THE IMPORTANCE OF THE VELOCITY TERM IN THE ELECTROMAGNETIC FORMING PROCESS

T.E. Manea[(1)], **M.D. Verweij**[(2)], **H. Blok**[(3)]

[(1)] ***Laboratory of Electromagnetic Research, Faculty of Information Technology and Systems Delft University of Technology, Mekelweg 4, 2628 CD Delft, The Netherlands. E-mail: t.e.manea@its.tudelft.nl***

[(2)] ***As (1) above, but E-mail: m.d.verweij@its.tudelft.nl***

[(3)] ***As (1) above, but E-mail: h.blok@its.tudelft.nl***

ABSTRACT

In electromagnetic forming a metal workpiece is shaped by the electromagnetic forces due to a pulsed electric current through a nearby forming coil. During the forming process, the workpiece moves with a certain velocity and the electromagnetic field should, in principle, be calculated with the use of Maxwell equations for moving media. Such equations contain additional terms as compared to those for stationary media. In this paper the motion of the workpiece is taken into account to get an insight in the order of magnitude of velocities that impose the use of Maxwell equations for moving media.

STATEMENT AND CONFIGURATION OF THE PROBLEM

In an electromagnetic forming process a metal workpiece is shaped by the electromagnetic forces that accompany the induced electromagnetic field from a pulsed electric current through a nearby forming coil. The electromagnetic forces act as the source of the mechanical deformation process and can shape workpieces without strong heat effects and tool marks. Since the electromagnetic forming process causes the workpiece to move with a certain deformation velocity, the electromagnetic field should be calculated with the use of Maxwell equations for moving media as in [1, 4, 5, 6]. As compared to stationary media, the Maxwell equations for moving media contain additional terms.

To focus on the fundamental aspects, in this paper the transient electromagnetic field is analyzed for a plane-layer configuration that consists of a sheet-antenna at a certain height above a conducting half-space, as shown in Fig. 1. This models the case in which there is an air-gap in between the current and the workpiece, while the workpiece is very thick. The configuration depends only on the z-direction. Its subdomains are: a lower half-space (workpiece) $\mathcal{D}_1 = \{\boldsymbol{r} \in \mathcal{R}^3 | -\infty < z < z_1\}$, a layer with finite thickness (air-gap) $\mathcal{D}_2 = \{\boldsymbol{r} \in \mathcal{R}^3 | z_1 < z < z_2\}$, and an upper half-space (air) $\mathcal{D}_N = \{\boldsymbol{r} \in \mathcal{R}^3 | z_2 < z < \infty\}$. In each layer, the medium is homogeneous, linear, time-invariant, locally and instantaneously reacting and isotropic in its electromagnetic behavior, with permeability $\mu = \mu_0$ and conductivity $\sigma = \sigma_n$ $(n = 1, 2, 3)$. Further, it is assumed that the conduction current is much larger than the displacement current, i.e. the electromagnetic field is diffusive in nature.

In an actual electromagnetic forming process, each particle inside the workpiece moves with a different particle velocity. For the sake of simplicity, in this paper it is assumed that each particle inside the half-space has the same particle velocity $\boldsymbol{v} = -v_z \boldsymbol{i}_x$. The surface of the conducting half space is initially placed at z_0 and its position at an arbitrary moment $t > 0$ is $z_1 = z_0 - v_z t$, as shown in Fig. 1.

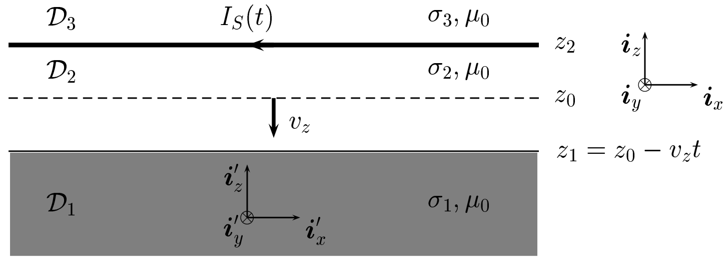

Figure 1: Sheet-antenna above a moving, conducting half space.

