DOMAIN DECOMPOSITION AND NON-UNIFORM POLAR/SPHERICAL GRID INTERPOLATION APPROACH TO FAST FIELD EVALUATION

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ABSTRACT

A novel algorithm is proposed for fast field evaluation, which is achieved via source domain decomposition and comprises two steps repeated for each subdomain. First, computation of the field produced by the currents residing within a subdomain over a sparse set of points surrounding the observation domain. Second, phase removal, interpolation, phase-restoration and aggregation of the field of the subdomain into the total field. The proposed approach is applied to fast iterative solution of problems discretized via the method of moments as well as to evaluation of multiple physical optics integrals.

INTRODUCTION

Rigorous analysis of scattering by arbitrary shaped bodies is often effected via numerical solution of integral equations by the Method of Moments (MoM) [1]. As a simple but representative example, consider the problem of analyzing time-harmonic scattering from an open two-dimensional perfectly conducting surface via the electric field integral equation. The number of field and current sampling points on the surface is proportional to the scatterer's electrical dimensions, i.e. of $O(N)$, where $N = kR$, and R and k being the radius of the smallest circle circumscribing the scatterer and the wavenumber, respectively. The $O(N^3)$ computational complexity of direct MoM solvers necessitates the use of iterative solvers when the scatterer is electrically large. When solving the EFIE, each iteration requires the evaluation of the electric field due to a given surface current. Straightforward evaluation of the field at $O(N)$ points by surface integration involving summation of $O(N)$ terms amounts to $O(N^2)$ operations. This high computational burden underscores the need for fast field evaluation techniques, such as the two-level Fast Multipole Algorithm (FMA) [2] or its multilevel cousin, the MultiLevel Fast Multipole Algorithm (MLFMA) [3].

In this paper, we present a novel scheme that facilitates the numerically efficient evaluation of fields produced by given current distributions. The algorithm is based on the observation that the field radiated by a finite size source behaves, locally, as an essentially bandlimited function of the angle and radial distance multiplied by a common phase factor. This behavior has been noted and subsequently exploited in devising efficient sampling schemes for far and near fields of antennas by Bucci et al. [4-5]. The angular bandwidth of the phase compensated field is proportional to the linear dimensions of the source, while the local bandwidth with respect to the radial distance decreases rapidly with distance from the source. Therefore, the radiated field can be sampled on a non-uniform polar grid on which the radial density decreases away from the source. Consequently, the radiated field on the surface can be interpolated from its samples at a number of polar grid points proportional to the source region dimensions. With this in mind, we decompose the scatterer surface into subdomains and compute the field produced by each of them separately. The field of each subdomain is directly evaluated at a small number of polar grid points and subsequently interpolated, thus providing computational savings. The phase common to all source points in a given subdomain is removed from the field prior to interpolation. Following the interpolation, the phase is restored and the partial fields are aggregated into the total field due to all subdomains combined. The two-level domain decomposition algorithm reduces the computational cost of evaluating the field (a single iteration) from $O(N^2)$ to $O(N^{3/2})$. The complexity estimate of our algorithm is similar to that of the two-level Fast Multipole Algorithm [2] while the implementation of the latter is considerably more complicated. Also note that the computational structure of this approach is closely related not only to the fast MoM analysis [6], but also, to the algorithms recently introduced in the context of the high frequency scattering [7-9] image processing [10], and radar imaging [11].

A multilevel algorithm is obtained by casting the above scheme in a hierarchical framework. By starting from small subdomains, for which the field is computed directly over a very coarse grid, fields of progressively larger domains are computed through recursive interpolation and aggregation to finer grids. The multilevel algorithm attains an asymptotic complexity of $O(N \log N)$, which is comparable with that of the MLFMA [3].

Application of our approach to three-dimensional scattering problems is conceptually simple and involves the use of non-uniform spherical rather than polar grids. The time-domain version of the proposed technique has been presented in [12]. It is to be noted that the above domain decomposition approach can be incorporated with relative ease in existing MoM codes.

