SPATIAL PROCESSING WITH LENS ANTENNA ARRAYS FOR DIRECTION-OF-ARRIVAL ESTIMATION

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ABSTRACT

In this paper, spatial processing with lens antennas arrays for direction-of-arrival (DOA) estimation is considered. In this approach, when using the Multiple Signal Classification (MUSIC) algorithm, the dimension is reduced by the front end which performs spatial processing, resulting in increased computational speed and decreased computational load. As a specific example, simulations of a 33-element lens array with DOA estimation for 7 simultaneous sources shows that the computational load is reduced to 68% of that for a 33-element linear uniform array. In this paper, we consider the case of narrowband uncorrelated signals, and uniform spacing of receivers on the lens image surface, but the analysis can be extended to broadband non-uniform scenarios.

INTRODUCTION

Direction-of-arrival (DOA) estimation is one of the main function requirements for direction-finding smart antennas in future-generation mobile communication systems [1]. This paper investigates the possibility of improving the resolution, while at the same time reducing the computational load for DOA estimation by using a lens antenna array front end in place of a more standard uniform antenna array. Discrete lens arrays (DLAs), Fig.1, are beam-forming multi-beam arrays which are implemented with 3 antenna arrays: the source-side array, the image-side array, and the receiver array. The first two arrays are fabricated on the same single or multi-layer substrate, while the third array is conformal at a distance determined by the focal-length-to-diameter (F/D) of the particular design. The source-side array samples the input wavefront, as in any two-dimensional array. Each antenna in this array transmits the received signals to the image-side antenna elements through waveguides (usually microstrip or CPW) of varying length across the array. This produces a spatial Fourier transform of the source space on the image (focal) surface of the DLA, and this image is sampled by the receiver elements. In contrast to dielectric lenses, DLAs can be fabricated using standard PCB technology, they can be designed to have good scanning properties for large angles [2] and low losses with no additional anti-reflection coatings, polarization is an additional design parameter, and active circuitry that provides gain can be integrated in the lens itself. Active DLAs for half-duplex [3], [4] and full-duplex [5] active transmit/receive arrays have been demonstrated over the past few years. In these arrays, power amplifiers and low-noise amplifiers are integrated in each array element, allowing for increased effective radiated power (ERP) in transmission and increased dynamic range in reception. More recently, the LMS adaptive algorithm applied to a DLA showed significant reduction of adaptation weights for non-optimal solutions [6]. A 10-GHz DLA was also integrated with an analog optical processor for broadband adaptive independent-component analysis. [7].

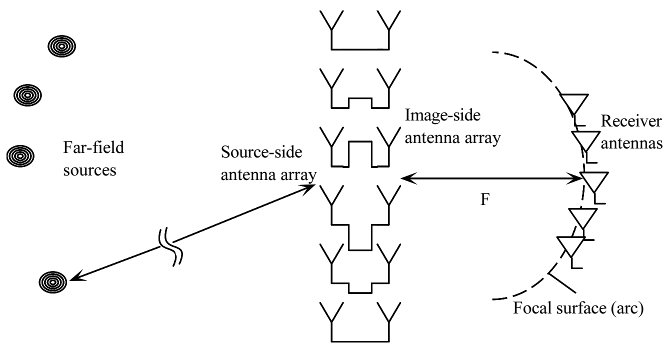

Fig. 1. Sketch of a discrete lens antenna array. In reception, the source-side antenna elements receive the incident waves, and transmit it through variable delay lines to the image-side elements. The delay lines are designed to provide focusing onto receiver antennas positioned on the focal surface. These signals are A-to-D sampled and input to the DOA algorithm.

In this paper, the Multiple Signal Classification (MUSIC) algorithm is applied to the signals received at the image (Fourier transform) surface of a DLA. The lens transforms the element-space (the output of the source-side array) to beam-space (the output of the receivers that sample the image) with equal or reduced dimension, allowing for reduced computational load. In addition, it is expected that the estimation bias may be decreased when applying MUSIC in beam-space, thereby improving the resolution. To evaluate the performance of a lens-array system and compare it to a uniform linear array, two parameters are calculated: the root-mean-square error (RMSE) and the resolution signal-to-noise (SNR) threshold (probability of resolving a source). In this paper, narrowband signals and uniform sampling of the DLA image are considered in the modelling. However, these are not fundamental limitations, as has been shown in [8], where an extension of MUSIC to broadband non-uniform arrays is presented. In the next section, the framework and notation for the simulations are presented; in particular, the narrow-band signal model, the source-direction matrix for arbitrary arrays, and a brief overview of the MUSIC algorithm are presented. In Section III, the spatial pre-processing by DLAs as applied to MUSIC is developed, and in Section IV, the simulation results are discussed.

