

# SENSITIVITY ANALYSIS OF NARROW-BAND PHOTONIC MICRO CAVITIES AND ARRAY WAVEGUIDES

**Ben Z. Steinberg**<sup>(1)</sup> and Amir Boag<sup>(2)</sup>

<sup>(1)</sup> Faculty of Engineering, Tel Aviv University, Tel-Aviv 69978, Israel, E-mail: [steinber@eng.tau.ac.il](mailto:steinber@eng.tau.ac.il)

<sup>(2)</sup> As (1) above, but E-mail: [boag@eng.tau.ac.il](mailto:boag@eng.tau.ac.il)

## ABSTRACT

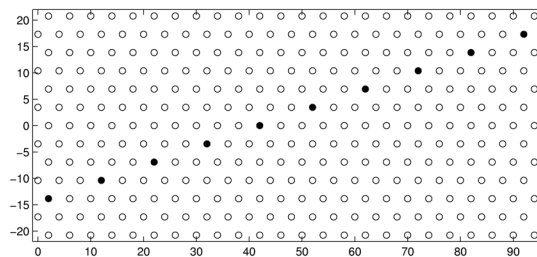
Coupled Cavity Waveguides (CCW) is a linear array of equally spaced identical local-defect microcavities situated within an otherwise perfect photonic crystal. Tunneling of light between adjacent microcavities forms a guiding effect, with a central frequency and bandwidth controlled by the local defect parameters and by the array spacing, respectively. In this work we develop perturbation theory to investigate sensitivity of photonic crystal micro-cavities and CCW devices to structure parameters and random inaccuracies. It is shown that the CCW appealing properties are practically not affected by random inaccuracies below a certain threshold value, and degrade significantly if the structural noise levels exceed this threshold.

## INTRODUCTION

It has been shown that line defects in photonic crystals can be used not only to guide but also to multiplex and demultiplex optical signals [1]. Most researchers studying the wave guiding by line defects employ photonic waveguides obtained by introduction of a continuous line of local defects. This results in a relatively wideband waveguides.

A new guiding device with a prescribed center frequency and narrow bandwidth—the Coupled Cavity Waveguide (CCW)—has been proposed recently by few researchers in different independent works, and for different applications [2]-[4]. In these devices a wave-guide is formed by widely spaced periodic defects in the photonic crystal. Each defect site with a resonant frequency in the band gap serves as a micro-cavity. Tunneling of radiation between the defect sites allows wave propagation along the line of defects. Sections of such wave-guides can be employed as ultra narrow band filters in optical routing devices. A schematic description of the CCW in a two dimensional photonic crystal is shown in Fig.1

**Figure 1.** A typical CCW in a 2D crystal. Local defects are shown by solid circles.



Let  $\omega_0$  be the resonant frequency of a single, isolated micro-cavity located within an otherwise perfect photonic crystal (the *perfect micro-cavity*). Weak coupling between the periodically located defects of an associated CCW causes this discrete resonance to turn into a narrow band of guided frequencies, centered essentially around  $\omega_0$ . The corresponding dispersion relation  $\omega(\beta)$  is given by [4]

$$\omega - \omega_0 \approx \omega_s + \Delta\omega \cos \beta. \quad (1)$$

The weak coupling perturbation theory applied in [4] facilitates an approximate calculation of both the frequency shift  $\omega_s$  and the half bandwidth  $\Delta\omega$ . It has been shown in [4] that  $\omega_s \ll \Delta\omega$ , and  $\Delta\omega$  decreases exponentially with the

inter-cavity spacing (see Fig. 1). Thus, by tuning the inter-cavity distance one can achieve extremely narrow band filters and routers [4], and by a proper design of the local defect one can obtain almost any prescribed central frequency. As a result of the above-mentioned appealing properties, effort has been devoted to use these devices in optical communication applications, where the operating wavelengths are in the  $1.5\mu\text{m}$  regime. Here, structural details of photonic crystal devices may have sizes in the deep sub-micron scale, which evidently approaches the accuracy limits of conventional fabrication technologies.

