Generalized Self-Calibration for LOFAR

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ABSTRACT

The LOw Frequency ARray (LOFAR) radio telescope will operate in a wavelength range between 10 and 200 MHz. This means that ionospheric phase variations in time and position will have a large impact on the quality of the observations. In addition, the shape of the electronic beams formed by the phased array stations will vary independently due to interference rejection (nulling). This means that traditional self-calibration, which essentially solves for only one instrumental parameter per station, will not be sufficient for LOFAR. It will be necessary to solve for some 20 parameters to characterize the main lobe of each station beam, and at least 20 more to characterize the ionospheric phase screen across it. In addition, the effects of bright sources like the Sun and the galactic plane will have to be estimated and removed. Fortunately, there will be sufficient information available to solve for these parameters, in the form of multiple bright sources in the field. This paper describes how self-calibration must be extended and *generalised* to do this.

INTRODUCTION

An aperture synthesis radio telescope samples the complex Visibility Function, which is the Fourier Transform of the observed brightness distribution. The samples are called *visibilities* or *uv-data*, after the coordinates (u, v) of the Fourier plane. The resulting image will be convolved by a Point-Spread Function (PSF), which is defined by the sampling function, and distorted by instrumental errors. Aperture synthesis calibration produce *residual* images, i.e. images from which the the brighter sources and their PSF have been 'subtracted'. If there are many (very) bright sources in the field of view, their contributions have to be subtracted with high accuracy if we want to study the faint sources that lie underneath.

The proposed LOw Frequency ARray (LOFAR) will observe at relatively low frequencies (10-200 MHz). It will consist of an array of about 100 'stations' of about 100 dipoles each. The stations will be spread out over 400 km, with the largest concentration in the centre. For more details, see the LOFAR web-page [1].

LOFAR will be more difficult to calibrate than existing radio telescopes [3]. The main reason is the ionosphere, which causes large and rapid phase variations at LOFAR observing frequencies. Another complicating factor is that the electronically formed beam of a phased array is less stable than the beam of a more traditional parabolic mirror. And last but not least, LOFAR fields will be very crowded with astronomical objects, which cause problems depending on their brightness, position and shape.

The central technique of radio aperture synthesis calibration is *self-calibration (selfcal)* [**?**]. It uses a tentative model of the observed brightness distribution to estimate parameters of the Measurement Equation (M.E.). This is illustrated and explained in fig 2. This very successful technique must be extended and generalised for LOFAR, which is the subject of this paper. The resulting calibration system will also be relevant for the future Square Kilometer Array (SKA), which operates at higher frequencies than LOFAR. Observations with existing telescopes may also profit from the extra calibration functionality discussed here.

SELF-CALIBRATION FOR LOFAR

For existing radio telescopes, it is usually sufficient to solve for a single complex gain parameter per antenna, which is valid for all sources in the field. The same is true for ionospheric effects, if they play a role at all. Moreover, it may be assumed that all antenna beams are more or less equal. Thus, the M.E. has a relatively small number of parameters, and all sources in the resulting image are convolved with the same Point Spread Function (PSF). Finally, the field often contains only a single dominating source.

For LOFAR, things are more complicated. First of all, the M.E. has many more parameters, because the instrumental effects vary significantly across the field of view. Therefore they are called 'image-plane effects', as opposed to 'uv-plane effects' which are valid for the entire field, and can thus be applied to the uv-data. Image-plane effects can only be applied to individual sources, e.g. in order subtracting them from the uv-data. Solving for all these parameters will require (much) more processing, and it also raises the question whether there is sufficient information available. Image-plane effects also cause the PSF to vary across the image, which makes the latter more difficult to interprete. Therefore it is advisable to subtract as many sources as possible from the uv-data, prior to making an image.

Secondly, LOFAR fields are very crowded, because of larger beams, greater sensitivity, and brighter sources at low frequencies. This means that up to 1000 bright (Category I) sources have to be individually predicted and subtracted with high accuracy. In addition, up to 100.000 fainter (Category II) sources have to be predicted in small groups, and subtracted from the uv-data with somewhat less accuracy. And finally, since uv-data can only be corrected for a single point in the field, the many remaining faintest (Category III) sources will be increasingly distorted towards the edge of the image, because of image-plane effects. This effect can be minimised by making a mosaic of smaller 'patch' images, each of which is corrected for its centre.