TRANSIENT ELECTROMAGNETIC FIELD IN THE CONFIGURATION

The transient diffusive electromagnetic field in the configuration consists of an incident field, a reflected field and a transmitted field. The Galilean transforms (obtained by neglecting the terms $O(v_z^2/c^2)$ in the formal Lorentz transforms) that apply to time and space coordinates in the current problem are

$$z' = z - v_z t, \qquad t' = t, \tag{1}$$

where primed symbols are used for all quantities in the co-moving reference frame. The transient incident field is the field given by the sheet-antenna when the half-space is not present. The expressions for the incident field components in the fixed reference frame and the co-moving reference frame are obtained as

$$H_y^i(z,t) = -\tfrac{1}{2} I_S[t - (z_2 - z)/c], \qquad E_x^i(z,t) = \tfrac{1}{2}\mu_0 c I_S[t - (z_2 - z)/c]. \tag{2}$$

$$H_y'^i(z',t') = -\tfrac{1}{2}(1+\beta) I_S[t' - (z_2' - z')/c], \qquad E_x'^i(z',t') = \tfrac{1}{2}\mu_0 c(1-\beta) I_S[t' - (z_2' - z')/c], \tag{3}$$

with $c = 1/(\varepsilon_0\mu_0)^{1/2}$ and $\beta = v_z/c$.

The transient reflected field and the transient transmitted field in the co-moving reference frame are expressed as convolution products between the kernel functions that also occur in the stationary case, see [2, 3], and time-dependent reflection or transmission factors. The expressions for the reflected and transmitted field components in the co-moving reference frame are obtained as

$$H_y'^r(z',t') = R(t') \overset{(t')}{*} \delta[t' - (z' - z_1')/c], \qquad E_x'^r(z',t') = \mu_0 c R(t') \overset{(t')}{*} \delta[t' - (z' - z_1')/c]. \tag{4}$$

$$H_y'^t(z',t') = T(t') \overset{(t')}{*} k^H[t', \gamma_1(z_1' - z')], \qquad E_x'^t(z',t') = -Z_1 T(t') \overset{(t')}{*} k^E[t', \gamma_1(z_1' - z')], \tag{5}$$

with

$$k^H(t,\alpha) = \frac{1}{2\pi^{1/2}}\frac{\alpha}{t^{3/2}}\exp(-\frac{\alpha^2}{4t})H(t), \qquad k^E(t,\alpha) = \frac{1}{2\pi^{1/2}}\frac{1}{t^{3/2}}(\frac{\alpha^2}{2t} - 1)\exp(-\frac{\alpha^2}{4t})H(t). \tag{6}$$

The time-dependent reflection or transmission factors have to be calculated from boundary and excitation conditions in the co-moving reference frame

$$\lim_{z\downarrow z_1}(H_y'^i + H_y'^r) - \lim_{z\uparrow z_1} H_y'^t = 0, \qquad \lim_{z\downarrow z_1}(E_x'^i + E_x'^r) - \lim_{z\uparrow z_1} E_x'^t = 0. \tag{7}$$

These conditions lead to the system of equations

$$-\tfrac{1}{2}(1+\beta) I_S[t'(1-\beta) - d_0/c] + R(t') - T(t') \quad = \quad 0, \tag{8}$$

$$\tfrac{1}{2}\mu_0 c(1-\beta) I_S[t'(1-\beta) - d_0/c] + \mu_0 c R(t') + Z_1 T(t') \overset{(t')}{*} k^E(t',0) \quad = \quad 0. \tag{9}$$

Under the condition $t > d_0/(c - v_z)$, the application of the Laplace transform to this system of equations yields

$$-\tfrac{1}{2}\frac{1+\beta}{1-\beta}\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S(\frac{s}{1-\beta}) + \hat{R} - \hat{T} \quad = \quad 0, \tag{10}$$

$$\tfrac{1}{2}\mu_0 c\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S(\frac{s}{1-\beta}) + \mu_0 c\hat{R} + Z_1 s^{1/2}\hat{T} \quad = \quad 0, \tag{11}$$

