A DISTRIBUTIONS-BASED APPROACH TO IMAGE STRONG SCATTERERS FROM PO DATA

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Abstract: The reconstruction of the shape of 2-d "strongly scattering" objects from the knowledge of the scattered electric field under the incidence of TM plane waves with varying frequency and fixed angle of incidence is investigated. With reference to perfectly conducting cylinders, the provided formulation accommodates the distribution nature of the induced surface current density. Thus, as unknown representing the objects' contour, a single layer distribution is chosen so that the boundary of the scatterers is described by its support. The nonlinear unknown-data mapping is linearized by means of the Kirchhoff approximation and the inversion is performed by the SVD approach.

INTRODUCTION

Microwave imaging is an inverse scattering problem consisting into determining the physical properties and/or the shape of unknown scattering objects starting from the knowledge of the measured scattered field data [1]. This problem is of interest in many applications. For example, ground penetrating radar [1], non-destructive testing or evaluation of materials [2], and geophysics [3], just to mention a few examples.

In this paper, we particularly concentrate on the problem of retrieving the shape of perfectly conducting infinitely long (2-d) cylinders. We derive a formulation accommodating the distribution nature of the induced surface current density through the use of volume integrals involving compact support distributions. In such a way, the objects' contour can be represented and searched for as the support of a single layer distribution [4, 5, 6, 7]. Also, the approach at hand does not require *a priori* knowledge of neither the number nor the location of the center of the scatterers. The unknown-data mapping arising from our formulation is then linearized with resort to the Kirchhoff approximation. This leads to recast the problem as the inversion of a linear operator $\mathcal{L}$ acting on a distribution space [5]. An extension of the Singular Value Decomposition (SVD) approach to solve the linearized problem, or in other words, to invert the involved distribution operator $\mathcal{L}$ was provided in [5] and the SVD approach is now applied in order to perform inversions [6]in the case when the scattered electric field is collected in the far zone under the incidence of plane waves with TM polarization, varying frequency and fixed angle of incidence.

FORMULATION OF THE PROBLEM

Let us denote by Ω the union of the cross sections (belonging to the xy plane) of N_c perfectly conducting infinitely long cylinders embedded in free space and by Γ the contour of Ω (see Fig. 1.a). Suppose that the cylinders are illuminated by TM-polarized time-harmonic plane waves with unitary amplitude (the electric field has only one component directed along the axis of the cylinders). The only component (along the z-axis) of the scattered electric field outside Ω is given by [6]

$$E_s(k_0,\underline{r}) = -\frac{\omega\mu_0}{4}\int_\Gamma H_0^{(2)}(k_0R)J(\underline{r}')d\Gamma \tag{1}$$

where ω is the working circular frequency, $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is free space wavenumber, ϵ_0 and μ_0 are the dielectric permittivity and the magnetic permeability of vacuum, respectively, $H_0^{(2)}(\cdot)$ is the Hankel function of zero order and second kind, $R = |\underline{r} - \underline{r}'|$ and J is the induced surface current density. The surface current density J satisfies the following *Magnetic Field Integral Equation* (MFIE)

$$J = 2J_i + \frac{j}{2}\frac{\partial}{\partial n}\int_\Gamma H_0^{(2)}(k_0 R) J(\underline{r}')d\underline{r}' \quad \text{on } \Gamma \tag{2}$$

where $J_i = \hat{n} \times \underline{H}_i \cdot \hat{\imath}_z$, $\underline{H}_i$ is the incident magnetic field, $\hat{n}$ is the outward unit contour normal vector (see Fig. 1.a) and j is the imaginary unit, that is $j^2 = -1$.

Note that, because of the TM polarization, the problem at hand is scalar, in the sense that the scattered field and the induced surface density current have only one component along z-axis and the problem is governed by the scalar equations (1) and (2). E_s represents the data of the problem which are assumed to be gathered at different wavenumbers k_0 (*multifrequency illumination*), whereas Γ is the unknown. The problem at hand thus consists into recovering the contour of the unknown scattering cylinders starting from the knowledge of the scattered electric field.

