# THEORETICAL STUDY OF WAVEGUIDING STRUCTURES CONTAINING BACKWARD-WAVE MATERIALS

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## ABSTRACT

We have considered theoretically the waveguide properties of a plane two-layered waveguide, whose one layer is a usual magnetodielectric (forward-wave medium), but another one is a slab of so-called *backward-wave material* (BW-material), whose both permittivity and permeability are negative. We have analyzed the properties of eigenwaves in this waveguide. In particular, it was found that there exist waves of both TE and TM polarizations, whose fields decay exponentially from the interface of the two slabs inside both layers, and their slow-wave factor tends to infinity at small frequencies. Thus, this waveguiding system supports super-slow waves with extremely short wavelengthes, as compared to the free-space wavelength and the cross section size. Other peculiarities of the spectrum are also discussed.

## INTRODUCTION

Recently, a lot of attention has been payed to new meta-materials which can be charactrized (in the microwave domain) by effective permittivity and permeability values with negative real parts. Electromagnetic waves in materials with negative material parameters have been theoretically studied in sixties by Veselago [1], and some very interesting features found. The main feature of plane waves in these media is the fact that they are backward waves, meaning that the direction of the Poynting vector is antiparallel to the phase vector. This leads to anomalous refraction of waves at planar interfaces between conventional and backward-wave materials: the tangential component of the refracted wave Poynting vector is directed opposite with respect to the same vector of the incident wave. The main recent developments in this field showed that the required parameters can be realized (in a certain frequency range, with some imaginary parts accounting for losses) as composite materials with specially shaped metal inclusions. Alternatively, realization can be possible with the use of active inclusions, in which case the main limitations on the bandwidth and dispersive properites can be dramatically relaxed.

If one combines two planar slabs, so that one of them is a normal medium but the other is a backward-medium slab, and a plane electromagnetic wave passes through the stack orthogonally to the interfaces, the total phase shift for the wave is the difference of the phase shifts gained in the propagation through the two slabs, not the sum of them as in the case of only conventional media. As was recently suggested by Engheta [2], this can be conceptually used to create very narrow resonant cavities, as the total phase shift for the wave can be made zero by a proper choice of the ratio between the thicknesses of two layers. Such interesting properties of backward-wave material layers suggest that waveguiding modes in planar waveguides with novel composite slabs can offer new interesting opportunities. At the URSI National Radio Science Meeting in January 2002 [3] N. Engheta presented the eigenvalue equation for a planar waveguide formed by two slabs (one of a usual material, the other of a backward-wave material) bounded by metal walls on the both sides, and discussed the properties of the eigenwaves for the case of thin slabs. In particular, he has found that for this case the propagation factor (under an approximation) drops out from the eigenvalue equation, which means that at a given frequency waves with virtually any propagation constant can travel along the guide.

In the presentation will first analyse surface waves along a planar conventional medium-backward-wave medium interface. This system can be viewed as a limiting case of the planar waveguide considered by Engheta in the case of very large thicknesses of both slabs. Certain analogy with the known case of dielectric-plasma interface can be established. In particular, there exist surface waves along the interface which exponentially decay in both media. We have found that for this interface the unique situation when waves with arbitrary propagation factors are all eigensolutions can be realized exactly, without any approximations. Next, we study eigenwaves in planar waveguides in detail, using appropriate analytical asymptotics and numerical solutions of the dispersion equation. The emphasis is on our finding that in this system there can exist exotic wave solutions, which might be very important for potential microwave applications.

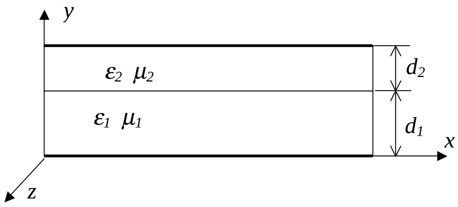


Figure 1: Geometry of the problem: a planar waveguide filled by two slabs of different materials, one of them is a metamaterial with negative parameters.

## EIGENWAVES IN TWO-LAYERED WAVEGUIDE

Let us consider a plane two-layered waveguide, infinite along $z$ and $x$ directions and bounded by electric walls in $z-y$ plane at distances $d_1$ and $d_2$ from the media interface (see Fig. 1). The media are characterized by relative permittivities $\varepsilon_1$, $\varepsilon_2$ and permeabilities $\mu_1$, $\mu_2$. We will discuss eigenwaves propagating in $z$ direction whose field depends on time and the longitudinal coordinate as $\exp(\omega t - k_z z)$. Assuming that $\partial/\partial x = 0$, eigenwaves can be separated into two classes, TE modes with $E_z = E_y = 0$, and TM modes with $H_z = H_y = 0$. The eigensolutions for this simple waveguide are well known.