The purpose of this work is to examine sensitivity aspects of the CCW filter design. More specifically, three goals are addressed. First, we use cavity perturbation theory [5], traditionally employed for microwave cavity tuning analysis, to study the shift of an isolated micro-cavity frequency  $\omega_0$  as a function of the local defect parameters. The theory is also extended to hold for the case of degenerate micro-cavity modes. Next, the above theory is used to investigate the influence of random structure inaccuracies on the single cavity center frequency, and compare its results to exact computations. Finally, we perform a study of the CCW sensitivity to random structure inaccuracies. This study is based on the fact that the microcavities are weakly coupled, and their (random) frequency shifts depend essentially on the local structure inaccuracy. Hence the entire CCW can be viewed as a linear array of micro cavities having independent random resonant frequencies.

### CAVITY PERTURBATION THEORY

Here we explore how the resonant frequency of a single isolated micro-cavity changes as the dielectric structure of the embedding crystal is varied, from a reference perfect structure  $\mathcal{E}(r)$  to  $\mathcal{E}(r) + \delta\mathcal{E}(r)$ . Let  $E_n, H_n$  be the electric and magnetic modes of the perfect micro-cavity associated with the resonant frequency  $\omega_n$ . We assume that the modes are non-degenerate, so these modes are unique. Due to the dielectric material variation, the latter modal field and resonant frequency change to  $E_n^*, H_n^*$  and  $\omega_n + \delta\omega_n$ , respectively. Applying the cavity perturbation procedure [5] to this eigenvalue problem we obtain for the frequency variation

$$\delta\omega_n \approx \omega_n \langle E_n, \delta\mathcal{E} E_n \rangle / 2\mu_0 \|H_n\|^2. \quad (2)$$

This last result, which applies to the non-degenerate case, can be generalized to hold also for cases in which the ‘‘initial’’ (i.e. before material variations take place) perfect micro-cavity possesses mode degeneracy. Towards this end, assume that  $\omega_n$  is associated with  $N$  degenerate modal fields  $\{E_n^{(j)}, H_n^{(j)}\}_{j=1\dots N}$ . Due to the  $\delta\mathcal{E}(r)$  variation, the resonance  $\omega_n$  may split to  $N$  different resonances  $\omega_n + \delta\omega_{nm}$ ,  $m=1\dots N$ . We expand now  $E_n^* \approx \sum_{j=1}^N a_j E_n^{(j)}$ , where  $a_j$  are yet to be determined coefficients. Substituting this expansion into the variational representation of the eigenvalue problem, and performing the standard variational procedure leads to the following result,

$$\mathbf{A}\vec{a} = 2\mu_0 \delta\omega_{nm} / \omega_n \vec{a}, \quad A_{i,k} = \langle E_n^{(k)}, \delta\mathcal{E} E_n^{(i)} \rangle, \quad \vec{a} = \{a_1, \dots, a_N\} \quad (3)$$

that is an  $N \times N$  matrix eigenvalue equation. Equations (2)-(3) can be used to study the effect of random structure inaccuracy on the micro-cavity resonance, by allowing  $\delta\mathcal{E}(r)$  to represent the random structure variations relative to the perfect micro-cavity. Since the perfect micro-cavity modal fields  $E_n, H_n$  are well localized, contributions to the frequency variations come essentially from the variations of the micro-cavity near neighborhood. Contributions from remote crystal cells are exponentially smaller.