At this point, we introduce the single layer distribution with density J, denoted by $J\delta_\Gamma$, as

$$\int\int_O J(x,y)\delta_\Gamma(x,y)dO = \int_\Gamma J(x,y)d\Gamma \tag{3}$$

where O denotes the investigation domain, that is the domain within which the scatterers are supposed to reside. Accordingly, equations (1) and (2) can be rewritten, respectively, as

$$E_s\left(k_0, \underline{r}\right) = -\frac{\omega\mu_0}{4}\int\int_O \mathcal{J} H_0^{(2)}(k_0 R)dO \tag{4}$$

$$\mathcal{J} = 2\underline{\mathcal{N}} \times \underline{H}_i \cdot \hat{\imath}_z + \frac{j}{2}\underline{\mathcal{N}} \cdot \underline{\nabla}\int\int_O H_0^{(2)}(k_0 R)\mathcal{J} dO \tag{5}$$

where the identity $\partial/\partial n = \hat{n} \cdot \underline{\nabla}$ has been exploited, $\underline{\nabla}$ being the gradient operator, $\mathcal{J}$ denotes the distribution $J\delta_\Gamma$, $\underline{\mathcal{N}}$ denotes the vectorial distribution $\hat{n}\delta_\Gamma$. Equations (4) and (5) form the basis of our formulation. Observe that $\underline{\mathcal{N}}$ depends only on the boundary of the cylinders' cross section Γ, whereas $\mathcal{J}$ depends also on the impinging radiation. Also, note that the boundary Γ coincides with the support of the distribution δ_Γ, where the latter one represents the single layer distribution with unit density, and that the union of the supports of $n_x\delta_\Gamma$ and of $n_y\delta_\Gamma$ coincides with Γ, where n_x and n_y are the x and y components of $\hat{n}$, respectively. Following Dirac's interpretation, the distribution $n_x\delta_\Gamma$ (and thus also $n_y\delta_\Gamma$) can be though of as a "function" satisfying

$$\begin{cases} n_x(x,y)\delta_\Gamma(x,y) = 0 \;\; \forall (x,y) \notin \Gamma \\ n_x(x,y)\delta_\Gamma(x,y) = +\infty \;\; \forall (x,y) \in \Gamma \end{cases}$$

We thereby assume the vectorial distribution $\underline{\mathcal{N}}$ as the unknown describing the cylinders' shape and formulate the inverse problem as follows: "determine $\underline{\mathcal{N}}$ from the knowledge of the scattered field".

It must be stressed that the relationship between the scattered electric field E_s and the unknown distribution $\underline{\mathcal{N}}$ is nonlinear. This can be inferred by observing that, owing to (4), E_s depends on $\mathcal{J}$ which is also unknown and, by virtue of (5), depends on $\underline{\mathcal{N}}$ and on itself.

THE LINEAR MODEL

In this section, we introduce a simplified model of the electromagnetic scattering that is based on two hypotheses. First, we assume that the spacing between the scattering objects is large compared to their extent and to the impinging wavelength, with the result that the scattered field due to the mutual interactions between the different scattering objects is negligible. This means that, from a point of view of the electromagnetic scattering, we can consider each object as it were standing alone in the free space. Second, we assume the local radius of curvature of the scatterers to be large compared with respect to the impinging wavelength, so that each cylinder may be locally considered as a plane (*physical optics or Kirchhoff approximation*) [5]. The above hypotheses lead to assume for the surface current density

$$J \simeq J_{PO} = \begin{cases} 2J_i & \text{on } \Gamma_i \\ 0 & \text{on } \Gamma_s \end{cases} \tag{6}$$

where the Γ_i and Γ_s denote the illuminated and shadowed sides of Γ.
Under the further assumption that $k_i = \hat{\imath}_x$ (i.e., the angle of incidence $\theta_i = 0$) and after some manipulations [6] we obtain

$$J_{PO}(\underline{r}) = -\frac{2}{\zeta_0} n_x e^{-jk_0 x} \mathbf{U}(-n_x) \tag{7}$$