SIGNAL AND ANTENNA ARRAY MODELS

In simulations presented in this paper, all M antenna elements are assumed to be uniform directional patterns and mutual coupling between elements is not taken into account. The signal and noise models are narrowband, and the P signals incident on the array from P different sources are assumed to be uncorrelated. The sources are assumed to be in the far field of the array, producing incident plane waves at angles $(\phi_1, \theta_1), \ldots, (\phi_P, \theta_P)$, centered around a frequency ω_0.

Narrowband signal model

For the narrowband problem, it is convenient to use models of signal and noise in time-domain. Using complex envelope representation, the $(M \times 1)$-vector of array-output signals can be expressed by

$$\mathbf{x}(t) = \sum_{i=1}^{P} \mathbf{a}(\phi_i, \theta_i) s_i(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{a}(\phi_i, \theta_i)$ is the $(M \times 1)$-*source-direction vector* (or *steering vector*) of the i-th source, $1 \leq i \leq P$,

$$\mathbf{a}(\phi_i, \theta_i) = \left[e^{-j\omega_0 \tau_1(\phi_i, \theta_i)}, ..., e^{-j\omega_0 \tau_M(\phi_i, \theta_i)}\right]^T \tag{2}$$

$\tau_j(\theta_i)$, $1 \leq j \leq M$, denote propagation delay between the reference point and the j-th antenna element for the signal source i-th; $s_i(t)$ denote the array-input signal of the i-th source, and $\mathbf{n}(t)$ denotes the $(M \times 1)$-vector of white noise at the antenna elements. In matrix notation, it follows

$$\mathbf{x}(t) = \mathbf{A}\ \mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{A} = [\mathbf{a}(\phi_1, \theta_1), ..., \mathbf{a}(\phi_P, \theta_P)]$ is the $(M \times P)$-*source-direction matrix*. The number M of elements is assumed to satisfy the condition: $M \geq P + D$ where $D = 1$, 2, 3 for 1-D, 2-D, 3-D estimations, respectively [9].

Suppose that the received vector $\mathbf{x}(t)$ is sampled N times (N snapshots), at $t_1, ..., t_N$. From (1), the sampled data can be expressed as

$$\mathbf{X} = \mathbf{A}\ \mathbf{S} + \mathbf{N} \tag{4}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $(M \times N)$-*array-output-signal matrix* and *array-output-noise matrix*, respectively; and $\mathbf{S}$ is the $(P \times N)$-*array-input signal matrix*. Then, an estimate $\hat{\mathbf{R}}_{\mathrm{xx}}$ of covariance matrix $\mathbf{R}_{\mathrm{xx}}$ is given by

$$\hat{\mathbf{R}}_{\mathrm{xx}} = \frac{1}{N} \sum_{k=1}^{N} \mathbf{x}(t_k) \mathbf{x}^H(t_k) = \frac{1}{N} \mathbf{X}\mathbf{X}^H \tag{5}$$

Arbitrary Three-Dimensional Antenna Arrays

A generalized three-dimensional (3-D) array model has been developed for applying the MUSIC algorithm. For the first results obtained in this paper, the model reduces to a simple one-dimensional uniform array. The generalized framework, however, enables us to continue investigating 2-D and 3-D arbitrary arrays, and is therefore briefly presented below.