MEASUREMENT EQUATION

The Measurement Equation (M.E.) enshrines the socalled *HBS-formalism*, named after Hamaker, Bregman and Sault [2]. It has been extended here to include image-plane effects.

The M.E. describes a generic radio telescope in full polarization. It can be used to predicts visibility values V_{ij}^{pred} as 'seen' by interferometers ij between stations i and j, using a Sky Model model with sources I_k, and an instrumental model enshrined in the socalled Jones matrices J_i:

$$V_{ij}^{pred}(f,t) = (J_{ic} \otimes J_{jc}) \sum_k (J_{ik} \otimes J_{jk}) * S * I_k = V_{ij}(p_1, p_2, p_{3,}, ..., p_n) \tag{1}$$

This prediction is needed to subtract sources from the uv-data. The station-based 2×2 Jones matrices can be written as a multiplication of 2×2 matrices, each of which describes a separate part of the instrumental model. Note that there is a position-dependent part J_{ik} that descrobes image-plane effects, and a position-inpependent part J_{ic} that describes uv-plane effects:

$$J_{ic} = E_{ic} P_i F_{ic} \quad and \quad J_{ik} = E_{ik} P_i F_{ik} K_{ik} \tag{2}$$

in which

- $E_{ic}(f,t)$ is the station (voltage) beam gain at the centre of the patch.
- $E_{ik}(f,t)$ is the station (voltage) beam gain in the direction $(l,m)_k$ of source I_k, relative to the patch centre. It is calculated by either interpolating the main lobe of the parametrised beamshape model $E_i(pp)$ (20 parameters per beam), or by using the known sidelobe gain in the direction of the 200 brightest sources.
- $P_i(t)$ describes the projected station dipole orientation as seen from the direction $(l,m)_k$ of source I_k.
- $F_{ic}(f,t)$ is the ionospheric phase at the centre of the patch.
- $F_{ik}(f,t)$ is the ionospheric phase in the direction $(l,m)_k$ of source I_k, relative to the patch centre It is calculated by either interpolating the parametrised ionospheric phase screen model $F_i(pp)$ (20-50 parameters per beam).
- $K_{ik}(u,v,w)$ represents the Direct Fourier Transform (DFT) kernel $e^{2\pi j(ul_k + vm_k + w\sqrt{1-l_k^2+m_k^2})}$ for source $I_k(l_k, m_k)$.

The $p_n(f,t)$ are parameters of the M.E., whose best known values are used in the calculation. One solves for incrementally improved values Δp_n of (subsets of) the p_n by solving sets of simultaneous equations of the form:

$$\Delta V_{ij} \;=\; V_{ij}^{meas} - V_{ij}^{pred} \;=\; \sum_n \frac{\partial V_{ij}^{pred}}{\partial p_n} \Delta p_n \tag{3}$$

Since V_{ij} is a vector of 4 complex numbers, the expression above actually represents 8 real equations. The solver only has to be supplied with the left-hand values ΔV_{ij} (the residual uv-data) and the gradients $\partial / \partial p_n$ to solve for the incremental

Δp_n, which are then added to the current values. The solver is generic in the sense that it provides general 'mathematical services', but does not have any knowledge about the M.E. The solver should be capable of detecting when the set of equations is ill-conditioned, and take suitable action. The optimum succession of parameter subsets to be solved for has to be learned by experience.

