SENSING GEOMETRY AND FREQUENCY BAND EFFECTS ON TOMOGRAPHIC 2-D RESOLUTION IN THE FRESNEL ZONE

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ABSTRACT

The problem of investigating resolution limits while reconstructing a dielectric object from the knowledge of its scattered field over a rectilinear and bounded domain located in the Fresnel zone with respect to the object itself is dealt with. We examine the case of multifrequency illumination and singleview-multistatic configuration . The analysis is performed by casting the problem as the inversion of the linear operator arising from the Born approximation by means of the Singular Value Decomposition. The achievable resolution is estimated by resorting the Rayleigh's criterion and the role of the sensing geometry and the adopted frequencies is highlighted.

INTRODUCTION

In the inverse scattering problem, resolution plays a significant role since it accounts for the finest reconstructable "details" of the unknown. Unfortunately, to determine it, is not an easy task since the mathematical relationship between the scattered field and the unknown dielectric permittivity profile is non-linear.

However, if the unknown object can be assumed as a "weak scatterer", then the Born approximation can be used to linearize the problem, but also in this framework, ill-posed nature [1] of the inverse problem and the presence of unavoidable noise make it not possible to reconstruct arbitrarily "small" details of the unknown function.

In this paper, we face the problem of determining the achievable resolution while reconstructing an unknown dielectric object from the scattered field within a two-dimensional and scalar geometry. The scattered field E_S is observed over a rectilinear domain located in the Fresnel zone with respect to the investigation domain. We cast the problem as the inversion of the linear operator arising from the Born approximation and we adopt the Singular Value Decomposition (SVD) [1] of the relevant operator to perform the analysis. Finally, the task of estimating resolution is pursued by applying the Rayleigh's criterion to the reconstruction of a pulse object. A similar analysis has already been undertaken for the case of strip objects; in those cases the analytical SVD of the relevant operator has been determined [2,3,4]. Herein, by following the same line of reasoning we extend the analysis to the more general case of a rectangular investigation domain.

MATHEMATICAL FRAMEWORK

We assume the unknown object laying within the rectangular investigation domain $[-a,a]\times[z_0,z_1]$, embedded in a homogeneous medium having dielectric permittivity ε_0 and magnetic permeability μ_0, respectively. As interrogating field we consider plane waves linearly polarized along the y-axis and propagating along the z direction with spatial frequency varying in the interval $\beta\in[\beta_{\min},\beta_{\max}]$, β being the wave number in the host medium The scattered field E_S is observed over the rectilinear domain of the x-axis $[-x_0,x_0]$ located in the Fresnel zone with respect to the investigation domain (see fig. 1). The problem amounts to invert the following integral scalar operator

$$E_S(x,\beta)=\frac{-\omega\mu_0}{4}\sqrt{2/(\beta\pi)}e^{j\pi/4}\int_{-a}^{a}\int_{z_0}^{z_1}\frac{1}{\sqrt{z}}e^{-j\beta\left(2z+\frac{(x-x')^2}{2z}\right)}\chi(x',z)dx'dz \tag{1}$$

with $-x_0<x<x_0$, $\beta_{\min}<\beta<\beta_{\max}$ and $\chi(\cdot)=\varepsilon(\cdot)/\varepsilon_0-1$ and $\varepsilon(\cdot)$ being the contrast function and the permittivity profile of the unknown object, respectively.

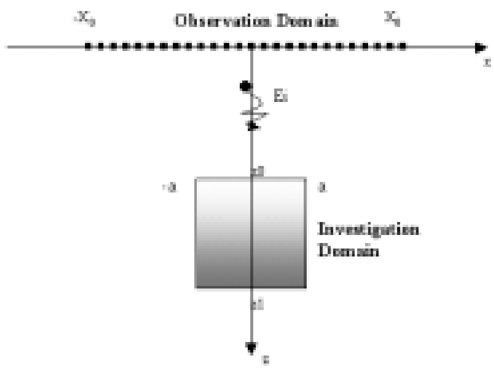

Fig. 1 Geometry of the problem

Let us denote as $X = L^2(\Omega = [-a,a]\times[z_0, z_1])$ and $Y = L^2(\Sigma = [-x_0, x_0]\times[\beta_{\min}, \beta_{\max}])$ the Hilbert spaces of square integrable functions within we search for the solution and suppose the data belong to, respectively. Since the kernel of the operator (1) is continuos on $\Sigma\times D$, (1) is a compact operator and thus its singular system $\{\sigma_n, u_n, v_n\}_{n=1}^{\infty}$ can be introduced. Furthermore, its singular values tend to zero as $n\to\infty$. This implies that solution does not depend continuously on data and, to restore stability, the problem can be regularized by truncating the SVD expansion of the unknown [1], obtaining thus

