MULTIGRID METHOD FOR LIGHTWAVE PROPAGATION IN TWO-DIMENSIONAL OPTICAL WAVEGUIDE

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ABSTRACT

Multigrid method is applied to the lightwave propagation in the optical waveguide. This method is one of the rapid methods because of speeding up the convergence of the relaxation method. In general, the electromagnetic fields are composed of fast and slow spatial variations. When the high and low frequency modes are solved at the fine and coarse levels of discretization using the iteration solvers, the electromagnetic fields can be obtained quickly. As the application models, the lightwave coupling in linear and nonlinear directional couplers is considered. It is shown that the multigrid method is very useful as the rapid method.

INTRODUCTION

Various numerical methods such as FD-TD method, Beam Propagation Method, Finite Element Method, Method of Moment have been proposed and developed in order to analyze the propagation phenomena in the optical waveguide. It is desirable to develop the rapid method to obtain the accurate solutions since the systems of equations are becoming large recently and many computer resources are needed.

Multigrid method, which has been developed by Brandt [1], is used widely in the field of the computational fluid dynamics and the structure analysis. This method is one of the rapid methods because of seeping up the convergence of the relaxation method [2, 3]. In general, the electromagnetic fields are composed of fast and slow spatial variations. When the high and low frequency modes are solved at the fine and coarse levels of discretization by using the iteration solvers, the electromagnetic fields can be obtained quickly. The multigrid method is known as the procedure which takes only $O(N)$ operations.

So far, the multigrid method has been applied to various problems such as antennas [4], rough surfaces [5], and microstrip line [6]. In this paper, the linear and nonlinear directional couplers, which are the application models, are examined by using Correction Scheme (CS) and Full Approximation Scheme (FAS), respectively. For the linear case, the discretized equation for the paraxial wave equation is reduced to be tridiagonal. This is solved by LU decomposition which takes $O(N)$ operations [3]. At first, the effect of the number of the levels and the iterations is examined by using the power conservation. Also, the computation time of the multigrid methods is compared with those of the Gauss-Seidel method and LU decomposition.

FINITE-DIFFERENCE BEAM PROPAGATION METHOD

Consider the nonlinear waveguide where the optical nonlinearity is of the Kerr type, isotropic and electrostrictive. The nonlinear wave equation for TE polarization is given by

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\left\{n_L^2(x) + \alpha(x)|E_y|^2\right\}\right]E_y = 0 \tag{1}$$

where $\alpha(x)$ is the nonlinear coefficient and $n_L(x)$ is the refractive index. After substituting $E_y = E_y(x,z)\exp(-j\beta z)$ for (1) and applying the Crank-Nicholson method, the following discretized equation is obtained in the implicit form.

$$-cE_{i+1,\ m+1} + aE_{i,\ m+1} - cE_{i-1,\ m+1} = cE_{i+1,\ m} + bE_{i,\ m} + cE_{i-1,\ m} \tag{2}$$

where

$$a = 2j\beta + 2c - \frac{\Delta z k^2}{2}\left\{n_L^2 + \alpha|E(i,\ \ m+1)|^2 - \left(\frac{\beta}{k}\right)^2\right\} \tag{3}$$

$$b = 2j\beta - 2c + \frac{\Delta z k^2}{2}\left\{n_L^2 + \alpha|E(i,\ \ m)|^2 - \left(\frac{\beta}{k}\right)^2\right\} \quad (4)$$

$$c = \frac{\Delta z}{2(\Delta x)^2} \quad (5)$$

and $E_{i,\ m}$ is the electric field at the grid $(i\Delta x, m\Delta z)$. Δx and Δz are the step size along the x and z direction, respectively. For linear waveguide, the discretized equation is obtained by placing with $\alpha = 0$ in (2).

MULTIGRID METHOD

Multigrid method exploit the property that solution is composed of fast and slow spatial variations. By using more than one level of discretization, the slow and fast solutions can be identified using the iteration solvers at the coarse and fine levels, respectively. In what follows,we give an outline of Correction Scheme (CS) for the linear problem and Full Approbation Scheme (FAS) for the nonlinear problem, briefly.

Correction Scheme

From (2) with $\alpha = 0$, the following equation is derived.

$$E_{i,m+1} = \frac{c}{a}[E_{i+1,m+1} + E_{i-1,m+1}] + \frac{d_i}{a} \quad (6)$$

where d_i is the right hand side of (1). Equation (6) is appropriate form for the relaxation method such as the Gauss-Seidel method.

