DYNAMICAL SPATIAL LOG-NORMAL SHADOWING MODELS FOR MOBILE COMMUNICATIONS

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Abstract - Large-scale wireless channel models are commonly used to describe the power-loss of signals, when the transmitter-receiver separation distance is large and the propagation environment is not heavily populated. In mobile communications, the relative movement between the transmitter and the receiver and dynamic variations in the propagation environment, contribute to the dynamical spatial characteristics of the channel. This paper introduces linear stochastic differential equations which describe the dynamical spatial characteristics of power-loss, which in turn give rise to log-normal densities for the signal attenuation coefficient. These models are easy to analyze and implement. Several simulation scenarios are presented to illustrate the concepts.

1 Introduction

Large-scale channel models are suited for describing the total power-loss of the signal when the transmitter-receiver separation distance is large and the propagation environment is not heavily populated such as suburban areas [1, 2]. They attempt to predict the reflection/diffraction and long distance power-loss as depicted in Figure 1.1 A. This power-loss is the cause for signal attenuation, generally known as shadowing. It has been reported in [1, 2] that the power loss in dB's along a single path is normally distributed, whereas the signal envelop is log-normally distributed. These models [3, 4, 5, 6], treat power loss as static. They do not take into consideration the relative motion between the transmitter and the receiver, or variations of the propagating environment which is a reality in mobile communications. The development of the dynamical spatial models, presented in this paper, is based on the traditional log-normal shadowing models and one way to model them is through stochastic processes.

Average power path-loss, at distance d, in decibels is given by

$$\overline{PL}_d(d)[dB] = \overline{PL}(d_0)[dB] + 10\alpha \log(d/d_0), \qquad d \geq d_0, \tag{1.1}$$

where $\overline{PL}(d_0)[dB] = 10 \log P_t - 10 \log P(d_0)$ is the power-loss in dB's at a reference distance d_0 [1, 2], P_t is the power of the transmitted signal and α is the path-loss exponent which depends on the propagation medium. Real measurements indicate further that this relationship is best described by adding to equation (1.1) a zero-mean Gaussian distributed random variable, $\tilde{X}$, ([2] p. 104), which can be interpreted as representing the variability of power-loss of transmitter-receiver configurations located at different places,

$$PL(d)[dB] = \overline{PL}_d(d)[dB] + \tilde{X}, \qquad d \geq d_0, \tag{1.2}$$

Signal Attenuation Coefficient. The signal attenuation coefficient represents how much the received signal magnitude is attenuated at a distance d, with respect to the magnitude of the transmitted signal [2]. It is defined by

$$r(d) \stackrel{\triangle}{=} \sqrt{P(d)/P_t} = e^{k\overline{PL}_d(d)[dB]} e^{k\tilde{X}}, \qquad k = -(\ln 10)/20. \tag{1.3}$$

From Static to Dynamical Log-Normal Shadowing Models. In converting the static models to dynamical models, we first note that in mobile communications, the distance d is necessarily a function of time, $d(t)$, and the random

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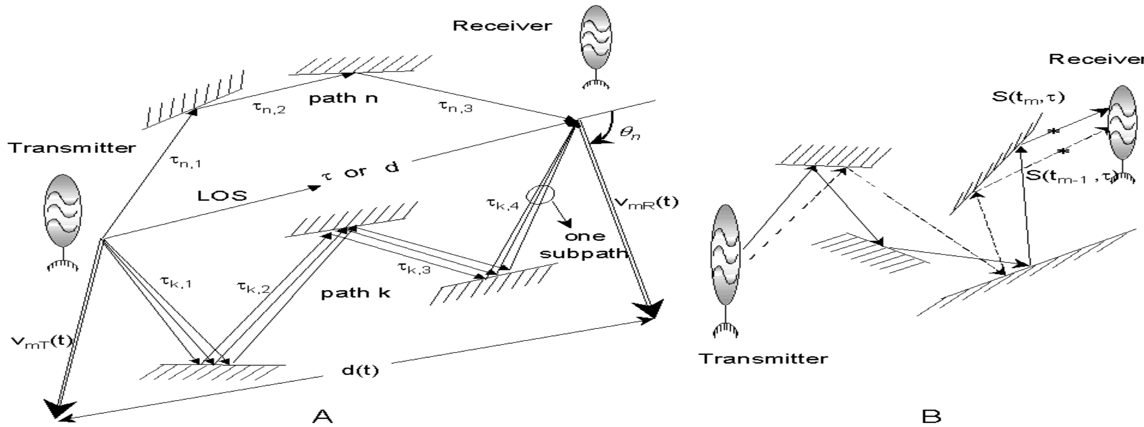