PROBLEM SPECIFICATION

Consider scattering from an open Perfectly Electrically Conducting (PEC) surface S depicted in Fig. 1. We assume that the surface can be circumscribed by a circle of radius R. For the sake of simplicity, we study a two-dimensional problem whereas the geometry and the incident field are uniform along the *z*-axis. The illumination is by a *z*-polarized electric field $\boldsymbol{E}^{inc}(\boldsymbol{r}) = \hat{z} E^{inc}(\boldsymbol{r})$ where $\hat{z}$ is the unit vector in z-direction and $\boldsymbol{r} = (x, y)$ denotes an observation point in the *xy*-plane. Harmonic time dependence $e^{j\omega t}$ is assumed and suppressed. The incident field induces *z*-directed surface current $\boldsymbol{J}(\boldsymbol{r}) = \hat{z} J(\boldsymbol{r}), \quad \boldsymbol{r} \in S$. The solution for the current distribution is facilitated via the electric field integral equation [1]

$$E^{inc}(\boldsymbol{r}) = \tfrac{k\eta}{4} \int_S \mathrm{H}_0^{(2)}(k\rho) J(\boldsymbol{r}') ds' \qquad \boldsymbol{r} \in S \tag{1}$$

where k and η denote the intrinsic wavenumber and impedance, respectively. Also in (1), $\mathrm{H}_0^{(2)}(\cdot)$ denotes the zero order Hankel function of the second kind and $\rho = |\boldsymbol{r} - \boldsymbol{r}'|$, $\boldsymbol{r}'$ being source point on *S*. Iterative solution of (1) calls for repeated evaluation of the field integral

$$E(\boldsymbol{r}) = \tfrac{k\eta}{4} \int_S \mathrm{H}_0^{(2)}(k\rho) J(\boldsymbol{r}') ds' \tag{2}$$

for candidate current distributions .

Let us consider the computational complexity of the field evaluation via application of (2). In this context, we define a large parameter providing a measure of the electrical size of the problem as $N = kR$. The integral in (2) can be evaluated numerically using $O(N)$ source points for each of $O(N)$ observation points on the boundary, thus amounting to $O(N^2)$ operations. The computational cost associated with the integration in (2) constitutes the major burden in the iterative solution of (1). The complexity of solving the scattering problem performing N_{it} iterations totals $O(N_{\mathrm{it}} N^2)$. Specific quadrature rule and the accuracy requirements determine the multiplicative constant factors implicite in these estimates. Numerically efficient evaluation of (2) forms the subject of the next section.

THE DOMAIN DECOMPOSITION FORMULATION

In this section, we show that the computational complexity of evaluating the electric field can be reduced below the $O(N^2)$ cost of the straightforward computation via (2). To this end, we subdivide surface S into P disjoint subdomains of roughly equal size, $S = \bigcup_{p=1}^{P} \overline{S}_p$ (see Fig. 1). Here, $\overline{\boldsymbol{r}}_p$ denotes the center of the smallest circle of radius

$\overline{R}_p$ circumscribing the *p*th subdomain $\overline{S}_p$. Based on this subdivision, the total electric field is computed by aggregating partial fields, i.e.,

$$E(\boldsymbol{r}) = \sum_{p=1}^{P} \overline{E}_p(\boldsymbol{r}) \tag{3}$$

where $\overline{E}_p(\boldsymbol{r})$ denotes the partial electric field due to restriction of the current J to subdomain $\overline{S}_p$. The partial field $\overline{E}_p(\boldsymbol{r})$ is evaluated in a way dependent on the distance of the observation point $\boldsymbol{r}$ from the source subdomain center $\overline{\boldsymbol{r}}_p$. In the near zone, $\overline{E}_p(\boldsymbol{r})$ is obtained by direct integration:

$$\overline{E}_p(\boldsymbol{r}) = \tfrac{k\eta}{4} \int_{\overline{S}_p} \mathrm{H}_0^{(2)}(k\rho) J(\boldsymbol{r}') ds' \qquad \rho_p < \Omega_R \overline{R}_p \tag{4}$$