Multilevel G^γ is introduced and the index γ denotes the level. The step size is given by h^γ, and then the step size of the coarser grid is given by $h^{\gamma-1} = 2h^\gamma$. Equation (1) is expressed in the following form

$$\mathcal{L}^\gamma E^\gamma = D^\gamma \quad (7)$$

where E^γ is the exact solution. The simplest case of a two-grid method is described by the following steps. The Gauss-Seidel (GS) method as the iteration method is chosen.

1) Using the GS method on (7) with a suitable initial guess, the approximate solution e_1^γ is obtained.
2) The definition of the correction $v^\gamma = E^\gamma - e_1^\gamma$ gives the residual equation as follows:
$$\mathcal{L}^\gamma v^\gamma = R^\gamma \quad (8)$$
where R^γ is the residual defined by
$$R^\gamma = \mathcal{L}^\gamma E^\gamma - \mathcal{L}^\gamma e_1^\gamma = D^\gamma - L^\gamma e_1^\gamma \quad (9)$$
3) Projecting the residual equation from G^γ to $G^{\gamma-1}$ gives
$$\mathcal{L}^{\gamma-1} v^{\gamma-1} = R^{\gamma-1} \quad (10)$$
where $\mathcal{L}^{\gamma-1}$ is the coarse-grid operator on $G^{\gamma-1}$. $R^{\gamma-1}$ is obtained by using the following restriction
$$R^{\gamma-1} = \mathcal{R}R^\gamma \quad (11)$$
where $\mathcal{R}$ is the restriction operator. Applying the GS method on (10) with a zero initial guess gives the approximate solution $v^{\gamma-1}$.
4) Projecting the correction $v^{\gamma-1}$ from coarse grid to fine grid as follows:
$$v^\gamma = \mathcal{P}v^{\gamma-1} \quad (12)$$
where $\mathcal{P}$ is the prolongation operator. By using the correction v^γ, the new approximate solution is given by
$$e_2^\gamma = e_1^\gamma + v^\gamma \quad (13)$$
Applying the GS method on (7) with the initial guess e_2^γ gives the updated approximate solution e_3^γ.

As the above procedure is applied recursively, the multigrid method for the linear problem is established.

Full Approximate Solution

For the nonlinear problem, the above procedure can not be applied, since the operator $\mathcal{L}^{\gamma}$ includes the unknown E^{γ}. FAS is described by the following steps.

1) The residual R^{γ} is defined by the same way as CS. By using (7) and (9), the following residual equation is derived.
$$\mathcal{L}^{\gamma}E^{\gamma} = R^{\gamma} + \mathcal{L}^{\gamma}e_1^{\gamma} \tag{14}$$
Projecting (14) from G^{γ} to $G^{\gamma-1}$ gives
$$\mathcal{L}^{\gamma-1}E^{\gamma-1} = R^{\gamma-1} + \mathcal{L}^{\gamma-1}e_1^{\gamma-1} \tag{15}$$
where
$$R^{\gamma-1} = \mathcal{R}R^{\gamma}\ , \qquad e_1^{\gamma-1} = \mathcal{R}e_1^{\gamma} \tag{16}$$
Applying the GS method on (15) with the initial guess $e_1^{\gamma-1}$ gives the approximate solution $e_2^{\gamma-1}$. By using $e_2^{\gamma-1}$, the correction $v^{\gamma-1}$ is obtained as follows:
$$v^{\gamma-1} = e_2^{\gamma-1} - e_1^{\gamma-1} \tag{17}$$
2) After the correction is interpolated from the coarse grid to the fine grid, the approximate solution e_2^{γ} is obtained by the same procedure as CS. The updated approximate solution e_3^{γ} is given by applying the GS method on (7) with the initial guess e_2^{γ}.

As the above procedure is applied recursively, the multigrid method for the nonlinear problem is established by the same way as CS.

NUMERICAL RESULTS

As the application models, the directional couplers are considered as shown in Fig. 1. TE fundamental guided mode is incident on waveguide II and the electric fields vanish at the edges of computational window, which is shown in the dash lines.