Under the assumption that $v_z \ll c$ ($\beta \ll 1$), the solutions for $\hat{R}$ and $\hat{T}$ can be approximated by

$$\hat{R} = \tfrac{1}{2}\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S(\frac{s}{1-\beta})\frac{s^{1/2} - a^{1/2}}{s^{1/2} + a^{1/2}} = \tfrac{1}{2}\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S\frac{s}{1-\beta})\hat{k}^R(s,a), \tag{12}$$

$$\hat{T} = -\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S(\frac{s}{1-\beta})\frac{a^{1/2}}{s^{1/2} + a^{1/2}} = -\exp(-\frac{d_0 s}{c - v_z})\hat{I}_S(\frac{s}{1-\beta})\hat{k}^T(s,a), \tag{13}$$

with $a^{1/2} = \mu_0 c/Z_1 = (\sigma_1/\varepsilon_0)^{1/2}$ and $d_0 = z_2 - z_0$. The time-domain counterparts of the new kernel functions, $k^T(t,\alpha)$ and $k^R(t,\alpha)$ are

$$k^T(t,a) = (\frac{a}{\pi t})^{1/2} - a\exp(at)\ \mathrm{erfc}\ [(at)^{1/2}], \qquad k^R(t,a) = -2(\frac{a}{\pi t})^{1/2} - a\exp(at)\ \mathrm{erfc}\ [(at)^{1/2}] = -2k^T(t,a). \tag{14}$$

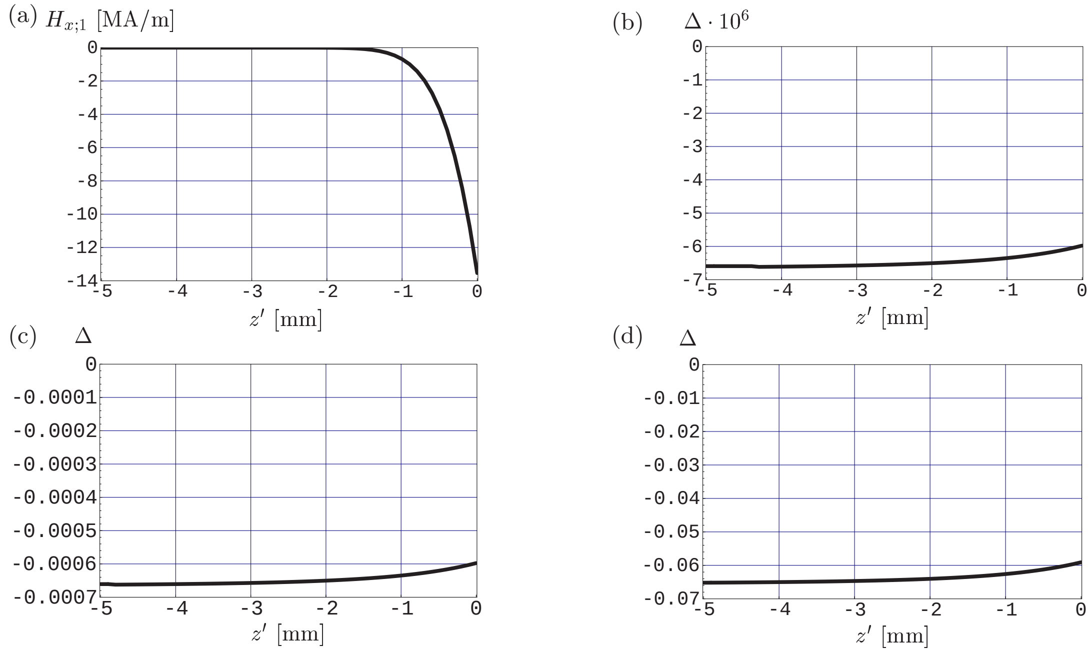

Figure 2: The magnetic field strength in a fixed workpiece at $t = 10$ μs (a), and the relative difference Δ occurring in a workpiece with $v_z = 10^3$ m/s (b), $v_z = 10^5$ m/s (c), and $v_z = 10^7$ m/s (d).