## MICRO-CAVITY ARRAY WAVEGUIDES WITH RANDOM INNACCURACY

This study is based on three observations; (a) The waveguide micro-cavities are weakly coupled, and apart from the variations induced by the random structural inaccuracy, they are identical. (b) The resonance frequency of the  $i$ -th micro-cavity in the array is  $\omega_0 + \delta\omega_i$  where  $\omega_0$  is the resonance of the perfect micro-cavity – a reference resonance that applies to all the array micro-cavities, and  $\delta\omega_i$  is a random shift that can be predicted by Eqs. (2)-(3). (c) Since  $\delta\omega_i$  depends essentially on the on the structure variations of the  $i$ -th micro-cavity closest neighborhood, the  $\delta\omega_i$  are independent. Therefore, the entire waveguide can be viewed as a linear array of weakly coupled independent micro-cavities, each having a resonant frequency slightly differing from  $\omega_0$ . The resonant mode spatial form is almost the same for each and all of the micro-cavities. Following essentially the same analysis as in [4], we express the total waveguide field as  $H(r) = \sum_m A_m H_0^m(r)$ , where  $H_0^m(r)$  is the fundamental mode of the isolated  $m$ -th micro-cavity, with the structural noise presence. Following essentially the same steps as in [4], we get an equation for  $A_m$ ,

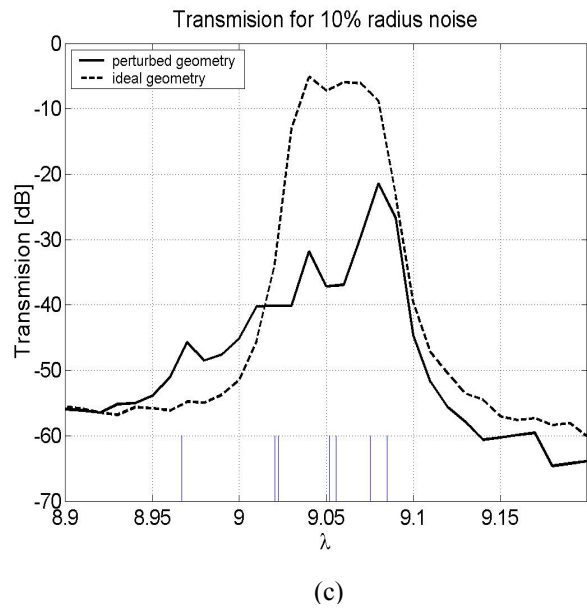
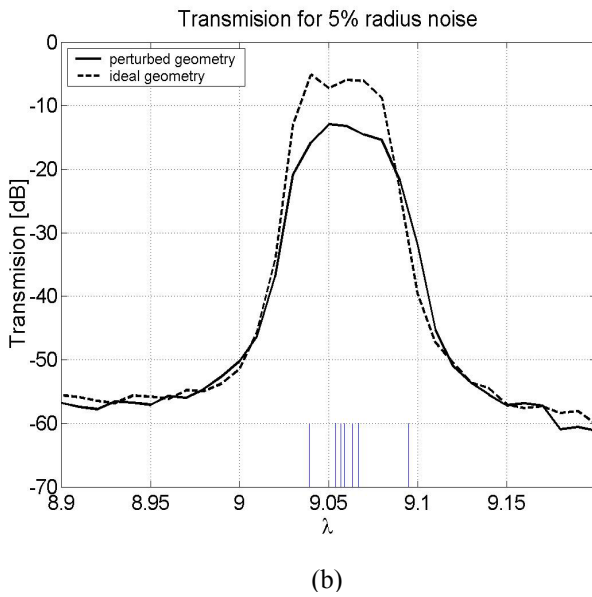
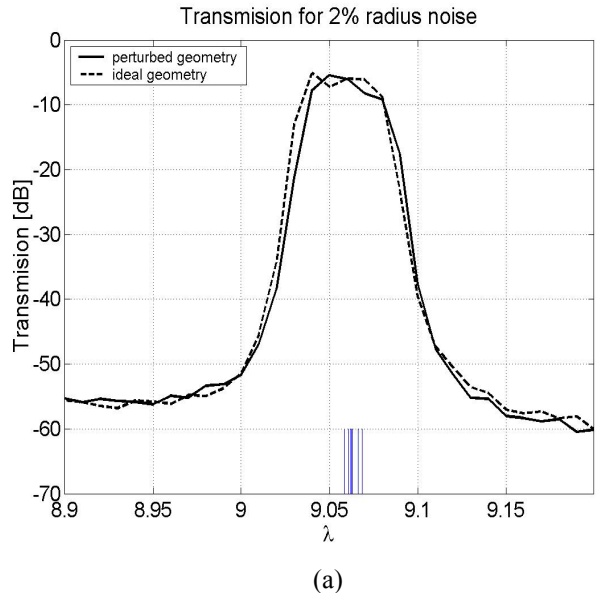
$$\sum_n [c^{-2}(\omega_0^2 - \omega^2)I_{n,k} + \tau'_{n,k}]A_n = 0 \quad \forall k, \quad I_{n,k} = \langle H_0^n, H_0^k \rangle, \quad (4)$$

