COMPARISON OF DIFFERENT IMPLEMENTATIONS OF THE FRESNEL PO SCATTERING MODELS

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ABSTRACT

The use of Physical Optics (PO) for computations of scattering phenomena has reduced the use of the classical Geometrical Optics (GO-UTD) methods for computations of propagation phenomena in mobile communication systems. However, there is an ever-increasing demand for efficient implementations of PO computations. So, even if the method used is already well developed and its implementation at the K.U.Leuven dating from a few years ago [1] it is useful to verify the relative importance of different elements used in the computations of the fields. This paper will be devoted to an analysis of the implementation of the Fresnel method, and its experimental verification.

INTRODUCTION

It is well known that PO methods describe more accurately the scattering phenomena than the GO methods, specially when the reflecting objects are not infinitely long, as assumed in the derivation of the canonical problems in GO/UTD. To compute the fields of magnetic, resp. electric current sources on a surface, we use eqns. 1 and 2.

\[
\vec{E} = -\frac{j\omega \mu}{4\pi} e^{-jRK} \left\{ \frac{1}{kR} \vec{F}_e - \frac{3j}{k^2R^2} i_R (\vec{i}_R \cdot \vec{F}_m) \right\}
\]

\[
\vec{E} = \frac{1}{4\pi} e^{-jRK} (1 + jkR)(i_R \times \vec{F}_m)
\]

where C is a point on the surface containing the currents and R is the position of the observer (receiver). If we suppose that the distance to the source is sufficiently large (we will discuss this later) we have to evaluate integrals of the kind:

\[
e^{-jRK} \int_S \left( \vec{F} \cdot \left( \frac{\hat{e}}{\hat{m}} \right) \right) e^{-jK} dS
\]

The classical approximation of Beckmann and Spizzichino [2] consists in supposing that the incident wave on the surface is plane. If the surface is also plane, (3) is a mere Fourier transform. Problems associated with this assumption are a/o. that the integral (3) is proportional with the reflecting surface. It may only be applied if BOTH source AND observer are in the far field of the reflecting surface, which is never the case for large surfaces in GHz propagation computations.

Therefor, we expanded the exponent of the integrand (consisting of two terms: the first from the source (denoted by T) and the second from the observer into a Taylor series (for a planar surface) as follows:

\[
\int_S e^{-jk\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} e^{-jk\sqrt{(x-x_e)^2+(y-y_e)^2+(z-z_e)^2}} dS
\]

The next step consists in expanding this integrand into a series around an arbitrary point C of the surface, usually taken as the centre of gravity of the surface, but that will have to be moved to another location if a mathematically correct expansion has to be pursued. Indeed, the expansion leads to an expression, consisting only of constants (a,b,c,d,e,f,g and h) that only depend on the location of the source, the observer and this up to now arbitrarily chosen centre point:
\[ R_{TC} \sqrt{1 + a \chi' x'^2 + b \chi' y'^2 + cy' + dy'^2} + R_{RC} \sqrt{1 + ex' + fx'^2 + gy' + hy'^2} \]

\[ \approx R_{TC} \left( 1 + \frac{1}{2} (ax' + bx'^2 + cy' + dy'^2) - \frac{1}{8} (a^2 \chi'^2 + c^2 y'^2 + 2ac x' y') \right) + R_{RC} \left( 1 + \frac{1}{2} (ex' + fx'^2 + gy' + hy'^2) - \frac{1}{8} (e^2 x'^2 + g^2 y'^2 + 2eg x' y') \right) \]

where the primed co-ordinates are the co-ordinates with respect to the centre point C. This allows reducing the integrals to simpler ones of the form:

\[ e^{-jk(R_{RC}+R_{RC})} \int_S e^{-jk\chi' x'^2} \cdot e^{-jk\chi' y'^2} \cdot e^{-jk\chi' y'} dS \]  

(6)

The parabolic approximations used in the evaluation of the phase require a good choice of an integration centre, leading to the nullification of the coefficient \( c_{xy} \) to allow to reduce (6) to a product of Fresnel integrals in the frequency domain, that can be efficiently evaluated. Since the Fresnel integral is bounded, no infinite fields, like in the classical theory can occur. One can see that for integration along the principal planes (\( x'=0 \) and \( y'=0 \)) the choice of C is completely arbitrary. For other points however, the separation of (6) into two Fresnel integrals is not so obvious. This paper will investigate how to choose C and the influence of this choice on the computations.