Consider a 3-D array of M antenna elements with $\mathbf{d}_j = [\mathrm{d}_{x_j}, \mathrm{d}_{y_j}, \mathrm{d}_{z_j}]^T$, $1 \leq j \leq M$, being the coordinate of the j-th element. Let the spatial frequencies in the x, y and z directions be defined as

$$\mu_i = -\omega_0 \frac{x_i}{c}\ ; \quad \upsilon_i = -\omega_0 \frac{y_i}{c}\ ; \quad \eta_i = -\omega_0 \frac{z_i}{c} \tag{6}$$

where $x_i = \cos\phi_i \sin\theta_i$, $y_i = \sin\phi_i \sin\theta_i$, $z_i = \cos\theta_i$ with c is propagation speed; $\phi_i, \theta_i, 1 \leq i \leq P$, are azimuth and elevation of i-th source, respectively; $-180^\circ \leq \phi_i \leq 180^\circ$, $0^\circ \leq \theta_i \leq 90^\circ$. If $\mathbf{\Omega}_i = [\mu_i, \upsilon_i, \eta_i]^T$ are the spatial frequency vectors, and the first element of the array is used as a reference, then the source-direction vector $\mathbf{a}(\phi_i, \theta_i)$ is expressed in terms of element coordinates and spatial frequencies as

$$\mathbf{a}(\phi_i, \theta_i) = \left[1, e^{j\mathbf{d}_2^T \mathbf{\Omega}_i}, \ldots, e^{j\mathbf{d}_M^T \mathbf{\Omega}_i}\right]^T \tag{7}$$

THE MUSIC ALGORITHM IN ELEMENT AND BEAM-SPACE

In the Multiple Signal Classification (MUSIC) algorithm [10], [11], the M-element-space is decomposed into signal and noise components using the eigenvectors of covariance matrix. The MUSIC spatial spectrum is written as

$$P_{MU}(\phi_i, \theta_i) = \frac{1}{\mathbf{a}^H(\phi_i, \theta_i)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\phi_i, \theta_i)} \tag{8}$$

where $\mathbf{a}(\phi_i, \theta_i)$ is the source-direction vector, $\mathbf{U}_n$ is noise-subspace matrix obtained from (5), and the superscript H is the hermitian transpose. If the value $\mathbf{\Omega}_i$ is equal to the spatial frequency of input signal, the denominator of $P_{MU}(\phi_i, \theta_i)$ has a minimum. Therefore the peaks of spectra present the directions of arrival of signals.

With a spatial pre-processor at outputs of array elements, we obtain the beam-space signals in matrix notation as:

$$\mathbf{y} = \mathbf{C}^H\mathbf{x} \tag{9}$$

where $\mathbf{y} = [y_1, ..., y_K]^T$ is vector of spatial pre-processor outputs; K is a subset of the M possible outputs and represents the reduced beam-space dimension, $P \leq K \leq M$; $\mathbf{x} = [x_1, ..., x_M]^T$ is vector of antenna array outputs. $\mathbf{C}$ is $(M \times K)$-preprocessing matrix of orthonormal columns and represents the functionality of the DLA. The matrix $\mathbf{C}$, which is designed to cover sector (Φ, Θ), transforms the element-space vectors into beam-space vectors. From (4), the outputs of the spatial pre-processor with N snapshots given by

$$\mathbf{Y} \quad = \quad \mathbf{C}^H\mathbf{X} \tag{10}$$

Following (5), obtained an estimate $\hat{\mathbf{R}}_{\text{yy}}$ of covariance matrix $\mathbf{R}_{\text{yy}}$ at outputs of pre-processor as

$$\hat{\mathbf{R}}_{\text{yy}} = \mathbf{C}^H\hat{\mathbf{R}}_{\text{xx}}\mathbf{C} \tag{11}$$

In the case of a discrete lens front end, the beam-space MUSIC algorithm can therefore be written as

$$P_{BS-MU}(\phi_i, \theta_i) = \frac{1}{\mathbf{b}^H(\phi_i, \theta_i)\mathbf{U}_{bs,n}\mathbf{U}_{bs,n}^H\mathbf{b}(\phi_i, \theta_i)} \tag{12}$$

where $\mathbf{U}_{bs,n}$ is (beam-space) noise-subspace matrix obtained from noise-eigenvectors of matrix $\hat{\mathbf{R}}_{\text{yy}}$ given by (11) and $\mathbf{b}(\phi_i, \theta_i) = \mathbf{C}^H\mathbf{a}(\phi_i, \theta_i)$ denotes $(K \times 1)$-vector of beam-space source-direction.