in which $\zeta_0 = \sqrt{\mu_0/\epsilon_0}$ is the wave impedance of free space and $\mathbf{U}(\cdot)$ is the Heaviside function accounting for the shadow boundary.
Under the Kirchhoff approximation, the scattered field (1), normalized to a factor $k_0/2$, can be rewritten, in terms of a single layer distribution with density $n_x \mathbf{U}(-n_x)$, as

$$E_s(k_0, \underline{r}) = \int\int_O \mathcal{K}(k_0, \underline{r}; \underline{r}') n'_x \mathbf{U}(-n'_x) \delta_\Gamma dO \tag{8}$$

where $\mathcal{K}(k_0, \underline{r}; \underline{r}') = H_0^{(2)}(k_0 R) \exp(-jk_0 x')$.
According to (8), the adoption of the physical optics approximation (6) leads to recast the problem as follows: *to determine the x-component of the unknown vectorial distribution $\underline{\mathcal{N}}$ over the illuminated region from the knowledge of the data*. So, the Kirchhoff approximation allows to obtain a simplified model of the electromagnetic scattering by leading to a linear relationship between unknown and data. However, it makes it possible to recover only the illuminated side of the scatterers.

THE INVERSION

In order to perform the inversion, we consider a scattering experiment in which the scattered field E_s is gathered over the angular sector $\theta \in [\pi - \theta_M, \pi + \theta_M]$ (with $\theta_M \leq \pi$) of the far zone circular observation curve $\underline{r} = (r\cos\theta, r\sin\theta)$ at frequencies yielding $k_0 \in [k_{0_{min}}, k_{0_{max}}]$. In such case, eq. (8) can be rewritten, after normalization to the nonessential factor $\sqrt{k_0/(2\pi r)}$, as the following relationship [6]

$$E_s(u, v) = \int\int_O n'_x \mathbf{U}(-n'_x) \delta_\Gamma e^{-j(ux' + vy')} dO \tag{9}$$

where $u = k_0(1 - \cos\theta)$ and $v = -k_0 \sin\theta$. The unavoidable limitations introduced by the measurement configuration along with the range of validity of the Kirchhoff approximation lead to an incomplete knowledge of $E_s(u, v)$ which is then available only over a finite domain of the spectral (u, v) plane. The problem of determining $n'_x \mathbf{U}(-n'_x)\delta_\Gamma$ from the incomplete knowledge of the scattered electric far field can be recast as the inversion of the linear operator

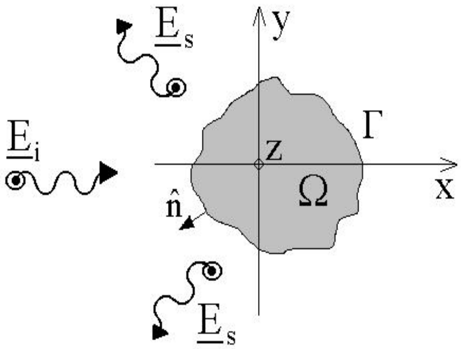
$$\mathcal{L} \; : \; n'_x \mathbf{U}(-n'_x)\delta_\Gamma \; \rightarrow \; \int\int_O n'_x \mathbf{U}(-n'_x) \delta_\Gamma e^{-j(ux' + vy')} dO \tag{10}$$

Note that, as remarked in [5], the convergence to zero of the singular values of $\mathcal{L}$ leads to a solution of the equation (9) which does not continuously depend on data. This means that the unavoidable presence of noise on data, due to the measurement and model errors, produces unreliable solutions. So in order to achieve stable solutions of (9), the employ of some regularization technique is mandatory. Regularization is here achieved via cut-off of the singular values (*Truncated SVD*) [5] as