What happens, if one of the layers is a BW material? Let us assume that $\varepsilon_1 < 0$, $\mu_1 < 0$. The dispersion relations for TE and TM modes become

$$\frac{|\mu_1|}{k_{y1}} \tan k_{y1} d_1 - \frac{\mu_2}{k_{y2}} \tan k_{y2} d_2 = 0 \tag{1}$$

and

$$\frac{k_{y1}}{|\varepsilon_1|} \tan k_{y1} d_1 - \frac{k_{y2}}{\varepsilon_2} \tan k_{y2} d_2 = 0. \tag{2}$$

Here $k_{yi} = \sqrt{k^2 \varepsilon_i \mu_i - k_z^2}$ $(i = 1, 2)$, and $k$ is the wavevector in vacuum. Now real solutions for the propagation factors are permitted with both $k_{y1}$, $k_{y2}$ being purely imaginary numbers. This means that the surface waves, whose fields decay exponentially from the interface between forward-wave (FW) and BW layers can propagate in such a waveguide and there are no upper restrictions for their propagation constants.

Let us study the low-frequency limit of the slow-wave factor for the fundamental mode. If the media parameters are all positive, this value is always within the limits

$$\sqrt{\varepsilon_2 \mu_2} \le \sqrt{\frac{d_1 \mu_1 + d_2 \mu_2}{d_1/\varepsilon_1 + d_2/\varepsilon_2}} \le \sqrt{\varepsilon_1 \mu_1}. \tag{3}$$

However, if we allow negative values of the material parameters, there are no limits at all:

$$0 \le \sqrt{\frac{d_1 \mu_1 + d_2 \mu_2}{d_1/\varepsilon_1 + d_2/\varepsilon_2}} \le \infty. \tag{4}$$

Very peculiar situations take place in the limiting cases. If $d_1/\varepsilon_1 + d_2/\varepsilon_2 \to 0$, we observe that the capacitance per unit length of our transmission line (we now consider the quasi-static limit) tends to infinity. This means that although the voltage drop between the plates tends to zero, the charge density on the plates remains finite. This can be understood from a simple observation that if we fix the charge densities (positive on one plate and negative on the other), the displacement vector is fixed and, in the quasi-static limit, it is constant across the cross section. However, the electric field vector is oppositely directed in the two slabs, if one of the permittivities is negative. In the above limiting case the total voltage tends to zero. Similarly, in the limiting case $d_1\mu_1 + d_2\mu_2 \to 0$, the inductance per unit lengths tends to zero.

### Interface between Two Media

Another interesting observation concerns the case when both layer thicknesses tend to infinity, that is, the case of waves travelling along a planar interface between two media. The dispersion equations reduce to

$$\frac{|\mu_1|}{k_{y1}} - \frac{\mu_2}{k_{y2}} = 0, \quad \text{TE modes}, \qquad \frac{k_{y1}}{|\varepsilon_1|} - \frac{k_{y2}}{\varepsilon_2} = 0, \quad \text{TM modes} \tag{5}$$

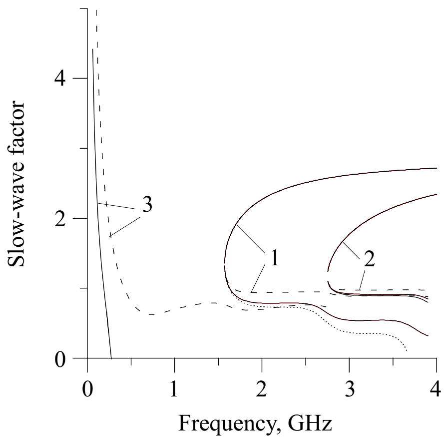


Figure 2: Dispersion of TE modes, when the first layer is a BW material and the second layer is a usual material: $\mu_1 = -2;\ \mu_2 = 1;\ \varepsilon_1 = -4;\ \varepsilon_2 = 1$, $d_1 = 0.5$ cm. The thicknesses of the second layer are: $d_2 = 1$ cm (dotted line), $d_2 = 1.1$ cm (solid line), $d_2 = 2$ cm (dashed line).

It is well known (and obvious from the above relations) that surface waves at an interface can exist only if at least one of the media parameters is negative, an obvious example is an interface with a free-electron plasma region. If both parameters are negative, both TE and TM surface waves can exist. A very special situation realizes if the parameters of the two media differ only by sign, that is, if $\varepsilon_1 = -\varepsilon_2$ and $\mu_1 = -\mu_2$. In this case the propagation factor cancels out from the dispersion relations, because $k_1 = k_2$. This means that waves with any *arbitrary* value of the propagation constant are all eigenwaves of the system at the frequency where this special relation between the media parameters is realized. A similar observation was made in [3] as an approximation in case of small heights $d_{1,2}$. For a media interface this result is exact.

## PROPERTIES OF EIGENWAVES

Let us consider dispersion of TE modes (see Fig. 2), calculated with different thicknesses of the second (FW) layer. One notable difference from the usual two-layered waveguide is a change of dispersion sign (see curves 1,2). It is caused by the opposite directions of the longitudinal components of the energy transport within FW and BW layers. The frequency point where dispersion changes sign, corresponds to the situation when the total energy flows are equal within the first and second layers.