SIMULATION RESULTS

As a simple example, we perform DOA estimation calculations for a 33-element linear uniform planar antenna array ($M = 33$), and compare it to DOA estimation for a 33-element lens array with varying number of receivers ($K = 33$, 18 and 11). The performance is evaluated by: the Root Mean Square Error (RMSE) of angles of DOA's, and the resolution SNR threshold, which is probability of resolution versus SNR for a given source configuration. Seven far-field sources are simultaneously detected, assuming a known spatial angle sector Φ from $-65°$ to $50°$. The sources are positioned at $-60°$, $-30°$, $-15°, 0°, 15°, 30°, 45°$ and are all of equal power. The linear (1-D) DLA array consists of antenna elements isotropic in the half-space and placed at a half of a free-space wavelength. The delay lines and position of the lens elements are designed for a F/D of 0.53. The receiver antennas are simulated as patch antennas with 7dB gain. The antenna elements in the lens are linearly polarized and the far-field sources are also assumed to be linearly polarized. The outputs of the 33 receivers on the DLA focal are calculated using electromagnetic field analysis are shown in Fig. 2. The number K of receivers in the DOA estimation simulations is selected by rank-ordering the relative intensities received at the different receiver antennas. A BPSK modulation was used for the sources, $N = 100$ time samples were used for the simulations and 100 trials were run for each SNR.

TABLE I

COMPARISON OF COMPUTATIONAL TIME FOR MUSIC-DLA.

Number of detectors	Relative processing time
K=33	1
K=18	0.68
K=11	0.55

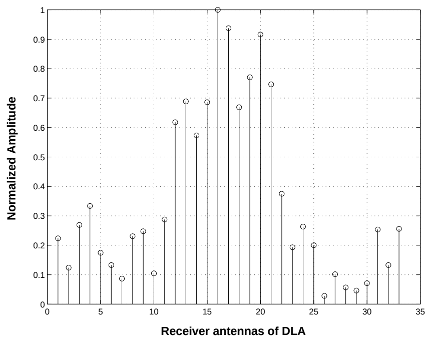

Fig. 2. Relative signal intensity received at the 33 receiving antennas of the DLA. The antenna aligned with the lens optical axis is #16 and in the center of the plot.

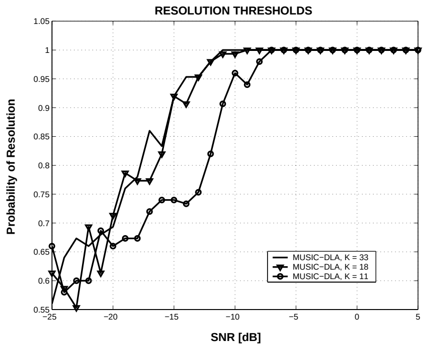

Fig. 3. Resolution threshold vs. SNR at position of $15°$, for MUSIC-DLA with the dimensions of $K = 33$, $K = 18$ and $K = 11$.

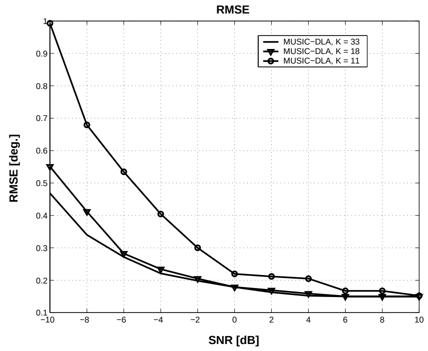

Fig. 4. RMSE vs. SNR at position of $15°$, for MUSIC-DLA with the dimensions of $K = 33$, $K = 18$ and $K = 11$.

Fig. 3 and Fig. 4 compare the resolution threshold and the RMSE, respectively, for MUSIC with DLA (MUSIC-DLA) at source position $15°$ in the given sector Φ. Similar results are obtained for the six other sources. Table I shows the relative computation time for DOA estimation using MUSIC-DLA when number of receivers is reduced. As illustrated in Fig. 3, for $K = 33$ and $K = 18$, the resolution SNR thresholds, which are the minimum SNR thresholds to resolve 7 sources, are almost similar. In Fig. 4, the difference of the RMSE for the cases of $K = 33$ and of $K = 18$ are small and can be acceptable. Therefore, the DLA with 18 receivers can be used instead with performance comparable to 33-receiver DLA. According to Table I, as a specific example, when using the DLA with 18 receivers, the computation load can be reduced to 68% of that of the $K = 33$ case. If one can tolerate a SNR of -8dB, then the computational time is reduced by almost a factor 2.

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