MODELLING OF RANDOM MIXTURES

WITH COMPLICATED INTERNAL GEOMETRY

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The literature concerning the effective modelling of heterogeneous materials contains several mixing rules that give the macroscopic permittivity of a mixture as a function of the constitution of the mixture. Maxwell Garnett and Bruggeman belong to the most famous ones. These mixing rules use the assumption that the mixture is discrete and simple-shaped inclusions are located, randomly or in an ordered fashion, in a homogeneous background medium. In the simplest case, the inclusions are spherical. Ellipsoidal inclusions can also be treated to the same degree of analytical exactness as spheres, within the framework of Maxwell Garnett and Bruggeman formalisms.

The purpose of this presentation is to show what can be done to generalize such basic mixing rules to the case when the inclusions are more complicated. In particular, the interest is in the electromagnetic modelling of heterogeneous materials where individual particles contain sharp corners and edges: Platonic polyhedra such as cubes, tetrahedra and octahedra. The electrostatic field solution of a problem involving such dielectric scatterers has to be solved numerically. Of course, from the point of view of random medium modelling, this non-analyticity is not a problem because the whole modelling is not amenable to exact electromagnetic treatment in the first place. In addition, the other way that an inclusion can be more complicated than through the external geometry is that its internal structure is inhomogeneous. Simple inhomogeneities, such as layered or continuously radially inhomogeneous spheres (and even ellipsoids) can be treated also analytically [1].

Indeed, it is cost-effective and pedagogically advantageous to try to use as much of analytical results as possible in building up the solution for the problem involving these complex scatterers. Essential is therefore to know the dielectric response of the inclusions themselves. Basic polarizability analysis follows from solving the internal and external electrostatic problems (the Laplace equation) involving the inclusions in uniform electric field [1,2,3]. But for cases with sharp edges, the analytical polarizability cannot be found. For cubes, the efforts of [4,5,6] have been complemented in [7] where the polarizability of a dielectric cube is given as a function of its permittivity (the dependence on volume is trivial). In [8], we have extended these results and calculated the polarizability of individual elements having sharp edges but which are more complicated in shape than cubes, such as tetrahedra, and octahedra, as functions of their permittivity. These numerical results can be exploited in the modelling analysis of mixtures containing such inclusions. The polarizability of a general inclusion is a dyadic but for symmetric polyhedra (as well as in the case of spheres) and isotropic permittivity, the response is independent of the field direction, hence a multiple of a unit dyadic and equivalent to a scalar response.

A sphere is an extremum shape in the sense that given a certain volume and permittivity, a scatterer with such a shape gives the minimum polarizability; any deviation from that form will increase the induced dipole moment (for theoretical considerations on this topic, see [9]). Therefore it is instructing to compare the electric response of individual scatterers as well as effective properties of mixtures containing such scatterers by normalising the response to that of a sphere or a mixture containing spherical inclusions. Results for such mixtures show that the effective permittivity falls in the following order: first above the minimum (spherical) geometry is the octahedric, then cubical, and tetrahedric shape gives the highest response of these four.

More quantitatively, a dilute mixture (a mixture where the volume fraction of the inclusion phase is small) has susceptivity in the following manner: in the limit case of high contrast between the inclusion and environment permittivities, the octahedron mixture is 1.18 times higher, the mixture with cubes is 1.21 times the corresponding amplitude in the spherical case, and this number for tetrahedra is 1.67. This asymptotic behaviour is the same for both Maxwell Garnett and Bruggeman mixing principles.

REFERENCES

[1] A. Sihvola, *Electromagnetic Mixing Formulas and Applications*, IEE Publishing, Electromagnetic Wave Series, London, 1999.

[2] J.D. Jackson, *Classical Electrodynamics*, Second Edition, Wiley, New York, 1975. (Third Edition, 1999)

[3] J.A. Kong, *Electromagnetic Wave Theory*, EMW Publishing, Cambridge, Massachusetts, 2000.

[4] T.W. Edwards, and J. Van Bladel, "Electrostatic dipole moment of a dielectric cube," *Applied Scientific Research*, Section B, vol. 9, pp. 151-154, 1961.

[5] D.F. Herrick, and T.A. Senior, "The dipole moment of a dielectric cube," *IEEE Transactions on Antennas and Propagation*, vol. AP-25, pp. 590-592, July 1977.

[6] L. Eyges, and P. Gianino, "Polarizabilities of rectangular dielectric cylinders and of a cube," *IEEE Transactions on Antennas and Propagation*}, vol. AP-27}, pp. 557-560, July 1979

[7] J. Avelin, R. Sharma, I. Hänninen, and A.H. Sihvola, "Polarizability analysis of cubical and square-shaped dielectric scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 49, No. 3, pp. 451-457, March 2001.

[8] J. Avelin and A. Sihvola, "Polarizability of polyhedral dielectric scatterers," *Microwave and Optical Technology Letters*, vol. 32, no. 1, pp. 60-64, 2002.

[9] D.S. Jones, "Low frequency electromagnetic radiation," *Journal Inst. Maths Applications*, vol. 23, pp. 421-447, 1979.