Another important new feature, already noted above, is a possibility of propagation of waves whose slow-wave factor exceeds $\sqrt{\varepsilon_1\mu_1}$ (we assume, as before, that $\varepsilon_1\mu_1 > \varepsilon_2\mu_2$). Dispersion characteristics of such a super-slow wave for different $d_2$ are shown by curves 3, Fig. 2. We observe that the slow-wave factor tends to infinity if $k \to 0$. It can be shown that the existence of super-slow waves is possible only if the condition

$$|\mu_1|d_1 < \mu_2 d_2, \qquad (\text{for} \quad |\mu_1| > \mu_2) \tag{6}$$

is satisfied.

Still another peculiarity of the spectrum of two-layered waveguides filled with FW-BW materials is the existence of a non-dispersive wave (under the assumption that both FW and BW materials are non-dispersive). To study this possibility, let us first note that non-dispersive solutions are only possible if the arguments of the two tangent functions in (1) are equal (so that there is no dependence on the wavenumber $k$). If this condition is satisfied, the tangent functions can be cancelled, and the eigenvalue equation (1) can be easily solved. The result for the slow-wave factor reads

$$n_c = \sqrt{\frac{|\mu_1|\mu_2(|\mu_1|\varepsilon_2 - \mu_2|\varepsilon_1|)}{\mu_1^2 - \mu_2^2}}. \tag{7}$$

The non-dispersive wave with the propagation factor given by (7) exists if the following two conditions are satisfied:

$$|\mu_1|d_1 = \mu_2 d_2, \qquad (|\mu_1|\varepsilon_2 - \mu_2|\varepsilon_1|)(\mu_1^2 - \mu_2^2) > 0. \tag{8}$$

The first condition guaranties the existence of a solution, and if the second condition is satisfied, the solution is a real number corresponding to a propagating mode. It can be seen also that such a solution describes a surface wave with an

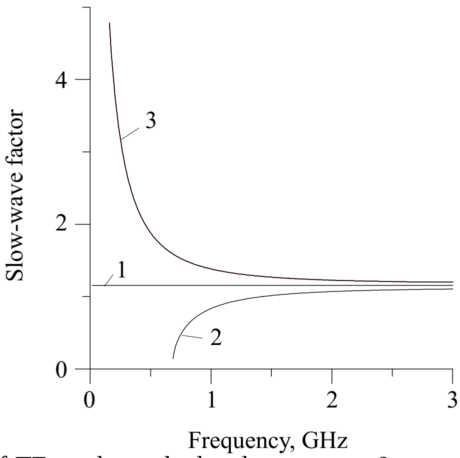


Figure 3: Dispersion characteristics of TE modes, calculated as $\mu_1 = -2;\ \mu_2 = 1;\ \varepsilon_1 = -4;\ \varepsilon_2 = 3$, $d_1 = 0.1$ cm, $d_2$=0.2 cm or $d_1 = 1$ cm, $d_2$=2 cm (curve 1), $d_1 = 1$ cm, $d_2$=1.95 cm (curve 2) and $d_1 = 1$ cm, $d_2 = 2.05$ cm (curve 3).

exponential field distribution in both of the media if

$$(\mu_1^2 - \mu_2^2)(\varepsilon_1\mu_1 - \varepsilon_1\mu_1) < 0. \tag{9}$$

In other cases the field distribution is described by trigonometric functions.

Let us discuss the properties of this non-dispersive wave. Its propagation constant does not depend on the frequency, which means that such a wave has no cutoff. Furthermore, its existence is not connected with the total thickness $d_1 + d_2$, but only with the relation $d_2/d_1 = |\mu_1|/\mu_2$. It is illustrated by curve 1, Fig. 3, calculated at $d_1 = 0.1$ cm. No other waves propagate within the spectral range presented in Fig. 3. This wave disappears under any small deviation of either $d_2$ or $d_2$ violating relation $|\mu_1|d_1 = \mu_2 d_2$, but it is not so sensitive to the values of $\varepsilon_1$ and $\varepsilon_2$, it is enough that inequality (9) is satisfied.

**CONCLUSIONS**

We have considered the eigenmodes in a layered waveguide containing a layer of a backward-wave metamaterial, which has negative and real material parameters. We have found important differences between the eigenmode spectra in ordinary and FW-BW two-layered waveguide. In a FW-BW waveguide both TE and TM modes can change the dispersion sign. This is possible because the energy transport directions are opposite in FW and BW layers, so there exists a spectral point, where the power flows in the two layers compensate each other. Under certain relations between the permeabilities and thicknesses of FW and BW layers there exists a non-dispersive TE mode without a low-frequency cutoff. There exist both TE and TM super-slow waves, whose slow-wave factor is not restricted by the values of the permittivities and permeabilities of the layers. The fields of these waves decay exponentially in both FW and BW layers from their interface in case of large propagation constants. It is remarkable, that such super-slow modes are caused not by large values of the permeability or permittivity, like it takes place near a resonance in ferrite or plasma, but by the layer thickness effects.

A more detailed report on this study which includes the analysis of TM waves, has been submitted to *Radio Science* [4].

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