**CHOICE OF THE INTEGRATION CENTRE**

Since the coefficient \( c_{x'y'} \) is given by:

\[ c_{xy} = \frac{(x_c - x_T)(y_c - y_T)}{R_{TC}^3} + \frac{(x_c - x_R)(y_c - y_R)}{R_{RC}^3} \]  

(7)

two trivial solutions for the centre exist, namely \( (x_c=x_T, y_c=y_T) \) and \( (x_c=x_R, y_c=y_T) \). The other solutions can be found analytically, by solving the 8th degree equation obtained from (7). An example of the loci of C is given in Fig. 1.

Fig. 1: Locus of all possible centres for TX=[0,0,6] and RX=[0,3]

The accuracy is higher if this centre is optimally chosen. Indeed, even if the integral (6) can be evaluated correctly when \( c_{xy}=0 \), we should not forget that the integral itself is an approximation that is only correct around one point of the reflecting surface, while the loci from Fig. 1 only depend on the relative positions of source and observer and hence can even be outside the reflecting surface. Therefore, one can choose to optimise:

\[ \sqrt{(x_c - x_{C1})^2 + (y_c - y_{C1})^2} + K \cdot Locus \]  

(8)

where \( C_1 \) is the real centre of gravity of the reflecting surface. Another possibility consists in fitting the derivatives so that the phase behaviour is osculating to the real behaviour as closely as possible. This can be done by choosing:
\[ \alpha = \frac{R_{TC}}{2} \]  
\[ \beta = -\frac{R_{TC}}{4}(y_c - y_C) \]  
\[ \gamma = \frac{\partial R_f(C_i)}{\partial x} - \left(x_{C_i} - x_C\right) \frac{\partial^2 R_f(C_i)}{\partial x^2} \]  
\[ \delta = \frac{R_{TC}}{2} \]  
\[ \epsilon = -\frac{R_{TC}}{4}(x_c - x_C) \]  
\[ \zeta = \frac{\partial R_f(C_i)}{\partial y} - \left(y_{C_i} - y_C\right) \frac{\partial^2 R_f(C_i)}{\partial y^2} \]

and finally (with extremely simple solutions if one chooses the gravity centre to be equal to the integration centre):

\[ c = \frac{-(\alpha \delta + \gamma \epsilon - \zeta \beta) \pm \sqrt{(\alpha \delta + \gamma \epsilon - \zeta \beta)^2 + 4 \alpha \beta \delta \zeta}}{2 \beta \delta} \]  
\[ a = \frac{\gamma}{\alpha + c \beta} \]

\[ b = \frac{\partial^2 R_f(C_i)}{\partial x^2} + \frac{a^2}{4} \]

\[ d = \frac{\partial^2 R_f(C_i)}{\partial y^2} + \frac{c^2}{4} \]

and afterwards minimising:

\[ \sqrt{(x_c - x_{C_i})^2 + (y_c - y_{C_i})^2 + K \cdot c_{sy}} \]  

The last method just minimises the phase difference between source and observer as shown in Fig. 2.

Fig. 2: Difference between surface and approximation.
A relative comparison of the CPU times of those 4 approximations is given in table 1. We note that using (11), we obtain a reasonable result (only 4 times more computations than the trivial solution is needed).
Table 1: Comparison of the number of FLOPS for nearly perpendicular incidence and a scan of 1015 points.

<table>
<thead>
<tr>
<th>OPTIMALISATION METHOD</th>
<th>NUMBER OF FLOPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trivial solutions</td>
<td>1.023.264</td>
</tr>
<tr>
<td>Optimal polynomial+distance</td>
<td>282.367.364</td>
</tr>
<tr>
<td>Optimal fitted ( c_{xy} )+distance</td>
<td>4.088.668</td>
</tr>
<tr>
<td>Minimalisation of phase-differences</td>
<td>11.521.514</td>
</tr>
</tbody>
</table>

It is obvious that a careful choice between a further subdivision of the reflecting surface and an optimal choice of the integration centre is important. It can be shown [3] that the distance to source or observer is reduced to

\[ R > \frac{D^3}{2 \sqrt{\lambda}} \]

instead of the classical Rayleigh distance \( R = \frac{D^2}{\lambda} \).

MEASUREMENTS

Many real surfaces, starting from flat, crenel, triangular, sinusoidal and random have been measured in the near-field facilities of the TELEMIC lab and compared with the simulations of the Fresnel model. One example for a flat PMMA plate of thickness 2 cm is given in Fig. 3. At the centre of the cut, only a few dB difference is noted.

![Fig. 3: Measured (left) and Fresnel simulated fields (right) of a flat plexiglass plate at nearly grazing incidence.](image)

CONCLUSIONS

A fast and accurate method for PO computations has been developed and has been extensively used in many different applications. Broadband alternatives have been considered both in the time domain and the frequency domain [4].

REFERENCES