Subsequently, the time-domain reflection and transmission factors are obtained as

$$R(t') = -(1-\beta)I_S[t'(1-\beta)-d_0/(c-v_z)] \overset{(t')}{*} k^T(t',a), \qquad T(t') = -(1-\beta)I_S[t'(1-\beta)-d_0/(c-v_z)] \overset{(t')}{*} k^T(t',a), \tag{15}$$

When at' is very large, the kernel functions may be approximated by delta functions, and the reflection and transmission factors become

$$R(t') = -\tfrac{1}{2}(1-\beta)I_S[t'(1-\beta) - d_0/(c-v_z)], \qquad T(t') = -(1-\beta)I_S[t'(1-\beta) - d_0/(c-v_z)]. \tag{16}$$

In the further calculations for the electromagnetic deformation process, we need the transmitted electromagnetic field in the conducting halfspace expressed in the co-moving coordinates, so

$$H'_{y;1}(z',t') = H'^t_y(z',t'), \qquad E'_{x;1}(z',t') = E'^t_x(z',t'), \tag{17}$$

while the electromagnetic field in the air-gap needs to be expressed in the fixed coordinates using the Lorentz transformations, giving

$$H_{y;2}(z,t) = H^i_y(z,t) + H^r_y(z,t), \qquad E_{x;2}(z,t) = E^i_x(z,t) + E^r_x(z,t), \tag{18}$$

$$H^r_y(z,t) = H'^r_y(z,t) + \varepsilon_0 v_z E'^r_x(z,t), \qquad E^r_x(z,t) = E'^r_x(z,t) - \mu_0 v_z H'^r_y(z,t). \tag{19}$$

NUMERICAL RESULTS

In this section, numerical results for a particular configuration relating to electromagnetic forming are presented for several velocities v_z of the conducting half-space. The conducting half-space, with conductivity $\sigma_1 = \sigma_{Al} = 3.6 \times 10^7$ S/m and permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m, (domain $\mathcal{D}_1$) is placed in a lossless medium (domains $\mathcal{D}_2$ and $\mathcal{D}_3$) with conductivity $\sigma_2 = \sigma_3 \to 0$ and permeability μ_0. The sheet-antenna is placed at the interface z_2 and carries a surfcae current density $\boldsymbol{J}^S_2 = -I_S(t)\boldsymbol{i}_x$, where $I_S(t)$ is a typical pulse encountered in the RLC-series circuit of a real electromagnetic forming device. In Fig. 2a the magnetic field strength at $t = 10$ μs in a fixed workpiece that is initially 5 mm below the sheet antenna is plotted versus the depth coordinate z'. Figs. 2b - 2d show the relative changes in the magnetic field strength when the velocity v_z of the workpiece is 10^3 m/s, 10^5 m/s, and 10^7 m/s, respectively. Figure 3 shows the same quantities as Fig. 2, but

