RECURSIVE ESTIMATION OF NOISE STATISTICS IN KALMAN FILTER BASED MIMO EQUALIZATION

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ABSTRACT

Estimating noise statistics is of great interest when using state-space models. Kalman filtering requires accurate values of state and measurement noise covariances to work optimally. Typically these two matrices are assumed to be known. However, in practice this is often not the case, and they need to be estimated from the received data. In our previous work we have proposed a batch method for estimating the noise statistics. The method was applied to single input single output (SISO) equalization of time-varying channels where the state space model was used to describe the system. In this paper we propose a recursive method for estimating the noise statistics with application to equalization of time-varying multi-input multi-output (MIMO) channels. The optimality of the estimates is tested using non-parametric runs test on innovation sequences. The accurate estimate of noise covariance matrices allows the Kalman filter to reliably estimate the state, thus leading to improved equalization performance.

1. INTRODUCTION

In wireless communications MIMO channel equalization is a difficult task due to the time varying nature of the channel. When the mobile station speed is high, deep fades occur in the received signal as well as rapid changes in the channel. In this case the traditional equalization schemes based on channel identification and decision-directed (DD) equalization cannot track the channel variations. Moreover, these techniques require that a significant percent of the transmitted symbols are training symbols used for equalization. An ideal equalizer would not need any training sequence and would be capable of tracking fast variations of the channel.

A semi-blind equalization algorithm for time varying MIMO channels based on Kalman filtering and Decision Feedback Equalizer (DFE) was introduced in [2]. This algorithm needs only a small training sequence for channel acquisition, after that Kalman filter uses the symbol decisions of a DFE for channel tracking and the algorithm runs blindly.

In many Kalman filter aplications the assumption that the process and measurement noise covariances are known is commonly used but not necessary valid. It has been recognized that in many practical implementations the erroneous estimation of the measurement and process noise variances degrades significantly the performance of the Kalman filter [8]. In this paper we introduce an online method for estimating these noise covariances for a MIMO system. A recursive non-parametric method for testing the whiteness of the innovation process is provided as well.

This paper is organized as follows. The system model is introduced in section 2 and the Kalman filter equations are presented based on the state-space model. Then a description of the proposed algorithm and optimality testing are given in section 3. In section 4 simulation results are presented.

2. SYSTEM MODEL

Let us consider a MIMO system with m transmitters and n sensors at the receiver. The received observations from sensor j (with j=1,...,n) at time t are given by:

$$y_j(t) = \sum_{i=1}^{m} \sum_{l=0}^{L_{ij}-1} h_{ij}(l) x_i(t-l) + v_j(t) \tag{1}$$

where $x_i(t-l)$ is the symbol drawn from a constellation $\mathcal{X}$ of the i-th user at time $t-l$, $h_{ij}(l)$ is the impulse response of the TVC, $y_j(t)$ is the received signal, and $v_j(t)$ is the additive Gaussian noise with variance σ_v^2. Setting $L = \max L_{ij}$, the channel length, we obtain the following vector form:

$$\mathbf{y}(t) = \sum_{l=0}^{L} \mathcal{H}_l(t) \mathbf{x}(t-l) + \mathbf{v}(t) \tag{2}$$

where $\mathbf{y}$ is a column vector of n received signals, $\mathbf{x}$ is a column vector of m transmitted signals, $\mathcal{H}_l(t)$ is a $n \times m$ matrix containing the channel taps and $\mathbf{v}$ is an additive noise vector. As an example, a simple 2×2 MIMO model is presented in Figure 1.

