_y(\omega) , \end{aligned} \tag{6}$$

where χ_p denotes the electric susceptibility, ω_p is the Lorentz resonant frequency, $\Delta\epsilon_p$ is the difference of the relative dielectric constant caused by the resonance, and δ_p is the damping factor. This is transformed into the differential equation by replacing $j\omega$ with $\partial/\partial t$ and $-\omega^2$ with $\partial^2/\partial t^2$,

$$\omega_p^2 P_L(t) + 2\delta_p \frac{\partial P_L(t)}{\partial t} + \frac{\partial^2 P_L(t)}{\partial t^2} = \epsilon_0 \Delta\epsilon_p \omega_p^2 E_y(t) , \tag{7}$$

which leads to the time difference form of P_L as

$$P_L^{n+1} = a_L P_L^n + b_L P_L^{n-1} + c_L E_y^n \tag{8}$$

with the coefficients

$$a_L = \frac{2 - \omega_p^2 \Delta t^2}{1 + \delta_p \Delta t} , \quad b_L = -\frac{1 - \delta_p \Delta t}{1 + \delta_p \Delta t} , \text{and} \quad c_L = \frac{\epsilon_0 \Delta\epsilon_p \Delta t^2 \omega_p^2}{1 + \delta_p \Delta t} . \tag{9}$$

The instantaneous Kerr nonlinear medium is simply given by

$$P_{NL}(t) = \epsilon_0 \chi_0^{(3)} E_y(t)^3 \ , \tag{10}$$

where $\chi_0^{(3)}$ is the strength of the nonlinearity related to the linear part of the refractive index n_0 and the nonlinear part to the refractive index n_2 (m^2/V^2) by $\chi_0^{(3)} = 2n_0 n_2$. The time difference expression is given by

$$P_{NL}^{n+1} = \epsilon_0 \chi_0^{(3)} \left(E_y^{n+1}\right)^3 \ . \tag{11}$$

By substituting (8) and (11) into (5), and solving with respect to E_y^{n+1}, we obtain the nonlinear equation

$$E_y^{n+1} = \frac{D_y^{n+1} - a_L P_L^n - b_L P_L^{n-1} - c_L E_y^n}{\epsilon_0 \epsilon_\infty + \epsilon_0 \chi_0^{(3)} \left(E_y^{n+1}\right)^2} \ . \tag{12}$$

Equation (12) can be solved by a simple iteration method as in [6].

Because we model the dispersive media by the relation between the polarization and the electric field, one can solve the nonlinear equation (12) with including arbitrary number of polarizations without solving a system of equations. The magnetic field can be computed by the usual non-magnetic formulation [3].

NUMERICAL RESULTS

We analyzed a pulse propagation in a 5 μm $\times$ 20 μm analysis region of nonlinear medium with the wavelet-collocation scheme based on the 10th-order interpolation function [3]. The space and the time increments were $\Delta x = \Delta z =$ 0.05 μm and $\Delta t =$ 0.023586 fs. The medium has an instantaneous Kerr nonlinearity of $\chi_0^{(3)} = 2.0 \times 10^{-20}$ (m^2/V^2), and a linear Lorentz dispersion of $\omega_p = 9.0 \times 10^{14}$ (rad/s), $\delta_p = 5.0 \times 10^9$ (1/s), $\Delta\epsilon_p = 3.0$, and $\epsilon_\infty = 6.05$.

With this choice, the numerical dispersion of the FDTD lattice is the order of 10^{-3}, thereby small compared to the Lorentz dispersion, and the carrier frequency is in the range of anomalous dispersion of the Lorentz medium; the group velocity dispersion parameter β_2 ranges from -100 to -5 (ps^2/m) over the -20 dB bandwidth from 190 to 270 THz, where temporal soliton pulse can be formed by an excitation of sufficient intensity.

We excited the field by a raised-cosine modulated wave packet with approximately 10 carrier cycles within the envelope. The carrier frequency of the pulse was 2.31×10^{14} Hz or the free-space wavelength $\lambda_0 = 1.3$ μm, and the maximum amplitude was 7×10^9 (V/m). The excitation pulse has the -20 dB bandwidth of approximately 80 THz. The transverse profile of the pulse is the hyperbolic secant function with its full width at half magnitude (FWHM) is 0.65 μm.

The propagating pulse waveforms and the time signal detected at $z = 0$ μm and 5 μm are shown in Fig. 1 (a) and (b), respectively. We observe the typical triangular waveforms in both spatial and temporal domain. The computation was performed on a portable PC with a CPU of 300 MHz clock rate and 96 MBytes of memory. The CPU time was about one hour and memory usage was 9.4 MBytes. When the standard FDTD was used for this analysis, we needed a very fine grid of $\Delta x = \Delta z =$ 0.0125 μm at largest, resulting in the CPU time and the memory requirement of approximately 8 hours and 50 MBytes.

CONCLUSIONS

We have modeled dielectric media with Kerr nonlinearity and the linear Lorentz dispersion in the wavelet-collocation time-domain analysis technique, and have demonstrated an efficient analysis of optical pulse propagation with reduced computational effort compared to the standard FDTD method.

