TLM SIMULATION

OF ELECTROMAGNETIC WAVE PROPAGATION

IN MOVING MEDIA

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ABSTRACT

A TLM (Transmission Line Matrix) model suitable to simulate the propagation of waves in moving continuous media is presented. As it is well known, an electromagnetic wave propagating in a general medium, moving with respect to its source, experiences a drag by the own medium, which involves a wave velocity dependent on the direction of propagation. In this work, the case of uniform movement of a medium with respect to an electromagnetic source is considered, and a special attention is devoted to the simulation of wave fronts (no concentric circular fronts, in an isotropic medium) because a kind of "anisotropy" appears.

INTRODUCTION

Space and time are on the basis of the description of every physical event. The TLM method is a numerical technique, which allows solving the wave equation of a propagation phenomenon in time domain, [1]. In TLM method, the space is discretized in an interconnected transmission lines network, each connection called « node ». The voltage and current pulses propagates through the transmission lines to the neighbour nodes in a specified time step.

The TLM algorithm is strongly dependent on the synchronism of pulse propagation and on the network geometry : in this way, the modelling of complex media is often a very uneasy task. In order to simulate the proposed electromagnetic problem, we start with a previous modelling technique suitable for anisotropic media : the permittivity tensor of such media means the coupling of electric field vector components. TLM allows modelling such a coupling by adding new components to the circuit representation of the transmission lines network. For an anisotropic medium, this is achieved by adding a voltage source to each node on the mesh, accounting for the coupling of fields.

Actually, the propagation speed in an anisotropic medium depends on the direction, the phenomenon being then different to the one, which happens, in an isotropic moving media : Let's consider a plane wave linearly polarised in an anisotropic medium. Its speed depends on the polarization of its fields. On the contrary, in an isotropic moving medium, with respect to the source the speed varies in both directions (forward and backward), [2].

PLANE WAVES IN MOVING MEDIA

The Lorentz Transformation (LT)

As it is well known from the special relativity, the space and time coordinates for different observers in inertial frames, are transformed through the Lorentz transformation, giving rise to the invariance of physical laws. Suppose that for an observer S, the electrodynamics laws are described by the usual Maxwell equations. For any other observer in an inertial frame S' moving with respect to S, the electrodynamics equations are still the same and the effect of the movement is included in the field transformation and constitutive relations. As a direct consequence of the invariance, the speed of light c is a constant in both S and S'.

Let's consider the simplest case where both coordinate systems S and S' lies parallels to each other and S' moves uniformly with respect to S with a speed less than *c*, expressed through β:

$$\bar{v} = \bar{\beta}\, c = \bar{\beta} \big/ \sqrt{\varepsilon_o \mu_o} \qquad (1)$$

If Maxwell equations are to be invariant under LT, the field vectors must be transformed following [2]:

$$\overline{E}' = \left(\overline{E} + \overline{\beta} \times c\overline{B}\right) \Big/ \sqrt{1-\beta^2} \qquad c\overline{D}' = \left(c\overline{D} + \overline{\beta} \times \overline{H}\right) \Big/ \sqrt{1-\beta^2}$$
$$c\overline{B}' = \left(c\overline{B} - \overline{\beta} \times \overline{E}\right) \Big/ \sqrt{1-\beta^2} \qquad \overline{H}' = \left(\overline{H} - \overline{\beta} \times c\overline{D}\right) \Big/ \sqrt{1-\beta^2} \tag{2}$$

It must be noted that the field components parallel to the direction of relative motion are not transformed at all.

Let's ε and μ denote the permittivity and permeability respectively of an isotropic medium, as measured in the rest frame S. The observer S' establishes then the following equations as constitutive relations :

$$\overline{D} + \varepsilon_o \mu_o \, \overline{v} \times \overline{H} = \varepsilon \, \overline{E} + \varepsilon \, \overline{v} \times \overline{B}$$
$$\overline{B} - \varepsilon_o \mu_o \, \overline{v} \times \overline{E} = \mu \, \overline{H} - \mu \, \overline{v} \times \overline{D} \tag{3}$$

Bidimensional Wave Equation (2D)

The resulting equations (2) and (3), allows to obtain the wave equation describing the propagation in the frame where the source fields are in rest. Now, let's consider the propagation in an isotropic medium, with permittivity ε and permeability μ, if the sources are moving uniformly along the X axis, as:

$$\overline{\beta} = \beta \, \hat{u}_x \tag{4}$$

For the observer S', the wave equation for each field component normal to X (lying in the YZ plane) are wrote in the same way, i.e. : for the z component of the electric field:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{1-\beta^2}{1-\varepsilon_r \mu_r \beta^2} \frac{\partial^2 E_z}{\partial y^2} - 2\beta \frac{\varepsilon_r \mu_r - 1}{1-\varepsilon_r \mu_r \beta^2} \frac{\partial^2 E_z}{\partial x \partial t} = \varepsilon_o \mu_o \frac{\varepsilon_r \mu_r - \beta^2}{1-\varepsilon_r \mu_r \beta^2} \frac{\partial^2 E_z}{\partial t^2} \tag{5}$$

Now we ask what are the consequences we can get from such equation related to the wave propagation. To do so, we start with the explicit equation for a rest medium, $\beta = 0$,:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \varepsilon\mu \frac{\partial^2 E_z}{\partial t^2} \tag{6}$$