Figure 1.1: A: Large-Scale Channel Model illustrating a number of possible propagation paths and a relative movement between the transmitter and the receiver. B: Mean-reverting log-normal model; for fixed τ, observe at different time instants.

attenuation coefficient, $r(d)$ in (1.3), is relaxed to become a random process, which is denoted by $\{r(t,\tau)\}_{t\geq 0,\tau\geq\tau_0}$, with respect to the observation time, t, and spatial location, d, or equivalently the time-delay, τ, since $d = v_c\tau$, where v_c is the speed of light. Similarly, the power path loss $PL(d)[dB]$ in (1.2), becomes a random process, which is denoted by $\{X(t,\tau)\}_{t\geq 0,\tau\geq\tau_0}$. Here we present the case where for fixed transmitter-receiver separation distance, d, the dependence of $\{r(t,\tau)\}_{t\geq 0}$ on t captures the random attenuation coefficient variations with respect to time. When expressed in dB's the power path loss, $\{X(t,\tau)\}_{t\geq 0}$, is described by an Ornstein-Uhlenbeck process with respect to the variable t, which has a mean-reverting structure. It is further described [7] by stochastic differential equation (SDE) whose solution, at every instant of time, represents the correlation properties both in time and space of the channel, and they correspond to the statistics of the static log-normal shadowing model [1, 2, 6, 8, 9].

2 Dynamical Temporal Log-Normal Model

Let $\big(\Omega,\mathcal{F},P\big)$ denote a basis probability space and $\{\mathcal{F}_t\}_{t\geq 0}$ denote a complete filtration, on which all random processes will be defined. The dynamical temporal propagation characteristics of the channel, capture the dependence of $\{r(t,\tau)\}_{t\geq 0}$ on t in the case where τ is fixed via a SDE as a function of t.

Consider the transmission of a continuous signal arriving at the receiver via a path of length d which is kept fixed, under the assumption that it undergoes many reflections along the way. We examine the attenuation coefficients at time instants $0 < t_1,\ldots,< t_m$. Let $S(t_j,\tau)$ denote the attenuation coefficient associated with time t_j, $1\leq j\leq M$, along that path as shown in Figure 1.1 B. Here $S(t_j,\tau)$ is proportional to the power-loss due to reflections, $\Pi_{m=1}^{M_j}P_{\ell r_m}$, where M_j is the number of reflections up to time t_j and $\sqrt{1/P_{\ell r_m}}$ is the attenuation coefficient corresponding to the mth reflection along path d. Suppose that the propagation environment from the transmitter up to location d is such that a subset of the attenuation coefficients, i.e. $\sqrt{1/P_{\ell r_m}}$, associated with $S(t_j,\tau)$, correspond to those of $S(t_{j-1},\tau)$, $1\leq j\leq M_j$. For fixed τ, define $\Delta S(t_j,\tau)\stackrel{\triangle}{=}S(t_{j+1},\tau)-S(t_j,\tau)$. Then (to a first-order approximation) the percent change of the attenuation coefficient $\Delta S(t_j,\tau)/S(t_j,\tau)$

$$\frac{S(t_2,\tau)-S(t_1,\tau)}{S(t_1,\tau)},\ldots,\frac{S(t_{m+1},\tau)-S(t_m,\tau)}{S(t_m,\tau)},$$

are independent random variables. It is thus reasonable to assume that these percent changes decompose into two components, a systematic part (a drift) and a random part (diffusion), the former representing the evolution of the local mean, while the latter the variability around the local mean respectively. This suggests that the evolution of the attenuation coefficients $\{S(t_j,\tau)\}$ follows the geometric (or exponential) Brownian motion (GBM [10] p. 62) for the attenuation coefficients with respect to the variable t. Further, for fixed τ, we can let $S(t,\tau)=e^{kX(t,\tau)}$, where $\{X(t,\tau)\}_{t\geq t_0}$, the power loss process, is a Brownian motion with non-zero drift, examined as a function of time.