and  $\tau'_{n,k} = \tau_{n,k} + 2\omega_0\delta\omega_k I_{n,k}$ , where  $\tau_{n,k} = \langle H_0^n, \sum_{j \neq k} \Theta_j H_0^k \rangle$ . Here  $\Theta_j$  is the well-known wave operator, acting only in the  $j$ -th local defect domain [4]. This equation is identical in form to that obtained for the unperturbed array waveguide, except for the presence of the random variable  $\delta\omega_k$  (see [4]). It is clear from the definition of  $I_{n,k}$  that it is exponentially small for  $n \neq k$ . Also, it has been shown that the dominant  $\tau_{n,k}$  are for  $n = k$  and  $n - k = \pm 1$ , and also  $|\tau_{n,n}| \ll |\tau_{n,n \pm 1}|$ . Thus, Eq. (4) can be rewritten as an infinite-matrix formulation,

$$(\boldsymbol{\tau} + \Delta)\bar{A} = -(\omega_0^2 - \omega^2)\bar{A}, \quad \Delta = \text{diag}(\delta\omega_k) \quad (5)$$

where  $\boldsymbol{\tau}$  is an essentially three-diagonal matrix. Without the presence of the noise it has an exact cyclic structure, as  $I_{n,k}$  and  $\tau_{n,k}$  are shift invariant. Using the properties of such matrices [6], the spectral properties of the CCW reported in [4] are immediately reconstructed. In this case, it has been shown that the waveguide bandwidth in (1) is given approximately by  $\Delta\omega \approx \tau_{0,1}$ . The presence of the random structural inaccuracy is expressed, predominantly, by the random  $\delta\omega_k$ . This reveals a “threshold” behavior: as long as the frequency variations  $\delta\omega_k$  are small compare to  $\Delta\omega$  of the unperturbed CCW, the perturbed CCW characteristics are similar to those of the ideal one. However, when  $\delta\omega_k$  approaches  $\Delta\omega$ , the CCW performance degrade significantly. To demonstrate this effect, we have simulated the transmission of a CCW in a 2D hexagonal lattice photonic crystal. The inter-cavity spacing is three periods in the horizontal direction, and the simulated CCW section consists a total of seven micro-cavities. The transmission of the CCW is depicted in Fig. 2 by a dashed line. In comparison, we show by a solid line the transmission corresponding to the same CCW, but with 2%, 5%, and 10% random radii perturbation. The solid vertical lines show  $\lambda_k$  - the individual resonant wavelengths of each of the seven micro-cavities. It is seen that the CCW performance degrades significantly when the variation of  $\delta\omega_k$  exceeds the unperturbed CCW bandwidth.

**Figure 2.** Transmission of the perfect CCW (dashed line) and the randomly perturbed CCW (solid line), for post radii random inaccuracy of (a) 2%, (b) 5%, and (c) 10%.



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