A SPATIAL-FREQUENCY FILTER FOR TIME-DOMAIN FAR FIELDS

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MOTIVATION

For antenna and scattering problems, most numerical methods do not directly compute the far field; this has to be calculated by post processing or -in some cases- parallel to the progress of the numerical algorithm. Usually the far field is obtained from the near-field data by employing the (vector) Green's second identity and the (dyadic) free-space Green's function in it's closed form, which is a function of the distance between the far-field observation point and the near-field source point. Consequently, in general any new observation point requires a new integration over all near-field points. It has been shown that the vector spherical-multipole (bilinear) expansion of the dyadic Green's function can be applied to perform a near-to-far-field transformation (NFT) suitable for numerically and for asymptotically determined near-field data in the frequency domain as well as in the time domain [2]. With once obtain multipole amplitudes we have the result in form of a simple multipole expansion valid at any far-field point. Moreover, as will be the topic in this presentation, the method allows the application of a spatial-frequency filtering technique, a convenient way to improve the far-field data by post processing. Exemplarily we will treat the time domain NFT for the Finite-Difference Time-Domain (FDTD) method and show how to improve the time domain far-field data.

TIME DOMAIN NEAR-TO-FAR-FIELD TRANSFORMATION

Outside a sphere which contains all the scattering sources any electromagnetic field can be represented by means of a spherical-multipole expansion. For far-field observation points we obtain at a time factor $e^{+j\omega t}$ the series

$$\vec{E}_\infty(\vec{r},\omega) = \frac{e^{-j\kappa r}}{\kappa r}\left[-\sum_{n=1}^{\infty}\sum_{m=-n}^{n} j^n A_{nm}(\omega)\vec{n}_{nm}(\vartheta,\varphi) + Z_0\sum_{n=1}^{\infty}\sum_{m=-n}^{n} j^n B_{nm}(\omega)\vec{m}_{nm}(\vartheta,\varphi)\right] \quad (1)$$

$$\vec{H}_\infty(\vec{r},\omega) = \frac{e^{-j\kappa r}}{\kappa r}\left[-\frac{1}{Z_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n} j^n A_{nm}(\omega)\vec{m}_{nm}(\vartheta,\varphi) - \sum_{n=1}^{\infty}\sum_{m=-n}^{n} j^n B_{nm}(\omega)\vec{n}_{nm}(\vartheta,\varphi)\right], \quad (2)$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance, $\kappa = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber, and $\vec{m}_{nm}$ and $\vec{n}_{nm}$ are orthogonal functions related to the surface spherical-harmonics $Y_{nm}(\vartheta,\varphi)$ by $\vec{m}_{nm} = -(1/\sin\vartheta)(\partial Y_{nm}/\partial\varphi)\hat{\vartheta} + (\partial Y_{nm}/\partial\vartheta)\hat{\varphi}$ and $\vec{n}_{nm} = +(\partial Y_{nm}(\vartheta,\varphi)/\partial\vartheta)\hat{\vartheta} + (1/\sin\vartheta)(\partial Y_{nm}(\vartheta,\varphi)/\partial\varphi)\hat{\varphi}$, respectively. The unknowns in the series, that are the multipole amplitudes A_{nm} and B_{nm}, are obtained using the spherical-multipole interface by the following procedure [1]: (a) Replace the equivalent electric and magnetic currents on an arbitrary surface which encloses the scattering object by appropriately chosen electric and magnetic elementary dipoles; (b) calculate the multipole amplitudes of each dipole by employing the bilinear form of the free-space dyadic Green's function; (c) add the related multipole amplitudes and obtain the multipole amplitudes of the field.

The first advantage of this NFT procedure is obviously, that it has to be performed only once, i.e., independent of the desired far-field observation points. Moreover, as can be deduced from the behavior of the spherical Bessel functions, the number of relevant multipole amplitudes only depends on the electrical size of the scatterer. As a rule of thumb we choose for the upper order n_{max} of the multipole expansion that

$$n_{max} > \kappa r_{max} + 5, \quad (3)$$

where r_{max} is the maximal distance of the scatterer's surface to a well-chosen origin. The multipole amplitudes with higher order than n_{max} are neglected, since they can correspond only to spurious or erroneous field parts. This omission of the higher-order multipole terms can be interpreted as a spatial-frequency filtering.