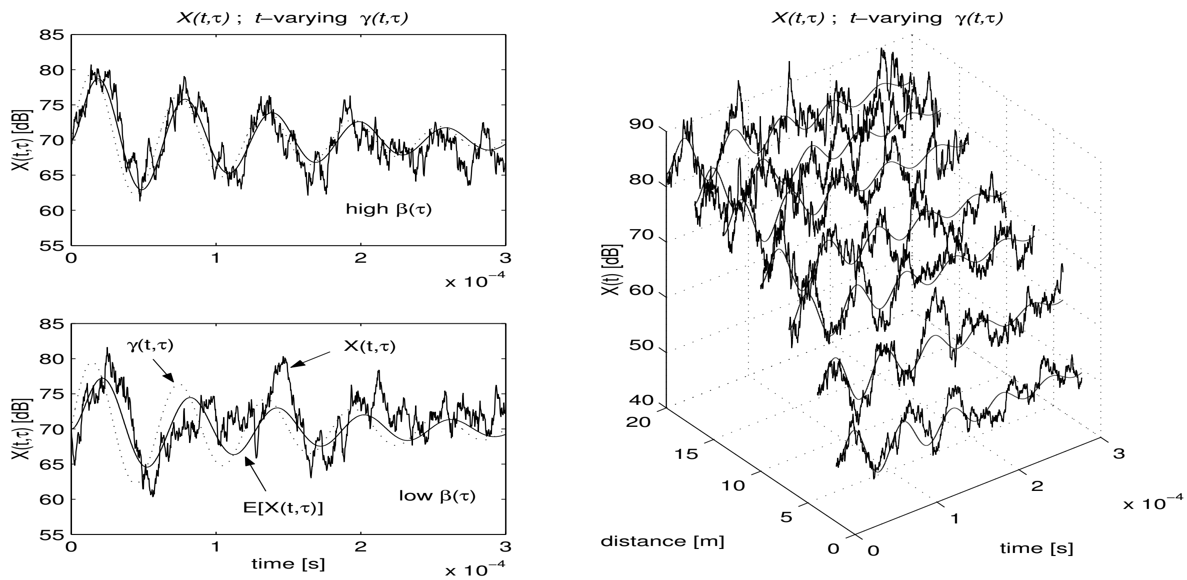

Figure 2.2: Left: Mean-reverting power path-loss and corresponding signal attenuation coefficient a function of time, t, for a given t-varying $\gamma(t,\tau)$ corresponding to an observation location $d = v_c\tau$; top graph: $\beta(\tau) = 225000$, lower graph: $\beta(\tau) = 105000$ Right: Mean-reverting power path-loss a function of t and τ, for a given t-varying $\gamma(t,\tau)$

$X(t,\tau)$ is the analog to $PL(d)[dB]$ in (1.2) for fixed $d = v_c\tau$ indexed by time, t. We thus choose to generate $X(t,\tau)$ by a mean-reverting version of a general linear SDE given by

$$dX(t,\tau) = \beta(t,\tau)\Big(\gamma(t,\tau) - X(t,\tau)\Big)dt + \delta(t,\tau)dW(t), \quad X(t_0,\tau) = N(\overline{PL}_d(d)[dB];\sigma_{t_0}), \tag{2.4}$$

where $N(\alpha;\beta)$ denotes a Gaussian random variable with mean α and variance β and $\{W(t)\}_{t\geq t_0}$ is a standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t,\tau_0)$. In particular, $\gamma(t,\tau)$ models the average time-varying power path-loss at a distance d from the transmitter, which corresponds to $\overline{PL}_d(d)[dB]$ indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)(\gamma(t,\tau)-X(t,\tau))$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift. The initial condition $X(t_0,\tau)$ can be obtained by estimating the average power path loss at distance d, say using (1.1). The random parameters $\{\theta(t,\tau)\}_{t\geq t_0} \triangleq \{\beta(t,\tau),\ \gamma(t,\tau),\ \delta(t,\tau)\}_{t\geq t_0}$ can be used to model the time-varying characteristics of the channel for the particular location, τ, i.e. variations in the number of reflections along the path up to d and corresponding power loss. A different location is characterized by a different set $\{\theta(t,\tau)\}$. The solution of (eq3.25a) and first and second moments can be easily obtained [7].

Example. Figure 2.2 corresponds to simulations of (2.4) for a fixed location d from the transmitter for the two different $\beta(\tau)$'s. The two graphs on the left of this figure illustrate, as expected, that the larger the value of $\beta(\tau)$ the faster the trajectories of power path loss $X(t,\tau)$ is pulled towards the mean of the process, and as time progresses towards $\gamma(t,\tau)$. On the right this figure presents simulations of the power loss as a function of t and τ. Each curve corresponds to simulations of (2.4) with a time varying $\gamma(t,\tau)$ of the form as presented above but with different γ's, thus each curve corresponds to a different transmitter-receiver distance.