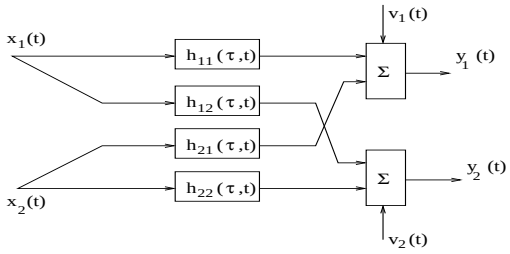

Figure 1: The (2×2) MIMO system model

Let assume that the j-th received signal is a superposition of N_p paths. The resulting channel impulse response can then be described using the Gaussian distributed *Wide-Sense Stationary with Uncorrelated Scattering* (WSSUS) model [3]:

$$h_{ij}(t,\tau) = \frac{1}{\sqrt{N_p}} \sum_{p=1}^{N_p} e^{j(2\pi f_{d,p} t + \theta_p)} h_{RF}(\tau - \tau_p) \tag{3}$$

where $f_{d,p}$ is the Doppler spread, θ_p is the angular spread, τ_p is the delay spread of the path p and $h_{RF}(t)$ is the impulse response of the receive filter.

Considering that the m-dimensional transmitted sequence is a white sequence drawn from a PSK constellation and if the n-dimensional received signal is sampled at symbol rate we get the following discrete-time model:

$$\mathbf{y}(k) = X(k)\mathbf{h}(k) + \mathbf{v}(k) \tag{4}$$

where X is a $n \times nmL$ data matrix defined as $X(k) = [x_1(k)I_m \ldots x_n(k)I_m \ldots x_1(k-L+1)I_m \ldots x_n(k-L+1)I_m]$, I_m is an $m \times m$ identity matrix and $\mathbf{h}(k) = \left[h_{11}^0(k) \ldots h_{1n}^0(k) \ldots h_{m1}^0(k) \ldots h_{mn}^0(k) \ldots h_{11}^{L-1}(k) \ldots h_{1n}^{L-1}(k) \ldots h_{m1}^{L-1}(k) \ldots h_{mn}^{L-1}(k)\right]^T$ is a vector of length nmL containing channel coefficients.

In matrix notation we have the following state space equations:

$$\mathbf{h}(k) = A\mathbf{h}(k-1) + \mathbf{w}(k) \tag{5}$$

$$\mathbf{y}(k) = X(k)\mathbf{h}(k) + \mathbf{v}(k) \tag{6}$$

where $X(k)$ contains transmitted symbols, $\mathbf{h}(k)$ are the channel taps at time instant k and A is the state transition matrix, in our case an identity matrix. Noises $\mathbf{v}$ and $\mathbf{w}$ are mutually uncorrelated, white noise sequences with covariance matrices R and Q. The goal of this paper is to estimate online the noise covariance matrices R and Q.

The Kalman filter equations can be summarized as follows:

$$\begin{aligned}
\hat{\mathbf{h}}(k|k-1) &= A\hat{\mathbf{h}}(k-1|k-1) \\
P(k|k-1) &= AP(k-1|k-1)A^T + Q \\
K(k) &= \frac{P(k|k-1)X}{X^T P(k|k-1)X + R} \\
\mathbf{r}(k) &= \mathbf{y}(k) - X(k)\hat{\mathbf{h}}(k|k-1) \\
\hat{\mathbf{h}}(k|k) &= \hat{\mathbf{h}}(k|k-1) + K(k)\mathbf{r}(k) \\
P(k|k) &= P(k|k-1) - K(k)X^T P(k-1|k-1)
\end{aligned} \tag{7}$$

where $P(k|k-1)$ is the prediction error covariance matrix, $P(k|k)$ is the correction error covariance matrix, $K(k)$ is the Kalman gain and $\mathbf{r}(k)$ is the innovation process. The equalization is based on a MIMO MMSE-DFE derived in [2].

3. NOISE COVARIANCES ESTIMATION

In typical Kalman filtering based channel tracking algorithms such as [13, 4], the noise statistics are assumed to be known. In the following we propose an algorithm for estimating Q and R in an online manner. The algorithm stems from the work in [6, 12]. It has two stages: covariance estimation and testing for the whiteness of the innovations.