where $\rho_p = |\boldsymbol{r} - \overline{\boldsymbol{r}}_p|$ and $\Omega_R > 1$ is a parameter determining the near zone radius relative to that of the source domain. On the other hand beyond the near field zone, we first compute the phase compensated partial field defined as

$$\tilde{E}_p(\boldsymbol{r}) = \tfrac{k\eta}{4} \sqrt{\tilde{\rho}_p} e^{jk\tilde{\rho}_p} \int_{\overline{S}_p} \mathrm{H}_0^{(2)}(k\rho) J(\boldsymbol{r}') ds' \qquad \rho_p \geq \Omega_R \overline{R}_p \tag{5}$$

where $\tilde{\rho}_p = \sqrt{\rho_p^2 + \overline{R}_p^2 / 2}$. Subsequently, $\tilde{E}_p(\boldsymbol{r})$ is employed to obtain

$$\overline{E}_p(\boldsymbol{r}) = \frac{e^{-jk\tilde{\rho}_p}}{\sqrt{\tilde{\rho}_p}} \tilde{E}_p(\boldsymbol{r}) \qquad \rho_p \geq \Omega_R \overline{R}_p \tag{6}$$

In (5), the phase factor preceding the integral is designed to cancel rapid oscillations due to the common phase present in the integrand. These oscillations are caused by the $\rho^{-1/2} e^{-jk\rho}$ asymptotic behavior of the Hankel function. Approximate removal of the phase factor allows interpolation of the partial field sampled at a rate related to the size of the subdomains. The computational savings are achieved since the partial fields in (5) are computed over a coarse sampling grid surrounding *S*. These fields are interpolated to the surface points at a substantially lower cost and only then inserted in (6).

With the goal of designing an optimal sampling grid for $\tilde{E}_p(\boldsymbol{r})$, it is advantageous to consider the partial field $\tilde{E}_p(\rho_p, \varphi_p)$ as a function of the polar coordinates ρ_p and φ_p of the local coordinate system centered at $\overline{\boldsymbol{r}}_p$. We have shown that the angular and radial sampling rate requirements are substantially different making the distinction very important. We also found that the number of coarse grid points $\overline{N}_p$ is bounded and, specifically, $\overline{N}_p = O(k\overline{R}_p)$. In order to estimate the computational cost of evaluating the electric field $E(\boldsymbol{r})$, we assume that all subdomains are approximately equal in size. Therefore for all subdomains, we use $\overline{N}$ to designate the number of coarse grid points. In (5), the $\tilde{E}_p(\rho_p, \varphi_p)$ is evaluated at $O(\overline{N})$ points at a cost of $c_1 \overline{N}^2$, $\overline{N} = O(k\overline{R})$, and, subsequently, interpolated to $O(N)$ points at a cost of $c_2 N$, c_1 and c_2 being constants. Near field computations for each subdomain via (4) also requires $O(\overline{N}^2)$ operations. The number of subdomains P for reasonably behaving surfaces is roughly proportional to the ratio $N / \overline{N}$. Thus, the computational burden of computing and aggregating P partial fields amounts to

$$C \propto \frac{N}{\overline{N}} \left(c_1' \overline{N}^2 + c_2 N \right) \tag{7}$$

where $c_1' \overline{N}^2$ terms accounts for both coarse grid and near field computations. We find that the lowest asymptotic complexity of $O(N^{1.5})$ is obtained for $\overline{N} \propto \sqrt{N}$. This result compares favorably with the $O(N^2)$ cost of the straightforward evaluation of (2). The obtained complexity estimate is similar to that of the two-level FMA algorithm [2].

CONCLUSION

A novel algorithm based on domain decomposition and non-uniform polar/spherical grid interpolation for fast computation of fields produced by given source distributions is presented. The multilevel computational sequence essentially reduces to interpolation and aggregation of fields of increasingly larger subdomains, starting with the direct evalution of fields of small subdomains. Efficient and accurate multidimensional interpolation scheme is a key to the implementation of the method. The two-level and multi-level algorithms can be applied with relative ease accelerating new and existing MoM solvers, serve as fast numerically exact boundary conditions for differential equation formulations, or facilitate efficient evaluation of PO integrals.