3 Dynamical Spatial Log-Normal Model

Consider next the case where both the transmitter and the receiver move relative to each other each with an arbitrary speed and in an arbitrary direction. Suppose also that we know their vector velocities given by where $\vec{v_{mR}}\ (t) = v_{mRx}(t)\ \vec{x}\ + v_{mRy}(t)\ \vec{y}$ and $\vec{v_{mT}}\ (t) = v_{mTx}(t)\ \vec{x}\ + v_{mTy}(t)\ \vec{y}$ denote the instantaneous vector velocities of the mobile receiver and transmitter respectively, at time t, relative to a fixed $x-y$ frame as shown in Figure 1.1. The

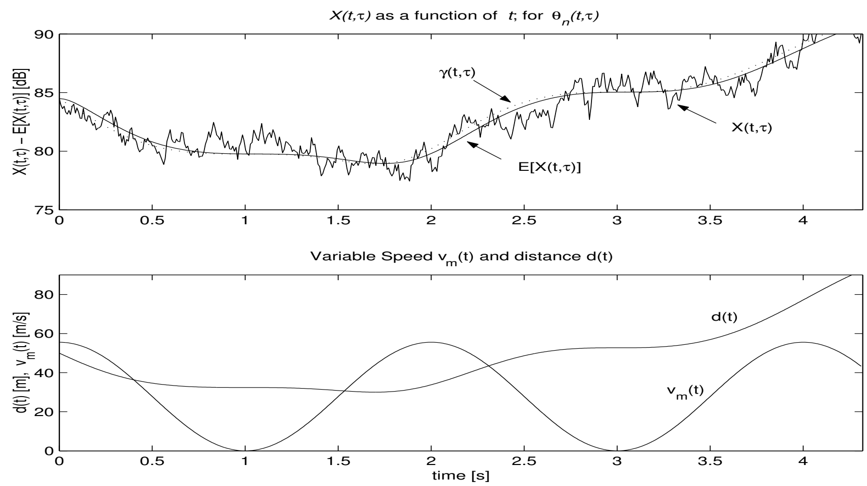

Figure 3.3: Mean-reverting power path-loss as a function of time, t, for a given t-varying $\gamma(t,\tau)$. Mobile moves away from point d with a constant angle of 143^o and a variable speed, $v_m(t)$, as shown at the bottom graph.

instantaneous distance is given by

$$d(t) = \left[\Big(d + \int_0^t \big(v_{mRx}(u) - v_{mTx}(u) \big) du \Big)^2 + \Big(\int_0^t \big(v_{mRy}(u) - v_{mTy}(u) \big) du \Big)^2 \right]^{1/2}. \quad (3.5)$$

According to (1.1), at each instant of time, the corresponding mean $\gamma(t,\tau)$ for the model is thus given by

$$\gamma(t,\tau) = \overline{PL}_d(d(t))[dB] = \overline{PL}(d_0)[\mathrm{dB}] + 10\alpha \log \big(d(t)/d_0 \big) + \xi(t) \quad (3.6)$$

where $\xi(t)$ could be any arbitrary function representing additional temporal variations of the environment. Figure 3.3 illustrates an example where the transmitter is fixed and the mobile moves away from point $d = 50$ m at an arbitrary angle of $\theta_n = 143^o$ with respect to the x-axis and with a variable speed, $v_m(t)$. The power loss is calculated using (2.4) with $\gamma(t,\tau)$ given by (3.6) where $d(t)$ is computed using (3.5) and $\xi(t) = 0$. The initial power loss and the average power loss, as the mobile moves, corresponds to that predicted by (1.1).

References

[1] K. Pahlavan and A. H. Levesque. *Wireless Information Networks*. New York, Wiley-Interscience, 1995.

[2] T.S. Rappaport. *Wireless Communications*. Prentice Hall, 1996.

[3] G. L. Turin, F. D. Clapp, T. L. Johnston, S. B. Fine, and D. Lavry. A statistical model of urban multipath propagation. *IEEE Transactions on Vehicular Technololy*, VT-21(6):1–9, 1972.

[4] A. D. M. Saleh and R. A. Valenzuela. A statistical model for indoor multipath propagation. *IEEE Journal on Selected Areas in Communications*, 5(2):128–137, 1987.

[5] A. J. Coulson, G. Williamson, and R. G. Vaughan. A statistical basis for lognormal shadowing effects in multipath fading channels. *IEEE Transactions on Communications*, 46(4):494–502, 1998.

[6] H. Suzuki. A statistical model for urban radio propagation. *IEEE Transactions on Communications*, 25:673–680, 1977.

[7] C.D. Charalambous and N. Menemenlis. Temporal and spatial log-normal shadowing models using stochastic differential equations. IEEE Transactions on Communications Systems (preprint), 2002.

[8] C.D. Charalambous and N. Menemenlis. Stochastic models for long-term multipath fading channels. *38th IEEE Conference on Decision and Control*, 5:4947–4952, December 1999.

[9] C.D. Charalambous and N. Menemenlis. General non-stationary models for short-term and long-term fading channels. *EUROCOMM 2000*, pages 142–149, April 2000.

[10] B. Oksendal. *Stochastic Differential Equations: An Introduction with Applications*. Springer-Verlag, New York, 5th edition, 1995.