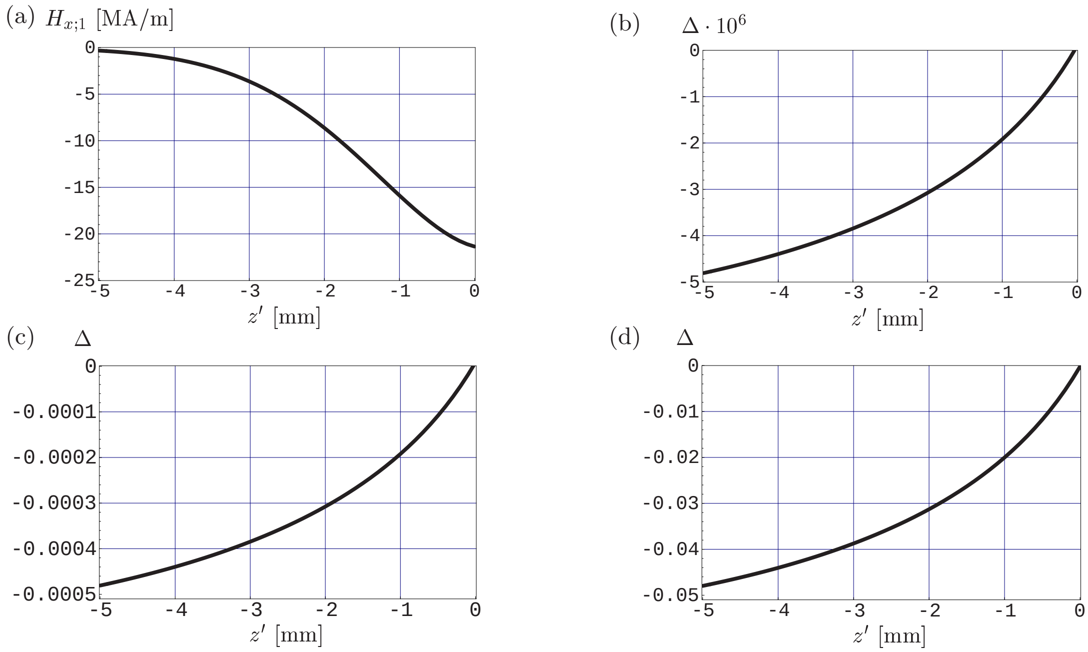

Figure 3: The magnetic field strength in a fixed workpiece at $t = 100$ μs (a), and the relative difference Δ occurring in a workpiece with $v_z = 10^3$ m/s (b), $v_z = 10^5$ m/s (c), and $v_z = 10^7$ m/s (d).

now for the time instant $t = 100$ μs.

CONCLUSIONS AND DISCUSSION

The numerical results show that the electromagnetic field in a moving, conducting half-space varies from the values obtained in a stationary, conducting half-space. At a time instant of 100 μs, the differences are only significant if the velocity is on the order of 10^7 m/s or more. Moreover, the differences decrease with with decreasing time. In electromagnetic forming, the deformation process takes place in a time interval in the order of 100 μs. The velocities are in the order of 10^3 m/s, i.e. far below the value for which motion of the halfspace becomes significant.Thus, for electromagnetic forming the electromagnetic field calculated in stationary media is a good approximation of the real electromagnetic field in the configuration.

References

[1] Bendjima, B., Srairi, K., Feliachi, M., “A Coupling Model for Analyzing Dynamical Behaviours of an Electromagnetic Forming System”, in *IEEE Transactions on Magnetics*, vol. 33, no. 2, March 1997.

[2] de Hoop, A.T., Oristaglio, M.L., “Application of the modified Cagniard technique to transient electromagnetic diffusion problems”, in *Geophysical Journal* Vol. 94, 1988.

[3] de Hoop, A.T., “Transient diffusive electromagnetic fields in stratified media: Calculation of the two-dimensional *E*-polarized field”, in *Radio Science* Vol. 35, No. 2, 2000.

[4] Lee, S.H., Lee, D.N., “Finite element analysis of electromagnetic forming for tube expansion”, in *Journal of Engineering Materials and Technology, Transactions of the ASM*, vol. 116, no. 2, April 1994.

[5] Maouche, B., Feliachi, M., “A discretized integral method for eddy current computation in moving objects with the coexistence of the velocity and time terms”, in *IEEE Transactions on Magnetics*, vol. 34, no. 5, September 1998.

[6] Takatsu N., Kato, M., Sato, K., Tobe, T., “High-speed forming of metal sheet by electromagnetic force”, in *JSME International Journal*, series III, vol. 31, no. 1, 1988.