ELECTRON ACCELERATION BY A STOCHASTIC ELECTRIC FIELD IN A THUNDERCLOUD

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ABSTRACT

The kinetic theory for runaway electrons is developed in a stochastic electric field, consisting of short-scale electric cells. The general kinetic equation for the isotropic part of electron distribution function is derived, which includes effects of electron acceleration and slowing-down of spatial diffusion of electrons in a region occupied by electric cells. The solutions are analyzed for some particular cases. The results are applied to conditions of a thunderstorm cloud. The change of flux density of energetic electrons, which is due to thunderstorm activity, is estimated in the case when the cosmic rays serve as a source of initial electron energetic component.

INTRODUCTION

The experiments on registration of γ and X emissions from thunderclouds [1–4] have revealed the correlation of these emissions with the electric field amplitude inside clouds. As it was shown by Gurevich et al. [5], Roussel-Dupre et al. [6], runaway effects play the very important role for electron acceleration, determining the advantage in appearance of electrons with near relativistic energies, whose collisional cross section with gas particles is minimal. The acceleration of electrons by a constant electric field seems to be a transient process, which terminates as a rule by air breakdown. At the same time the experiments show that γ and X emission is a long process testifying a continuous presence of energetic electrons inside a thundercloud. Some additional possibilities in production of energetic electrons appear if one takes into account electron acceleration by stochastic electric field. Actually in large volumes of a cloud atmosphere regions with different orientation of electric field are present, so such a formulation of the acceleration problem is quite natural.

The important accompanied problem is the life-time of accelerated electrons in the volume, occupied by an electric field. A stochastic electric field can essentially influence this life-time.

At first the problem of electron acceleration by stochastic in time electric field with runaway and relativistic effects taken into account was considered by Trakhtengerts et al [7]. Below we will investigate the electron acceleration by a constant in time but changing in space short-scale electric field of occasional orientation, taking into account limited dimension of an acceleration region.

BASIC EQUATIONS

The basic suggestion is that the electric field $\boldsymbol{E}$ is stochastic in space and has the isotropic orientation. The kinetic equation for the velocity distribution function f of electrons with high energies (energy $W \gtrsim 10$ kW) can be written for an arbitrary oriented electric field $\boldsymbol{E}$ in the following form:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\,\frac{\partial f}{\partial \boldsymbol{r}} - e\boldsymbol{E}(\boldsymbol{r})\,\frac{\partial f}{\partial \boldsymbol{p}} = \left(\frac{\partial f}{\partial t}\right)_c \qquad (1)$$

where $\boldsymbol{p}$ is the electron momentum, $\boldsymbol{r}$ and t are the coordinate and time, and $(\partial f/\partial t)_c$ is the collisional integral for the electron-air interaction. The collisional integral can be written as follows:

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{1}{p^2}\,\frac{\partial\left(p^2\Phi f\right)}{\partial p} + \nu_{\text{eff}} f \qquad (2)$$

where

$$\Phi(p) = \frac{4\pi e^4 N_m}{mc^2} \frac{\gamma^2}{\gamma^2 - 1} \left\{ \ln\left(\frac{p^2}{mI\sqrt{2(\gamma+1)}} \right) - \left(\frac{2}{\gamma} - \frac{1}{\gamma} \right) \frac{\ln 2}{2} + \frac{1}{2\gamma^2} + \frac{\gamma - 1}{16\gamma^2} \right\} \tag{3}$$

Here $Z = 14.5$ is the mean molecular charge for the air, $I = 80.5$ eV is the ionization potential for the air, e is the electron charge, c is the light speed, m is the electron mass, N_m is the molecular density, $\gamma = (p^2/m^2c^2 + 1)^{1/2}$; the operator $\hat{\nu}$ describes collisional isotropization of the distribution function. The collisions in our problem are principal because the considerable and irreversible change of the electron energy by a stochastic electric field is possible only under the joint action of electric field acceleration and collisions. It will be suggested further that the change of f on the one special stochastic period of the electric field is small. It means that the distribution function f can be presented in the form:

$$f = F(\boldsymbol{p}, \boldsymbol{r}, t) + f_{\sim} \tag{4}$$

where F is the regular part of distribution function averaged over an electric field stochastic ensemble, and $f_{\sim}$ is a small addition ($|f_{\sim}| \ll F$). We will apply further the iteration procedure in the form (4) to find the contribution of a stochastic electric field to evolution of the regular F. This contribution can be found from the kinetic equation (1) averaged over electric field ensemble. The details of this averaging are described in the paper [7]. Here we give the final result: the kinetic equation for the regular part F of distribution function is written in the following form:

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial z}\left(\frac{v^2}{6(D_1 + D_2)} \frac{\partial F}{\partial z} \right) + \frac{1}{p^2} \frac{\partial \left(p^2 \Phi F\right)}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}\left\{ \frac{e^2 I_0}{k_0^2 v^2} \Phi \frac{(Z+2)p}{4\gamma} \frac{\partial F}{\partial p} - \frac{e^2 I_0}{k_0^2 v^2} \frac{\partial}{\partial p}\left(\frac{p^2 \Phi}{v} \frac{\partial F}{\partial p} \right) \right\} + J \tag{5}$$

where J is the source of electrons, k_0^{-1} is the characteristic scale of electric cell, $I_0 = \langle \boldsymbol{E}(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}) \rangle$ is the averaged over the ensemble electric field intensity, the coefficient D_1 can be written as

$$D_1 \simeq e^2 I_0 / k_0 v p^2 \tag{6}$$

D_2 is equal to

$$D_2 = \nu_{\text{eff}} = \frac{Z+2}{8\gamma p} \Phi \tag{7}$$

DISCUSSION

Equation (5) is valid when a layer occupied by electric cells is sufficiently thick, and an electron changes many times its direction of motion under the influence of collisions and stochastic electric field. The additional condition should be fulfilled for validity of (5), it is [7]

$$I_0 \gtrsim (\Phi_{D\,\min}/e)^2 \tag{8}$$

where the minimum value of the friction force $\Phi_{D\,\min}$ is equal to

$$\Phi_{D\,\min} = (44\pi Z e^4 N_m / mc^2) \tag{9}$$

The condition (8) coincides actually with the condition for appearance of runaway electrons in a constant electric field [5] The corresponding to (9) relativistic factor $\gamma_m = 3.8$. According to (5) the characteristic scale p_0 of the distribution function F in p space is of the order

$$p_0 \sim e I^{0.5} / k_0 c \tag{10}$$

The nonstationary solution of the equation (5) is shown in Fig 1 for the case, when the first term at the left side of (5) can be neglected. The dimensionless variables x and y in this picture are taken in the form

$$x = p/p_0, \qquad y = \Phi_{D\,\min} \cdot t / p_0^3 \tag{11}$$

According to [7] the distribution function in Fig 1 acquires the universal dependence on x under $x \gg 1$, which corresponds to the stationary solution of (5) without a source, and its amplitude is growing lineary in time. The asymptotic behavior of F is $x^{-1} \cdot \exp(-x)$, when $x \mapsto \infty$.