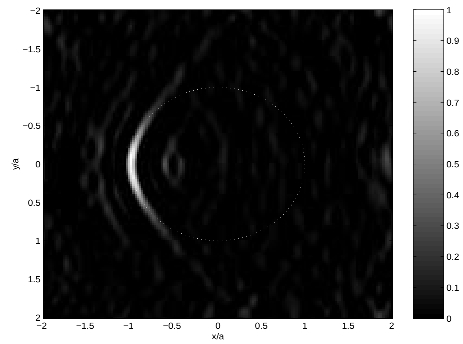
$$\mathcal{R} n'_x \mathbf{U}(-n'_x)\delta_\Gamma = \sum_{n=0}^{N-1} \frac{1}{\sigma_n} < E_s, v_n >_{L^2(S)} u_n \tag{11}$$

where $\mathcal{R} n'_x \mathbf{U}(-n'_x)\delta_\Gamma$ is the regularized solution and $\{\sigma_n, u_n, v_n\}$ is the singular system of $\mathcal{L}$. For the details about the numerical implementation of the extended SVD approach see [6].
Finally, a numerical result concerning the reconstruction of a cylinder with a circular cross section of radius a is shown. We consider a square investigation domain $O = [-x_M, x_M] \times [-y_M, y_M]$, where $x_M = y_M = 2a$, $[k_{0_{min}} a, k_{0_{max}} a] = [2\pi \cdot a, 2\pi \cdot 3a]$ and $\theta_M = 70°$. The Signal to Noise ratio on data equals $10dB$.
Fig.1.b depicts the positive part of $-Re\{\mathcal{R} n'_x \mathbf{U}(-n'_x)\delta_\Gamma\}$ normalized with respect to its maximum since the

density of $n'_x \mathbf{U}(-n'_x)\delta_\Gamma$ is a priori known to be real and negative. The singular values of operator $\mathcal{L}$ have been cut-off at a threshold put at $10dB$ below the maximum singular value.



(Fig. 1.a)



(Fig. 1.b)

Figure 1: (a) Pictorial view of the geometry of the problem; (b) Reconstruction of the perfectly conducting cylinder with circular cross section.

FUTURE DEVELOPMENTS

In this paper, the problem of reconstructing the shape of perfectly conducting 2-d objects starting from the knowledge of the scattered electric field has been faced throughout a SVD approach in free space.

This work is preliminary and can be extended to cases of interest in subsurface prospection [1], namely when the unknown cylinders are buried below an air-soil interface.

Also, it is a starting point for treating the case of ”strongly scattering objects”, that is objects having dielectric permittivity much different from that of the host medium. Indeed, such problem can be formulated according to equivalent electric and magnetic surface current densities which can again be dealt with as distribution as done in this paper [7]. For strongly scattering objects having radius of curvature large with respect to the impinging wavelength, a physical optics approximation for the equivalent current densities can be further derived [7].

References

[1] R. Pierri, A. Liseno, and R. Solimene, “Key points in microwave imaging,” in *Surface Penetrating Radar*, D.J. Daniels Ed. IEE Press, 2003, in press.

[2] K.J. Langenberg, “Elastic wave inverse scattering as applied to nondestructive evaluation,” in *Inverse problems in scattering and imaging*, Bertero and Pike Eds. Bristol: Adam Hilger, 1992.

[3] A.J. Devaney, “Geophysical diffraction tomography,” *IEEE Trans. on Geosc. and Remote Sens.*, vol. GE-22, pp. 3-13, Jan. 1984.

[4] V.S. Vladimirov, *Distributions in the mathematical physics*, Moscow: Mir publishers, 1981.

[5] A. Liseno, and R. Pierri, “Imaging perfectly conducting objects as support of induced currents: Kirchhoff approximation and frequency diversity,” *J. Opt. Soc. Am. A*, vol. 19, n. 7, Jul. 2002, in press.

[6] R. Pierri, A. Liseno, and F. Soldovieri, “Shape reconstruction from PO multifrequency scattered fields via the Singular Value Decomposition approach,” *IEEE Trans. Antennas Prop.*, vol. 49, n. 9, pp. 1333-1343, Sept. 2001.

[7] A. Liseno, “Shape reconstruction of strong scatterers by distributional models,” *Second University of Naples*: Ph.D. Thesis, Dec. 2001.