Now the inverse Fourier transform of (1) und (2) yields with respect to (3)

$$\vec{e}_\infty(\vec{r},t) = -\sum_{n=1}^{n_{max}}\sum_{m=-n}^{n} a_{nm}(t-\frac{r}{c})\vec{n}_{nm}(\vartheta,\varphi) + Z_0\sum_{n=1}^{n_{max}}\sum_{m=-n}^{n} b_{nm}(t-\frac{r}{c})\vec{m}_{nm}(\vartheta,\varphi) \quad (4)$$

$$\vec{h}_\infty(\vec{r},t) = -\frac{1}{Z_0}\sum_{n=1}^{n_{max}}\sum_{m=-n}^{n} a_{nm}(t-\frac{r}{c})\vec{m}_{nm}(\vartheta,\varphi) - \sum_{n=1}^{n_{max}}\sum_{m=-n}^{n} b_{nm}(t-\frac{r}{c})\vec{n}_{nm}(\vartheta,\varphi), \quad (5)$$

where the time-domain multipole amplitudes a_{nm} and b_{nm} can be calculated via a convolution integral from the time-domain electromagnetic field. This integral can be calculated 'on the fly' during each time step of an FDTD algorithm. Since the FDTD method only yields the electric field at discrete time-steps, we employ a linear time-domain interpolation, similar to the procedure described for the FDTD analysis of fields in dispersive media [5]: For instance, the convolution integral for the multipole amplitudes a_{nm} is given with respect to the causality principle by

$$a_{nm}(k\Delta t) = a_{nm}^k = \int_0^t \vec{\alpha}_{nm}(t')\cdot\vec{c}(k\Delta t - t')\,dt', \quad (6)$$

where the first convolution partner $\vec{\alpha}_{nm}$ can be calculated analytically and is related to the Fourier transform of the spherical Bessel function [2], while the second convolution partner $\vec{c}$ denotes the 'current moment' of an equivalent electric or magnetic elementary dipole which is linearly related to the field at discrete time-steps and which is provided by the FDTD algorithm. With the following linear approximation, valid in the interval $l\Delta t \le t' \le (l+1)\Delta t$

$$\vec{c}(k\Delta t - t') = \vec{c}^{\,k-l} + \frac{1}{\Delta t}\left(\vec{c}^{\,k-(l+1)} - \vec{c}^{\,k-l}\right)(t' - l\Delta t)\,,$$

we are lead to the expression

$$a_{nm}^k = \sum_{l=0}^{k}\vec{c}^{\,k-l}\cdot\int_{l\Delta t}^{(l+1)\Delta t}\vec{\alpha}_{nm}(t')dt' + \frac{1}{\Delta t}\left(\vec{c}^{\,k-(l+1)} - \vec{c}^{\,k-l}\right)\cdot\int_{l\Delta t}^{(l+1)\Delta t}(t'-l\Delta t)\,\vec{\alpha}_{nm}(t')dt'\ . \quad (7)$$

Both integrals in (7) can be evaluated analytically. Analogous expressions are obtained for the multipole amplitudes a_{nm}^k.

NUMERICAL RESULTS

Consider a perfectly electrically conducting (PEC) half-sphere (radius 0.2 m) illuminated by a Gaussian-type modulated plane electromagnetic wave (centre frequency is 505 MHz; damping is 25 dB at 10 MHz and at 1 GHz), which is symmetrically incident on the spherical side of the half sphere. The problem has been discretized into 51x51x51 cubes with an edge length of each 0.01 m and solved with an FDTD code, which provides a PML. The far field was obtained by the spherical-multipole interface, where a cubic surface spaced 3 cells from the PML was chosen for the equivalent-dipoles' locations.