In comparing (5) and (6), we find two remarkable points in (5).
Firstly, the second space order space-time derivative (cross-derivative) accounts for the inertia of the medium. The origin of the wave fronts is dragged in time.
Secondly, the coefficients of the second order time derivative are differently « weighted » in (5) and (6) as the coefficients of the second order space derivative, normal to X, are too. This is equivalent to an « effective elasticity » and anisotropy giving rise to elliptic wave fronts.
The solution of the wave equations is a combination of characteristic waves with different phase speeds, corresponding to the propagation forward and backward. The propagation is equivalent to the superposition of waves with speeds depending on the propagation direction :

$$\frac{1 \pm \sqrt{\varepsilon_r \mu_r}\,\beta}{\sqrt{\varepsilon_r \mu_r} \pm \beta} \frac{1}{c} \tag{7}$$

TLM MODELING

We will extend the TLM method to simulate the wave propagation when its source moves uniformly with respect to an isotropic medium. The starting point is to discretize the field equations (either Maxwell or wave equations) for the field component normal to X, (5), then to compare it with the equations modelled by the TLM algorithm. The terms in the

discretized wave equation having no representation in the mesh equations, will then be accounted with new elements added to the network nodes as voltage sources, its value being updated each time iteration.
Let's go back to the propagation of a wave in an isotropic uniformly moving medium parallel to X-axis, this is the already presented problem. The X field components are not transformed at all, as shown, then we deal with the other components normal to X. We point, for instance to the Ez electric field component, normal to XY plane.
The basic 2D TLM node is a parallel connection of transmission lines with characteristic admittances Y_x and Y_y, with a permittivity stub with admittance Y_o, and a series connected voltage source V_s [3]. Then, the field component Ez is represented in the TLM mesh by the total voltage V_z at the node. From the equations of voltage and current at the TLM node, we can write an equation for the total voltage V_z depending on the voltages at neighbouring nodes and the time variation of the source [4]. In this way the values of the elements of the equivalent circuit are related with the values of the electromagnetic problem through a relation that can be easily found by comparing it with the discretized form of (5).

The results of this anisotropic modelling are the different values of propagation speeds in the transmission lines, which depends on the rate of the real constitutive parameters to the effectives. The arms and stub normalized admittances are:

$$Y_x = \frac{1-\varepsilon_r\mu_r\beta^2}{1-\beta^2} \qquad Y_y = 1 \qquad Y_o = 4\left(\frac{\varepsilon_r\mu_r-\beta^2}{1-\beta^2} - \frac{Y_x+Y_y}{2}\right) \tag{8}$$

The voltage source, which includes the cross-derivatives of the total voltage at the node, and is denoted by the symbol Δ^2, is updated each time iteration following the rule, at time step $k+1$:

$$_{k+1}V_s = {}_{k-1}V_s - \sqrt{2}\beta\,\frac{\varepsilon_r\mu_r - 1}{\varepsilon_r\mu_r-\beta^2}\,\Delta^2_{xt}\left({}_{\mathrm{k}}V_z\right) \tag{9}$$

When modelling the propagation in an unbounded medium we need to add an artificial absorbing boundary condition in the numerical limits of the simulation domain. This is achieved by introducing, in the limiting nodes, a reflection coefficient, which matches the admittances. So, different values are needed for the YZ and YX planes. The effective speeds of propagation are given by:

$$\left.\frac{1}{\sqrt{\varepsilon_r\mu_r}}\right|_{xef} = \sqrt{\frac{2Y_x}{Y_x+Y_y+Y_o/2}} \qquad \left.\frac{1}{\sqrt{\varepsilon_r\mu_r}}\right|_{yef} = \sqrt{\frac{2Y_y}{Y_x+Y_y+Y_o/2}} \tag{10}$$

The calculation of the reflection coefficients in those nodes follows the same general sketch than in the TLM modelling of anisotropic media.

There is still a very important question that has been not treated yet, which regards the limits of relative speeds β to be simulated, and which must be always less than c. Due to the numerical dispersion intrinsic to TLM method, the realistic situation where β is greater than the speed of light in the isotropic medium cannot be studied with this modelling. This is not a problem for electromagnetic waves, but can be in the modelling of other wave phenomenon such as acoustics.:

$$\beta^2 < (\varepsilon_r\mu_r)^{-1} \tag{11}$$

This condition ensures the positive values of the transmission lines admittances and stubs in the TLM mesh, as is needed for a correct march-in-time algorithm.

RESULTS

We have simulated the wave propagation in an isotropic uniformly moving medium, with respect to the electromagnetic source, using a 2D–TLM model.

In an isotropic medium in rest, with respect to the source, the wave fronts are circles around the point source, always with the same space point. When there is relative movement, the wave fronts are no longer circles and moreover, the origin of each wave front is dragged. A time domain analysis has been performed, in order to verify the TLM results.

They are shown in the following figures (Fig. 1, Fig. 2 and Fig. 3), with $\varepsilon_r\mu_r = 2$ and β values as indicated. The source is located at the cross point of the lines, and the excitation is a harmonic signal.

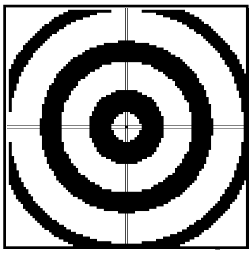

Fig. 1. Wave fronts with $\beta = 0$.

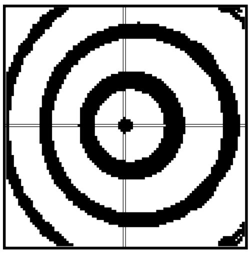

Fig. 2. Wave fronts with $\beta = 0'2$.

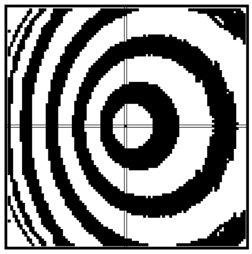

Fig. 3. Wave fronts with $\beta =0'4$.

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