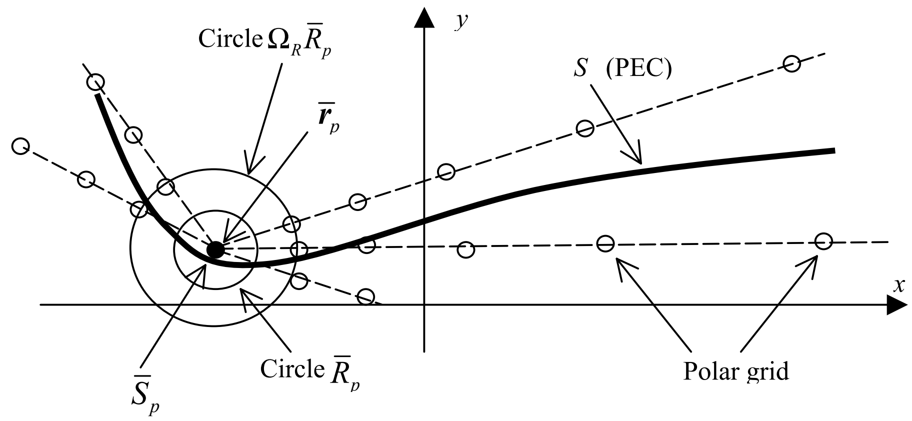

Fig. 1. Geometry of the scatterer and polar grid points corresponding to the *p*th subdomain.

REFERENCES

[1] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagetics*, IEEE Press, New York 1998.

[2] C. C. Lu and W. C. Chew, "A fast algorithm for solving hybrid integral equation," *IEE Proceedings-H*, vol. 140, no. 6, pp. 455-460, Dec. 1993.

[3] J. M. Song and W. C. Chew, “Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering,” *Microwave Opt. Technol. Lett*., vol. 10, no. 1, pp. 14-19, Sept. 1995.

[4] O. M. Bucci and G. Franceschetti, “On the spatial bandwidth of scattered fields,” *IEEE Trans. Antennas and Propagation,* vol. 35, no. 12, pp. 1445-1455, Dec. 1987.

[5] O. M. Bucci, C. Gennarelli, and C. Savarese, “Representation of electromagnetic fields over arbitrary surfaces by a finite and nonredundant number of samples,” *IEEE Trans. Antennas and Propagation,* vol. 46, no. 3, pp. 351-359, March 1998.

[6] E. Michielssen and A. Boag, "Multilevel Evaluation of Electromagnetic Fields for the Rapid Solution of Scattering Problems," *Microwave Opt. Technol. Lett*., vol. 7, pp. 790-795, Dec. 5 1994.

[7] A. Boag, "A Fast Physical Optics (FPO) Algorithm for High Frequency Scattering," *URSI Radio Science Meeting*, Salt Lake City, Utah, July 2000.

[8] A. Boag and E. Michielssen, "A Fast Physical Optics (FPO) algorithm for double-bounce scattering," unpublished.

[9] A. Boag, "Fast Iterative Physical Optics (FIPO) Algorithms for Multiple Bounce Scattering," *URSI Radio Science Meeting*, San Antonio, TX, June 2002.

[10] A. Boag, Y. Bresler, and E. Michielssen, “A multilevel domain decomposition algorithm for fast re-projection of tomographic images," *IEEE Trans. Image Proc.*, vol. 9, no. 9, pp. 1573-1582, September 2000.

[11] A. Boag, "A fast multilevel domain decomposition algorithm for radar imaging," *IEEE Trans. Antennas and Propagation,* vol. 49, no. 4, pp. 666-671, April 2001.

[12] A. Boag, V. Lomakin, E. Heyman, and E. Michielssen, , "Fast Evaluation of Time Domain Fields by Domain Decomposition and Non-Uniform Spherical Grid Interpolation," *URSI Radio Science Meeting*, San Antonio, TX, June 2002.