$$R\chi = \sum_{n=1}^{N}\frac{1}{\sigma_n}\left\langle E_S, v_n\right\rangle_Y u_n \tag{2}$$

where $\langle\cdot,\cdot\rangle_Y$ denotes the scalar product in the set Y and $R\chi$ is the stable reconstruction of the unknown.
The key point is how to choose the truncation index N. This depends on the behaviour of the singular values and on the tolerable noise level on data. Indeed, the bigger N, the finer resolution, the higher the effect of noise. Thus, a trade-off between accuracy and stability must be established. Fixed N, resolution can be estimated by applying the Rayleigh's criterion to the stable reconstruction of a Dirac pulse object , that is by evaluating the half-width of the main "lobe" of (2) with $\chi = \delta(\cdot)$.

THE CASE OF STRIP OBJECTS

This section is devoted to resume some results on the achievable resolution limits already obtained by analyzing strip objects [3,4] in a 1-D geometry. In this way, we start with simpler configurations to derive a guideline to estimate resolution and determine the role played by the parameters of the measurement configuration for the full 2-D configuration addressed in next section. In particular, in [3,4] we have examined the problem of reconstructing strip dielectric objects both orthogonal and parallel to the observation domain to focus attention on depth resolution and transverse resolution, respectively.

The relevant SVD has been analytically worked out and, under rather general circumstances about the measurement configuration, it has been observed that the singular values are nearly constant until their index reaches a critical value beyond which they decay exponentially fast to zero. The truncation index N has been evaluated as this critical index which is "virtually" independent from noise.

As far as depth strip object is concerned, we have found the following estimates for the truncation index Nz and resolution Δz, respectively

$$Nz = \frac{(\beta_{\max} - \beta_{\min})}{\pi}(z_1 - z_0) \tag{3}$$

and

$$\Delta z = \frac{\pi}{(\beta_{\max} - \beta_{\min})} \tag{4}$$

It is worthwhile to remark that by virtue of the Fresnel paraxial approximation whereas in the single-frequency case resolution degrades in depth and ameliorates quadratically with the extent of the measurement domain [2], in the multi-frequency case it remains almost uniform along depth and mainly depend on the adopted frequency diversity [3]. Conversely, for transverse resolution, frequency diversity is useless and the best resolution can be achieved by using only the highest one, thus leading to the well known estimate for the truncation index Nx and resolution Δx, respectively

$$Nx = \frac{2x_M a\beta_{\max}}{\pi z} \tag{5}$$

and

$$\Delta x = \frac{\pi z}{x_M \beta_{\max}} \tag{6}$$

We conclude this section by observing that transverse resolution are favourably affected by an enlargement of the measurement domain and in a negative way by the distance between the latter and the scattering object. On the contrary, for depth resolution this dependence is weaker since it mainly depends on the adopted frequency diversity. We expect that this behaviour holds also in the case of two-dimensional objects.

THE CASE OF RECTANGULAR INVESTIGATION DOMAIN

In this section we extend the above discussion to the case of a two-dimensional unknown object depicted in Fig. 1. To perform this task we find numerically the SVD of the operator (1).

In Fig. 2 the normalized behaviour of the singular values is reported for the case of $\beta \in [2\pi/1.5, 2\pi]$, $a = 4\lambda_{\min}$, $z_0 = 70\lambda_{\min}, z_1 = 85\lambda_{\min}$ $x_0 = 25\lambda_{\min}$ (solid line), $x_0 = 35\lambda_{\min}$ (dashed line), $\lambda_{\min} = 2\pi/\beta_{\max}$ being the smallest working wavelength. Since in the realistic situation the scattered field is affected by a number of factors, a rather high level of uncertainty should be expected. Therefore, in all the following examples we cut the singular values at –10 dB.

In particular, this leads to a truncation index $N = 59$ ($x_0 = 25\lambda_{\min}$) and $N = 78$ ($x_0 = 35\lambda_{\min}$). Note that, for these same configurations, the truncation indexes provided by (3) and (5) are $Nz = 10$ and $Nx = 5.7$ (evaluated in correspondence of z_0) for the first case, and $Nz = 10$ and $Nx = 8$ for the second case, respectively. This allows to state that the truncation index N for the 2-D case can be well estimated as the product between Nz and Nx of the depth and transverse 1-D case. This also allows to conjecture that the above reported results about resolution still hold.

This is confirmed by Fig. 3 and Fig. 4 which show, for $x_0 = 25\lambda_{\min}$, the (stable) reconstruction of two impulse objects located at $(-2\lambda_{\min}, 77\lambda_{\min})$ and $(2\lambda_{\min}, 77\lambda_{\min})$, and at $(0, 72\lambda_{\min})$ and $(0, 83\lambda_{\min})$, respectively .