To demonstrate the versatility of the method, Fig. 1 shows the time-angle representation of the scattered electric far field, which has been obtained purely by a post processing of the time-domain multipole amplitudes.

Since an analytic reference solution to the problem is available [4], the same example is considered for demonstrating how the accuracy of the numerical results can be improved by means of the spatial-frequency filtering technique. If an upper bound for the time spectrum band of 1.0 GHz is assumed, the maximal order of the multipole expansion given by $n_{max} = 6$. Figures 2 and 3 show the normalized forward scattered far field calculated by using 6 and 16 multipole amplitudes, respectively. Compared to the analytical solution and to the result obtained by the standard NFT technique based on the same near-field data, it is seen that the multipole solution corresponds better to the analytical result in case of $n_{max} = 6$, while -as expected- it compares better to the standard result if $n_{max} = 16$.

CONCLUSIONS

A procedure has been described to perform a time-domain near-to-far-field transformation by means of a spherical-multipole technique. The method allows to systematically manipulate the spatial spectrum of the far-field results by

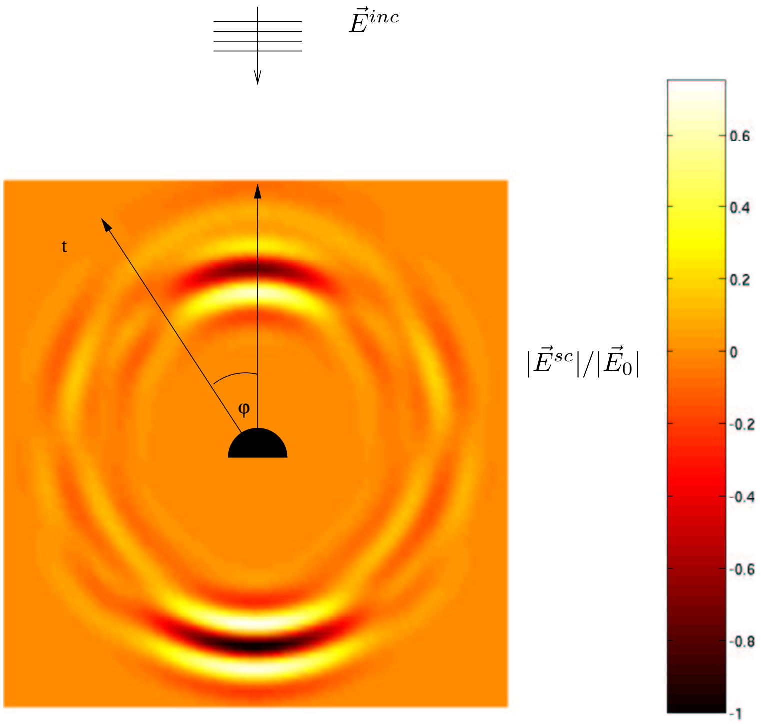

Fig. 1: Time-angle representation of the scattered far-field for a Gaussian-type modulated plane electromagnetic wave scattered by a PEC half sphere; FDTD-calculation with multipole-interface postprocessing

neglecting the unphysical higher-order terms. First numerical results from the application to an FDTD algorithm confirm the proposed behavior. Among other items, future work will focus on the determination of the origin of these higher-order terms in the numerical process.