GENERALISED SELF-CALIBRATION

Summarizing, self-calibration has to be extended for LOFAR with the following main features:

- **Use a proper Measurement Equation,** i.e. an accurate mathematical description of how a particular Sky Model leads to the actual values of the measured visibility data. Of the existing packages, only AIPS++ is based on the correct full-polarisation matrix formalism. For LOFAR, this has to be extended to include image-plane effects.
- **Solve for many more parameters** of the M.E. Traditional selfcal only solves for one parameter per antenna, usually the gain in the direction of the dominating source. With LOFAR there will be many more bright sources in the field, and there will be much larger image-plane effects, i.e. effects that depend on direction.
- **Solve for coefficients of smooth functions (f,t).** Most M.E. parameters smoothly depend on frequency in a known way. Most of them also vary in time, and we may often assume that they do so in a smooth way. This *a priori* knowledge should be used as a constraint on the solution.
- **Use parametrised Sky Model components.** They are much more convenient to represent frequency and time dependence than the traditional (cubes of) CLEAN components. Since their parameters are also parameters of the M.E., any of them can be solved for if necessary. Extended sources may be parametrised with shapelets or pixons, but some very extended sources must still be represented as images.
- **Solve for arbitrary sequences of arbitrary subsets** of M.E. parameters. Since the M.E. contains thousands of parameters, they must be solved for in suitable subsets, while the others are kept constant. In some cases, extra constraints will be needed to obtain a solution. In any case, the solver must be able to detect when there is not enough information available to solve for particular parameters, and take suitable action. The correct sequence of subsets will have to be learnt by experience.
- **Use judicious subsets of uv-data** for efficiency. Some data samples do not contribute much to a particular solution, but they are as expensive to include. Skipping them can lead to large savings in processing, especially in the first few iterations. The trick is to distinguish the most useful ones.
- **Allow large non-uniform uv-bins**. Since prediction of the values of uv-data is an expensive process, it should be done for the minimum number of uv-bins. One way of achieving this is to make the uv-bins as large as possible. Therefore, uv-data used for selfcal should be integrated over as much bandwidth and time as is allowed by parameter variation. Short baselines (the majority in LOFAR!) may often be integrated more than long ones. Of course this is only useful if we can cheaply and accurately integrate the highly variable visibility function over a large uv-bin (see below).
- **Use a Global Sky Model** to combine images to a single larger one. These can be sub-images from the same observation (e.g. patches, or facets), or images from different observations that are part of a mosaic.

It goes without saying that generalised selfcal will be very expensive in processing. It might be said that LOFAR has only become possible because of the advent of very powerful parallel computers. But only if all possible computational corners are cut.

PREDICTING VISIBILITIES

A visbility sample represents the integrated value of the socalled *Visibility Function* over a uv-bin, i.e. an area of the uv-plane. Since this area is defined by the bandwidth and the integration time, it is an integral over frequency and time. In order to subtract (very) bright sources with high accuracy, their frequency and time dependence must be properly taken into account. This may be done by calculating their contributions for very small uv-bins, over which the Visibility Function can be assumed to be constant, or at least linear. But since prediction is a very expensive process it is important to be able to calculate the integrated visibility function over a large uv-bin analytically. Fortunately this is possible, since the visibility function may be approximated by the following function for a wide range of cases:

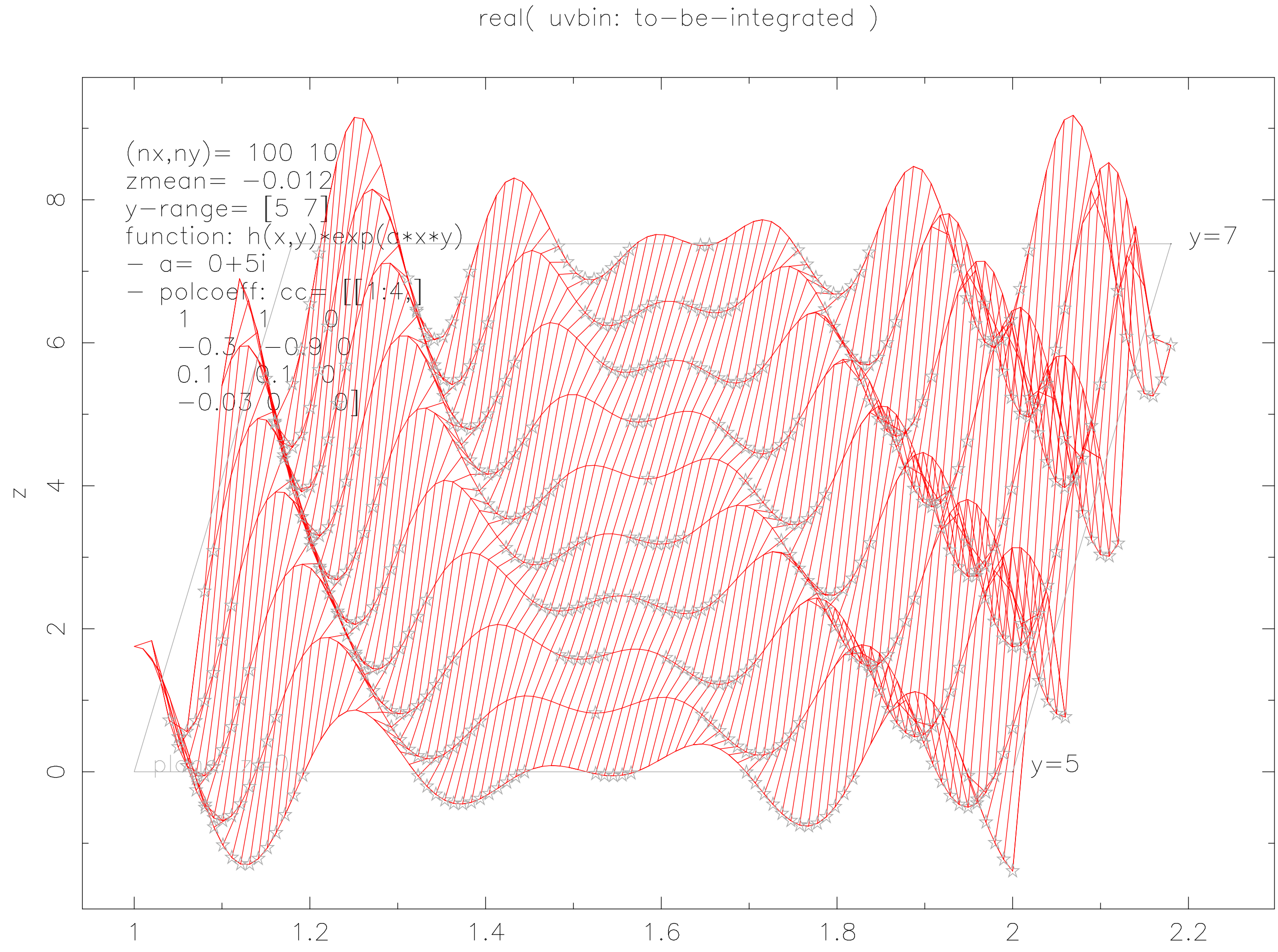

Figure 1: *The visibility function of a single source varies over a uv-bin. This can be expressed as a function frequency(x) and time(y), and integrated analytically. A predicted visibility sample is the sum of the integrated values of all sources in the Sky Model. Since there are many of them, the integration should be as cheap as possible, and we should strive for a small number of large uv-bins.*

$$V(f,t,l,m) \;=\; \int H(f,t)e^{2\pi i\beta f(l_k cos(\alpha t)\;+\;m_k\gamma sin(\alpha t))}\;df\;\,dt \qquad (4)$$

in which $H(f,t)$ is a 2D polynomial in frequency and time. The exponential is the Fourier kernel for a source k at position (l_k, m_k) w.r.t. the field centre. The kernel is also a function of f and t. This function may be readily evaluated analytically over large uv-bins with very modest processing cost. α, β and γ are constants.