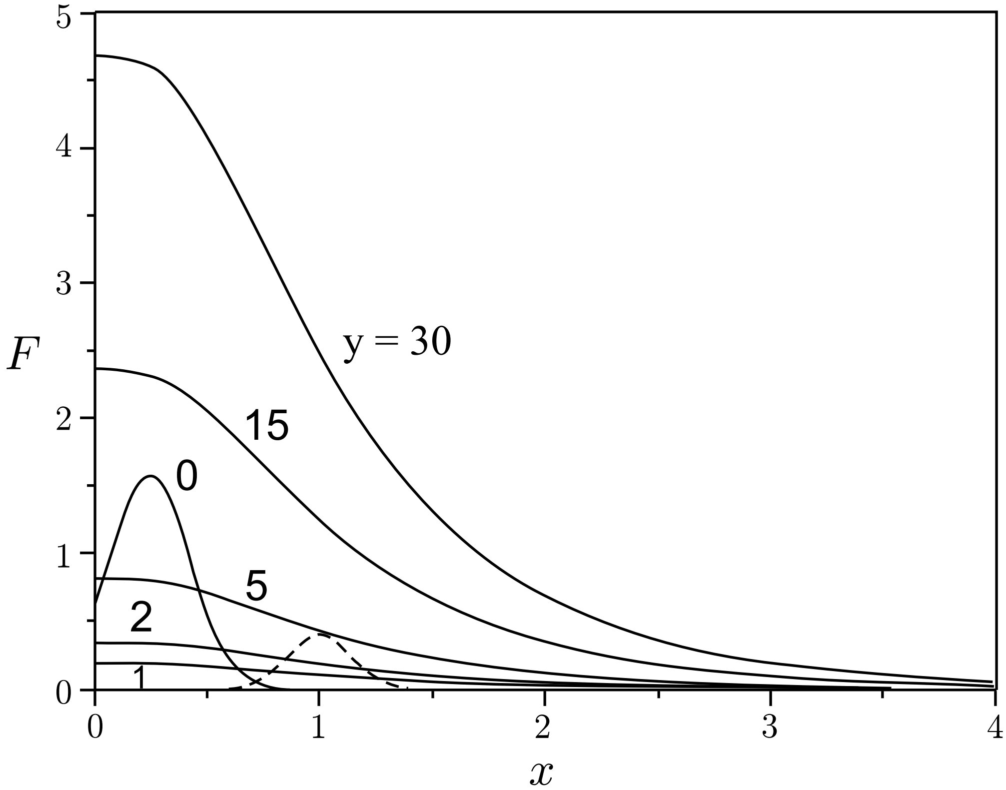

Fig. 1. Numerical simulation results. Distribution function $F(x,t)$ in the relativistic case and with the presence of a source (dashed line)

The life time of energetic electrons in a thundercloud is determined by the relation

$$\tau \simeq 6z_m^2(D_1 + D_2)/c^2 \tag{12}$$

where z_m is the vertical scale of a cloud. The ratio D_1/D_2, characterizing slowing-down of the electron diffusion in a cloud, is determined from (6) and (7) ($\gamma \gg 1$)

$$D_1/D_2 = 8e^2 I_0/k_0c^2m(Z+2)\Phi \tag{13}$$

QUANTITATIVE EXAMPLE

Let us consider the quantitative example for the height $h = 5$ km. For the air $Z = 14$ and $N_m = 1.32\cdot10^{19}$ cm^{-3}, and we have from (8) and (9) the threshold electric field intensity $\sqrt{I_{0\min}} = 104$ kv/m. Using the relation (10) and the equality $(p_0/mc) \geq \gamma_m = 3.8$ we find the scale of electric cells

$$k_0^{-1} \lesssim \gamma_m(mc^2/\Phi) \sim 20m \tag{14}$$

In this case the ratio $(D_1/D_2) \sim \gamma_m \cdot (mc^2/\Phi_{\min}) > 1$. The life time in the case $D_1/D_2 \gg 1$ is growing with I_0 as

$$\tau \sim 6z_m^2 e^2 I_0/k_0c^3p^2 \tag{15}$$

So, if we fix the energy of electrons, the flux density in this energy channel is growing proportionally to I_0. Simultaneously the width of distribution function is growing according to (10). Substitution of p_0 into (15) gives

$$\tau \sim 6k_0z_m^2/c \tag{16}$$

In the case $z_m \sim 3$ km and $k_0^{-1} \sim 20$ m we have $\tau \sim 10^{-2}$ sec. The considered particular example shows that a thunderstorm electric field, which reaches values (1–2.5)$\cdot10^2$ kv/m and demonstrates rather complicated and multi-layer structure, can essentially influence on the intensity of an energetic electron component. More detailed consideration of the case, when the source J is due to the cosmic rays, gives the increase of the electron flux density with energies $\sim$ MeVs by $(1 + D_1/D_2)$ times, that is equal to 5 for $I_0^{0.5} \sim 200$ kv/m. New transient sources of energetic electrons appear during thunderstorm activity, such as electrical micro and macrodischarges, but consideration of these sources is beyond of this paper.

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