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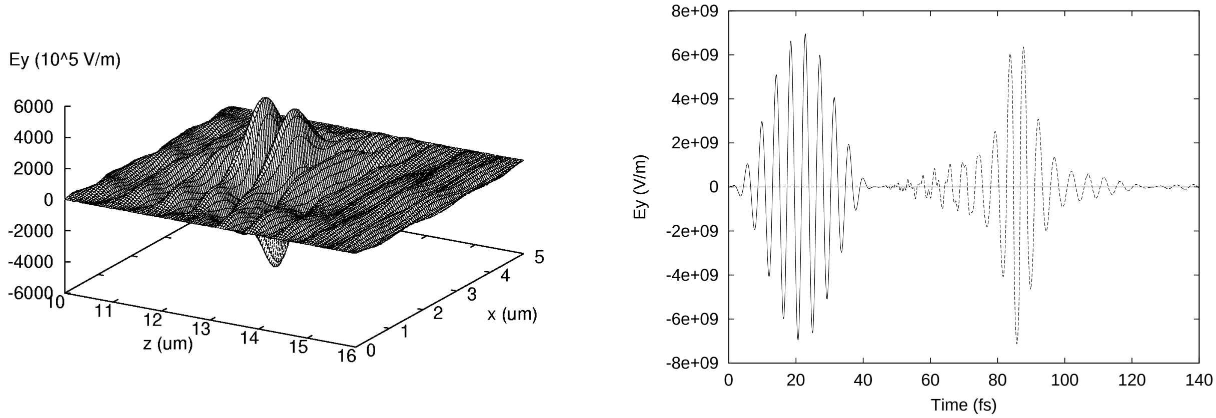

(a) (b)

Fig. 1. (a) A snapshot of the pulse waveform taken at 180 fs. (b) Time signal detected at z=0 (——) and at z=5 μm (- - -).

REFERENCES

[1] K.S.Yee, “Numerical solution of initial boundary value problems involving Maxwell’s equation in isotropic media”, *IEEE Trans. Antennas Propagation*, vol. 14, no. 5, pp. 302–307, May 1966.

[2] A.Taflove and S.C.Hagness, *Computational electrodynamics: the finite-difference time-domain method. –2nd ed.*, Artech House, 2000.

[3] M.Fujii and W.J.R.Hoefer, “A wavelet formulation of finite difference method: Full vector analysis of optical waveguide junctions”, *IEEE Journal of Quantum Electronics*, vol. 37, no. 8, pp. 1015–1029, Aug. 2001.

[4] R.Y.Chiao, E.Garmire, and C.H.Townes, “Self-trapping of optical beams”, *Phys. Rev. Lett.*, vol. 13, no. 15, pp. 479–482, Oct. 1964.

[5] P.Chernev and V.Petrov, “Self-focusing of light pulses in the presence of normal group-velocity dispersion”, *Optics Lett.*, vol. 17, no. 3, pp. 172–174, Feb. 1992.

[6] R.M.Joseph and A.Taflove, “Spatial soliton deflection mechanism indicated by FD-TD Maxwell’s equations modeling”, *IEEE Photon. Tech. Lett.*, vol. 6, no. 10, pp. 1251–1254, Oct. 1994.

[7] P.M.Goorjian and A.Taflove, “Direct time integration of Maxwell’s equations in nonlinear dispersive media for propagation and scattering of femtosecond electromagnetic solitons”, *Optics Lett.*, vol. 17, no. 3, pp. 180–182, Feb. 1992.

[8] R.W.Ziolkowski and J.B.Judkins, “Full-wave vector Maxwell equation modeling of the self-forcusing of ultrashort optical pulses in a nonlinear Kerr medium exhibiting a finite response time”, *J. Opt. Soc. Am. B*, vol. 10, no. 2, pp. 186–198, Feb. 1993.

[9] M.Fujii and W.J.R.Hoefer, “Time-domain wavelet-Galerkin modeling of two-dimensional electrically large dielectric waveguides”, *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 5, pp. 886–892, May 2001.

[10] M.Krumpholz and L.P.B.Katehi, “MRTD: New time-domain schemes based on multiresolution analysis”, *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 4, pp. 555–571, Apr. 1996.

[11] R.M.Joseph, S.C.Hagness, and A.Taflove, “Direct time integration of Maxwell’s equations in linear dispersive media with absorption for scattering and propagation of femtosecond electromagnetica pulses”, *Optics Lett.*, vol. 16, no. 18, pp. 1412–1414, Sep. 1991.

[12] R.M.Joseph and A.Taflove, “FDTD Maxwell’s equations models for nonlinear electrodynamics and optics”, *IEEE Trans. Antennas Propagat.*, vol. 45, no. 3, pp. 364–374, Mar. 1997.

[13] M.Okoniewski, M.Mrozowski, and M.A.Stuchly, “Simple treatment of multi-term dispersion in FDTD”, *IEEE Microwave Guided Wave Lett.*, vol. 7, no. 5, pp. 121–123, May 1997.