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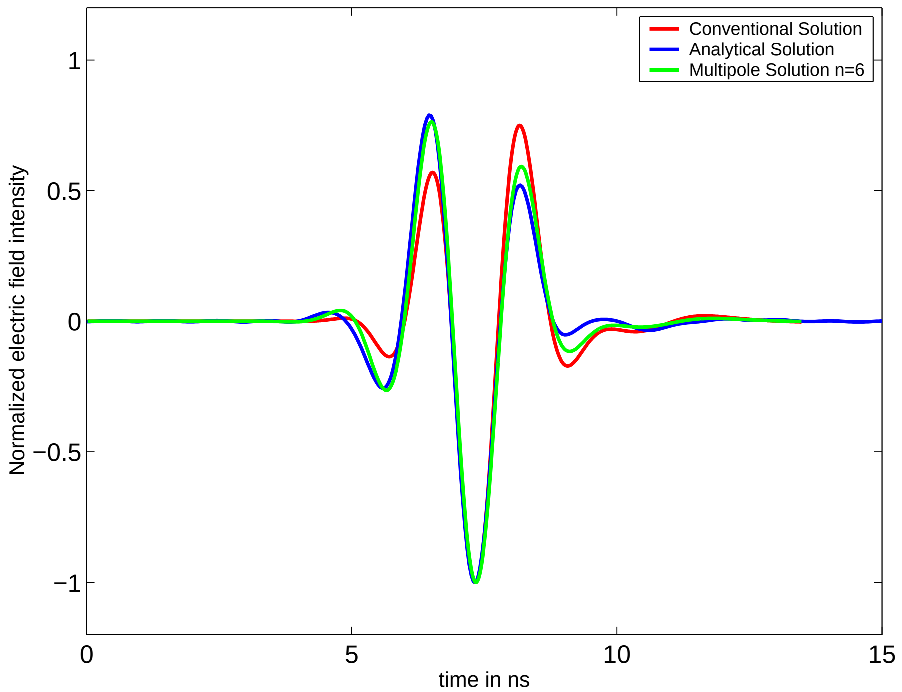

Fig. 2: Normalized forward-scattered far-field for a Gaussian-type modulated plane wave, symmetrically incident on the spherical part of a PEC half-sphere; Maximal order of the multipole-interface solution: n_{max} =6

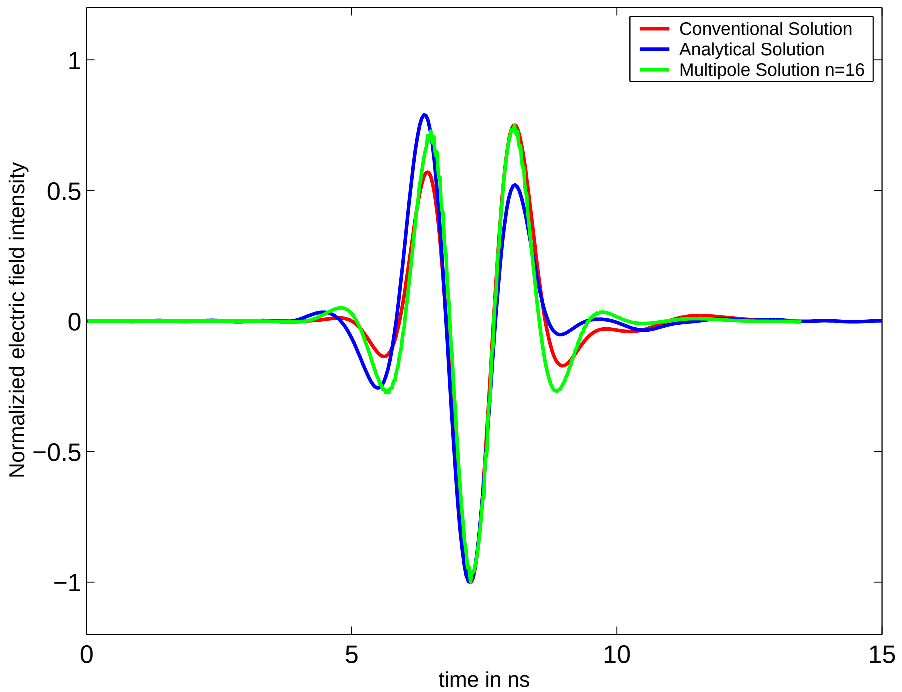

Fig. 3: Normalized forward-scattered far-field for a Gaussian-type modulated plane wave, symmetrically incident on the spherical part of a PEC half-sphere; Maximal order of the multipole-interface solution: n_{max} =16