3.1. Recursive covariance estimation

The noise statistics computation is based on covariance matching method. The measurement covariance matrix is found by using the theoretical and estimated covariances of the innovation process while the process noise covariance matrix is found by using the theoretical and estimated covariances of the residual process. The recursive formulas for estimated the noise statistics are presented in Table 1. In these equations $\bar{\mathbf{r}}(k)$ and $\bar{\mathbf{q}}(k)$ are the mean of the innovation and

Table 1: Noise statistics estimation

	R estimation	Q estimation
process	$\mathbf{r}(k) = X(k)\left[\hat{\mathbf{h}}(k\|k) - \hat{\mathbf{h}}(k\|k-1)\right] + \mathbf{v}(k)$	$\mathbf{q}(k) = A[\hat{\mathbf{h}}(k) - \hat{\mathbf{h}}(k+1\|k)] + \mathbf{w}(k)$
theoretical cov. matrix	$S_r = X(k)P(k\|k-1)X^H(k) + R$	$S_q = AP(k\|k)A^H + Q$
estimated cov. matrix	$\hat{S}_r(k) = \frac{k-1}{k}\hat{S}_r(k-1) +$ $+\frac{1}{k}\left[\mathbf{r}(k) - \bar{\mathbf{r}}(k)\right]\left[\mathbf{r}(k) - \bar{\mathbf{r}}(k)\right]^H$	$\hat{S}_q(k) = \frac{k-1}{k}\hat{S}_q(k-1) +$ $+\frac{1}{k}\left[\mathbf{q}(k) - \bar{\mathbf{q}}(k)\right]\left[\mathbf{q}(k) - \bar{\mathbf{q}}(k)\right]^H$
estimated noise statistics	$\hat{R}(k) = \hat{S}_r(k) - \frac{1}{k}\sum_{i=1}^{k} X(i)P(i\|i-1)X^H(i)$	$\hat{Q}(k) = \hat{S}_q(k) - \frac{1}{k}AP(k\|k)A^H$

residual process computed recursively at time instance k. Adaptive methods for estimating the noise statistics were also introduced in [7, 9]. The method in [9] was based on a Bayesian type of approach and the method derived in [7] on a correlation matching method. Both methods found the same adaptive update for the observation noise covariance matrix but different formulas for the state noise covariance.

3.2. Optimality testing

A necessary and sufficient condition for Kalman filter to operate optimally is that innovations sequence is white (zero mean, uncorrelated). In this purpose a randomness test has to be performed on the innovation processes. A suitable solution for our recursive algorithm is the runs test [11]. New sequences are formed from the innovation processes taking the sign of their samples. A run is defined as a set of identical symbols contained between two different symbols A sequence would be considered non-random if there are either too many or too few runs and random otherwise.

We collect the number of positive values in N_1, the number of negative values in N_2 and the number of runs in V. Then we have a sampling distribution of the statistic V. It can be shown than this sampling distribution has the mean and variance given by:

$$\mu_V(k) = \frac{2N_1(k)N_2(k)}{N_1(k) + N_2(k)} + 1 \qquad \sigma_V^2(k) = \frac{2N_1(k)N_2(k)[2N_1(k)N_2(k) - N_1(k) - N_2(k)]}{[N_1(k) + N_2(k))^2(N_1(k) + N_2(k) - 1]} \tag{8}$$

By using (8), we can test the hypothesis of randomness at appropriate levels of significance. If N_1 and N_2 are both bigger than 8, then the sampling distribution of V is nearly a normal distribution and then

$$z(k) = \frac{V(k) - \mu_V(k)}{\sigma_V(k)} \tag{9}$$

is normally distributed with zero mean and variance 1. At each time instance k the the absolute value of the z-score obtained with (9) is compared with the 5% level of significance. the hypothesis of randomness is rejected if the threshold is exceeded.