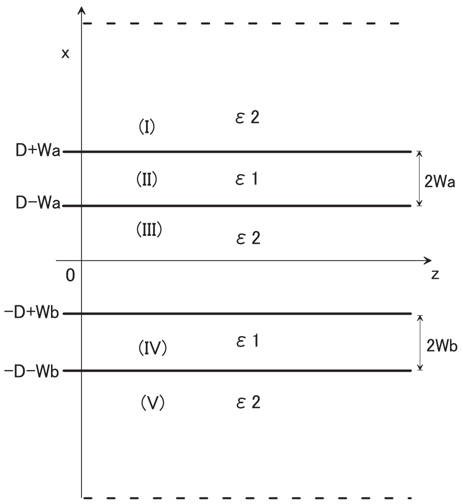

Fig. 1. Directional coupler

Linear Directional Coupler

The parameters of the linear directional coupler are $\sqrt{\varepsilon_1} = 1.5$, $\sqrt{\varepsilon_2} = 1.3$ $D = 1.0[\mu\text{m}]$, $2W_a = 2W_b = 0.5[\mu\text{m}]$, and the wavelength$\lambda = 1.5$ [μm] [7]. The discretized equation (2) with $\alpha = 0$ can be solved by using LU decomposition, since a system of the equation is tridiagonal.

In order to examine the effect of the number of levels and the iterations, the relative error δ of the power of the guided wave is defined as $\delta = |P_{out} - P_{in}|/P_{in}$. P_{in} is the incident power and P_{out} is the power at the output plane $z = z_o(= 40\mu\text{m})$. Fig. 2 shows the relative error δ for the number of iterations l as function of the level γ. The number of grids N is 513. As the number of iterations increases, the relative error δ becomes small and it is enough for this structure that $\gamma = 2$ and $l = 5$ are chosen. Table 1 shows the comparison of computation time for the number of grids. The number of levels and iterations of CS are $\gamma = 2$ and $l = 5$, respectively. The number of iterations of the Gauss-Seidel method is $l = 50$. The computation time of CS is almost the same order as that of LU decomposition and the difference between these two methods depends on the calculation of the residual and the interpolation.

Nonlinear Direction Coupler

The nonlinear region is assumed to be region III. The parameters of the nonlinear directional coupler are $\sqrt{\varepsilon_1} = 1.57$, $\sqrt{\varepsilon_2} = 1.55$, $W_a = W_b = 1\mu\text{m}$, $D = 2.2\mu\text{m}$, $\lambda = 1.064\mu\text{m}$, $\alpha = 6.377 \times 10^{-12}\text{m}^2/\text{V}^2$ [8]. The tendency about the relative error of the power and the computation time in the nonlinear directional coupler is almost the same as those of the linear directional coupler. Fig. 3 shows the contour maps of the nonlinear directional couplers. It is found that the critical power obtained by the improved coupled mode theory (65 [W/m]) is different with that of the present method (72 [W/m]).

CONCLUSIONS

Multigrid method has been applied to the beam propagation method and the effect of the number of the levels and the iterations on the relative error of the power have been examined. It is shown that the multigrid method is so useful for the lightwave propagation problem.

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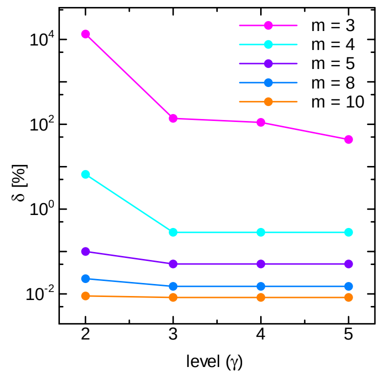

Fig. 2. Relative error of the power for the levels

Table 1. Comparison of computation time for the number of grids of the linear directional coupler (sec.)

Number of grids	CS	*LU* Decomp.	GS method
257	0.45	0.25	4.32
513	0.99	0.39	8.52
1025	2.32	0.78	16.80

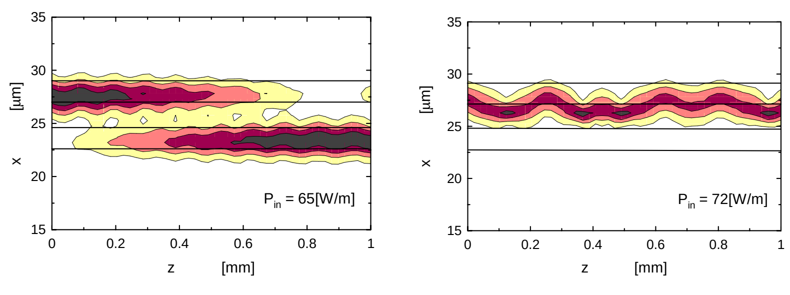

Fig. 3. Contour maps of nonlinear directional couplers for the incident power (a) P_{in} = 65[W/m] and (b) P_{in} = 72[W/m]