References

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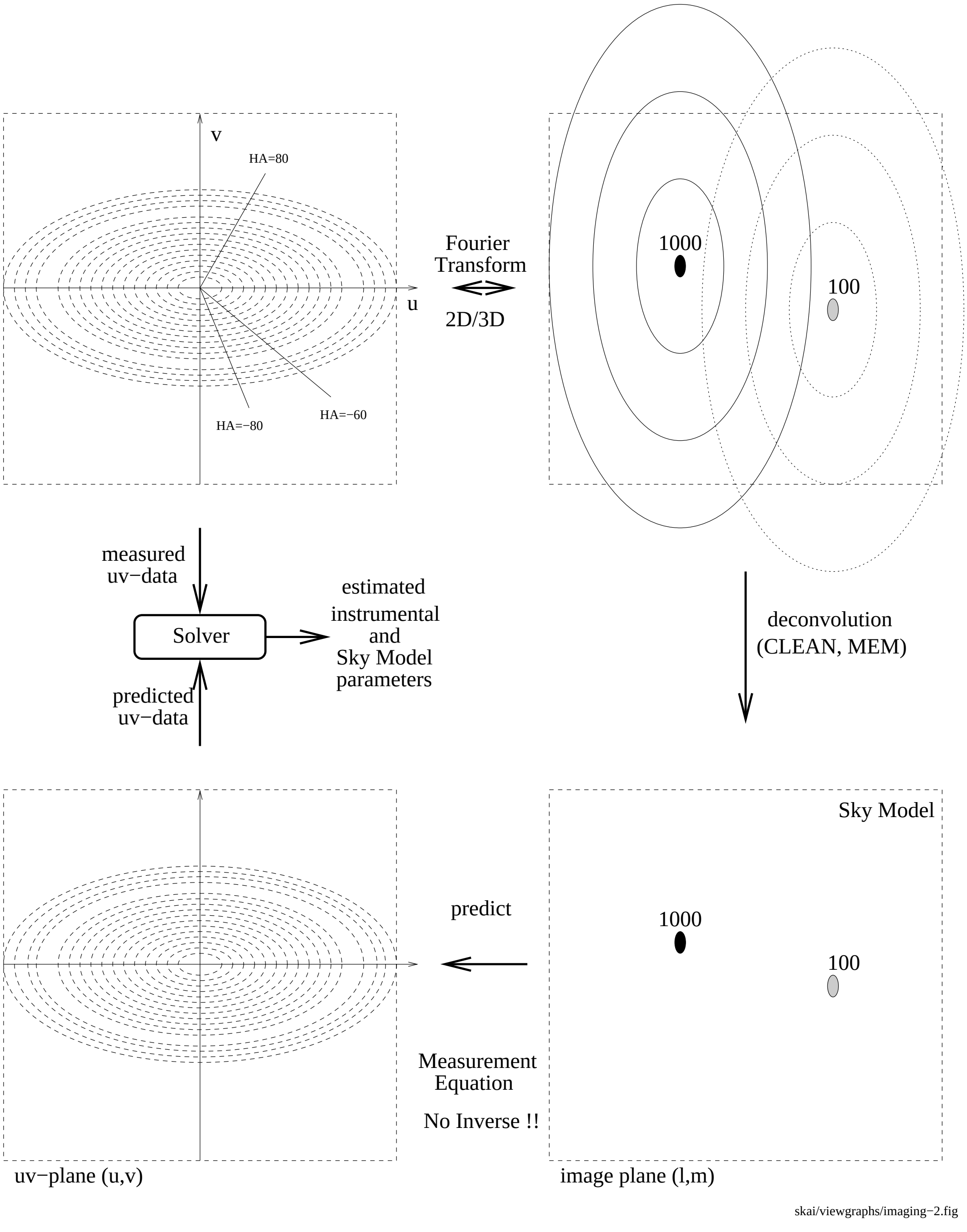

Figure 2: *Schematic diagram of self-calibration. An aperture synthesis telescopes samples the visibility function, i.e. the Fourier Transform of the sky brightness distribution (top left). The resulting image after transformation (top right) is convolved with a Point-Spread Function (PSF) that is defined by the sampling function, and distorted by instrumental errors. This image can be used to generate an approximate Sky Model (bottom right), which is used to predict the values of all measured samples by means of the socalled Measurement Equation (M.E.). From the residuals, i.e. the differences between measured and predicted values, better values may be estimated for the M.E. parameters. The latter include the parameters of the sources in the Sky Model as well as instrumental parameters. This process can be repeated iteratively, until the residuals are minimaul noise-like. Note that the residual image, i.e. the image made from residuals, can be used to find ever fainter sources for inclusion in the Sky Model.*