4. EXAMPLE

In this section we present simulation results illustrating the performance of the algorithm. In the simulations we use linearized GMSK signals [1]. The pulse shape of this modulation is used as the receive filter impulse response. The pulse shape is the same for all the channels, however, due to the nature of (3) all the channels are independent.

The input signal is a binary sequence of $N = 1000$ symbols. Two users are considered, i.e. we have a (2×2) MIMO system. The length of each channel is L=2, hence, the state vector is a 8×1 vector. In this experiment we considered a HT channel with a mobile speed of 100 km/h and 1000 symbols were transmitted at SNR=10 dB. The data matrix X is known to the receiver.

The estimates of $\hat{R}$ and $\hat{Q}$ are presented in Figures 2 a) and b). We note that $\hat{Q}$ converges to a steady state value even if the true value it is not known. The results of the whiteness test are presented in Figure 3 where the formula (9) has been applied. The results indicate that the innovation sequences are white and hence the filter performs reliably.

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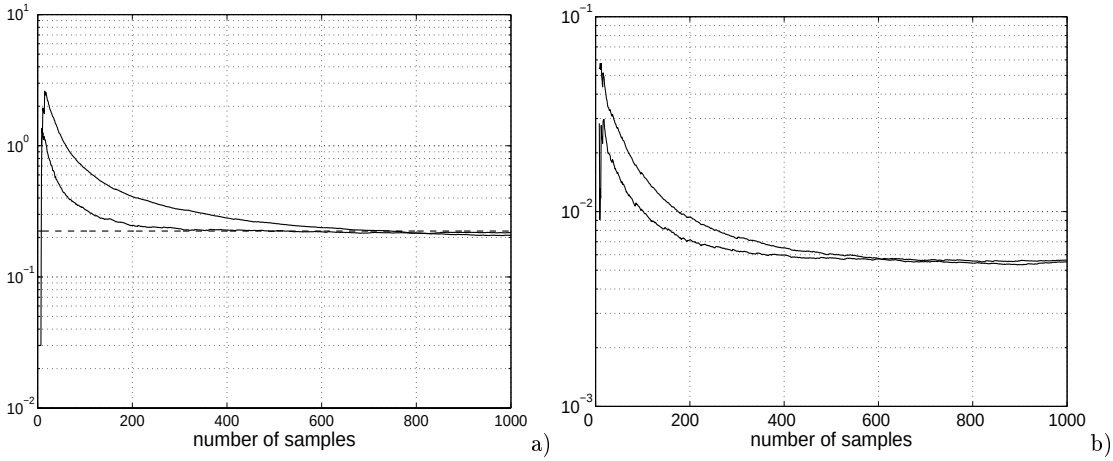

Figure 2: a) The on-line estimated diagonal elements of measurement noise variance, where dotted line is the true variance. Each line represents the time evolution of the diagonal elements of $\hat{R}$ corresponding to each user.b) The on-line estimated process noise variance. Each line represents the trace of the elements of $\hat{Q}$ corresponding to each user.

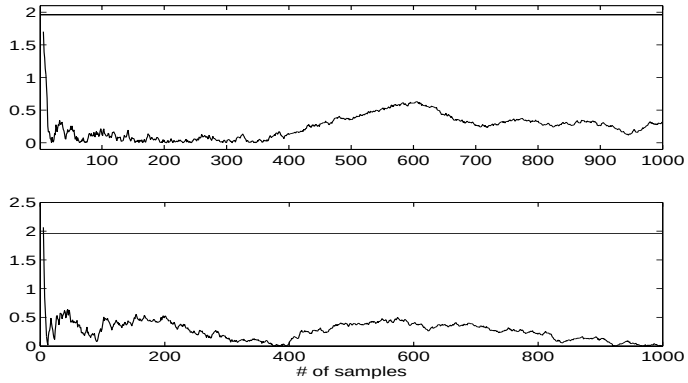

Figure 3: Results of whiteness test. z-score values and 5% significance level.

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