It can be observed that transverse resolution is uniform along the x-axis and degrades in depth; on the contrary depth resolution is practically the same for each point. These results are in strict accordance with the ones already obtained in the 1-D cases.

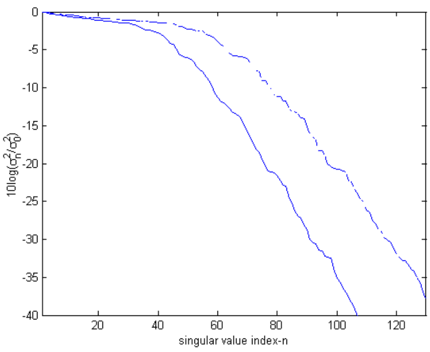

Fig. 2. Normalized behaviuor of the singular values; solid line a=25$\lambda_{\min}$; dashed line a=35$\lambda_{\min}$

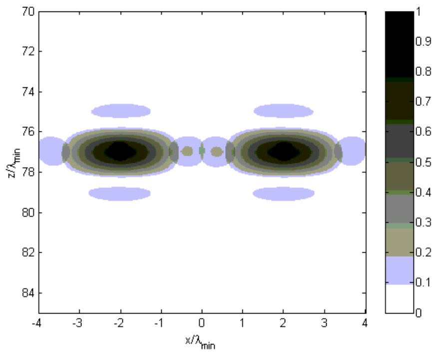

Fig. 3. Amplitude reconstruction of two pulses with a=25$\lambda_{\min}$ and located at (- 2$\lambda_{\min}$,77$\lambda_{\min}$) and (2$\lambda_{\min}$,77$\lambda_{\min}$)

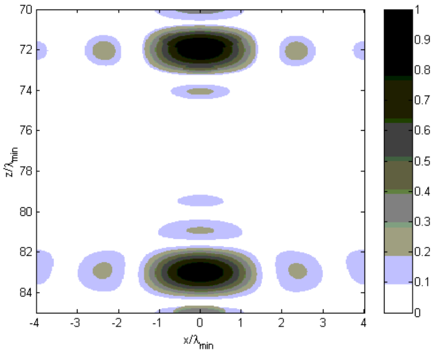

Fig. 4. Amplitude reconstruction of two pulses with a=25λ_{min} and located at (0,72λ_{min}) and (0,83λ_{min})

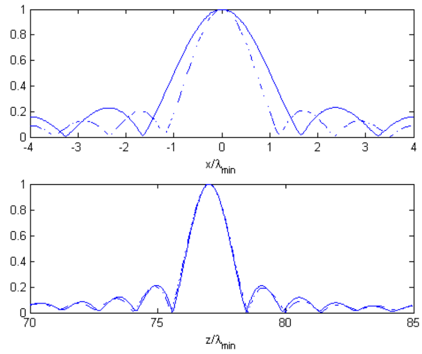

Fig. 5. Amplitude recontruction of a pulse located at (0,77λ_{min}). Cut along x (top); Cut along z (bottom); a=25λ_{min}(solid line); a=35λ_{min} (dashed line)

In order to make this accordance also quantitative we refer to Fig. 5 (solid line $x_0 = 25\lambda_{\min}$, dashed line $x_0 = 35\lambda_{\min}$) showing the cut view along the x-axis (top) and along the z-axis (bottom) of the stable reconstruction of a pulse object located at $(0{,}77\lambda_{\min})$. By applying the Rayleigh's criterion to estimate resolution we obtain $\Delta x = 1.6\lambda_{\min}$ and $\Delta x = 1.2\lambda_{\min}$ for transverse resolution, and $\Delta z = 1.46\lambda_{\min}$ for depth resolution in both cases, as it can be appreciated from the cut along the z-axis. Instead, (6) returns $\Delta x = 1.54\lambda_{\min}$ and $\Delta x = 1.1\lambda_{\min}$, and (4) $\Delta z = 1.5\lambda_{\min}$. This confirm the effectiveness of estimates (4) and (6) also in the case of two dimensional objects. In particular, transverse resolution is favourably affected by an enlargement of the measurement domain whereas depth resolution mainly depends on the adopted frequency diversity.

CONCLUSIONS

In this paper we have analyzed the achievable resolution limits for the sensing configuration depicted in Fig. 1. We have found that transverse resolution is favourably effected by the extent of the measurement domain and by the highest adopted frequency, whereas degrades as depth increases. On the contrary, depth resolution mainly depends on the adopted frequency diversity and is almost uniform along depth. Furthermore, we have found that the estimates analytically derived for the cases of 1-D objects [3,4] hold also for the case of a 2-D object.

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