A Model for the Solitons Observed in Upper Ionosphere

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Abstract An electrostatic model for the solitons observed by Freja scientific satellite in upper ionosphere is established. The MHD equations are used in a cylindrical coordinate system and the "Sagdeev potential" is derived to study the nonlinear waves. The results show the existence not only of soliton with a density hump and soliton with a density dip, but also of periodic density wave and density shock wave. The theoretic amplitudes of the solitons are consistent with the Freja's observation, some waveforms of the solitons are very similar to that observed by the Freja, too.

I. INTRODUCTION

The nonlinear density waves were observed by Freja scientific satellite in the upper ionosphere[1-3]. The existence of ion acoustic solitons has been demonstrated in an unmagnetized plasma[4-6]. *Shukla, Yu*[7] and *Ray*[8] showed that finite amplitude ion acoustic solitons propagating obliquely to an external magnetic field can occur in a plasma. *Witt* and *Lotko*[9] studied ion-acoustic solitary waves in a low-β magnetized plasma with arbitrary electron equation of state. Nonlinear electrostatic waves propagating nearly perpendicular to the magnetic field have been studied by *Chaturvedi*[10] and *Temerin et al*[2]. *Lee* and *Kan*[11] used the exact ion dynamics formulation to show that nonlinear ion-acoustic periodic waves and solitons can occur in a low-β magnetized plasma. *Kalita* and *Bhatta*[12] investigated ion-acoustic solitary waves in a warm magnetoplasma with electron drift. Nonlinear density periodic waves in a cylindrically symmetric magnetic tube have been studied by *Molotovshchikov* and *Ruderman*[3]. *Maxon* and *Viecelli*[13] studied small amplitude cylindrically symmetric ion acoustic waves in plasma. *Wu et al*[14,15] studied the Solitary Kinetic Alfven Waves (SKAWs) in a plama.

In this paper, the "Sagdeev potential" in a low-β magnetized plasma is exactly derived from the MHD equations and the solutions of nonlinear periodic waves, solitons with a dip and a hump, and shock waves are found. We apply to use the solutions of solitons with a dip and a hump to interpret the Freja's observation.

II. BASIC MODEL

We assume the waves are excited in magnetized plasma with cylindrical symmetry and conditions as follows are satisfied. (1) The fluid consists of electrons and ions, and plasma pressure is much smaller than magnetic pressure (i.e., β <<1), so we only consider electrostatic waves. (2) Magnetic field $\boldsymbol{B} = B_0 \boldsymbol{e_Z}$ (B_0 is a constant, $\boldsymbol{e_Z}$ is the unit vector along

Z axis). (3) Phase velocity satisfies $v_{Ti} << v_p/\gamma << v_{Te}$ (here $v_\alpha = (2T_\alpha/m_\alpha)^{1/2}$ is particle thermal velocity, T_α and m_α are particle energy and mass, subscript " i " denotes ions and " e " denotes electrons, v_p is the phase velocity, θ is the angle between the wave vector $\boldsymbol{k}$ and $\boldsymbol{e_Z}$, $\gamma = \cos\theta$), so the Landau damping can be neglected. (4) The wave scale $\rho_i >> \lambda_D$ (λ_D is the Debye radius, $\rho_i = C_s/\Omega_i$ is the ion gyro radius, $C_s = (T_e/m_i)^{1/2}$ is the ion acoustic velocity, $\Omega_i = eB_0/(m_i c)$ is the ion gyro frequency, e is the elementary charge, c is the velocity of light), so charge separation effects can be neglected and the quasi-neutrality condition is satisfied, i.e., $n \approx n_e \approx n_i$ (n is the ion number density).

Taking a cylindrical coordinate system, and neglecting electron inertia because the electron mass is much smaller than ion mass, the MHD equations for ions can be written as

$$\frac{\partial n}{\partial t} + \frac{\partial (n v_r)}{\partial r} + \frac{\partial (n v_z)}{\partial z} = -\frac{n v_r}{r} \tag{1}$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{m_i n}\frac{\partial p}{\partial r} - \frac{e}{m_i}\frac{\partial \phi}{\partial r} + \frac{v_\theta^2}{r} + \Omega_i v_\theta \tag{2}$$

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} = -\frac{v_r v_\theta}{r} - \Omega_i v_r \tag{3}$$

$$\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{m_i n}\frac{\partial p}{\partial z} - \frac{e}{m_i}\frac{\partial \phi}{\partial z} \tag{4}$$

with

$$p = n T_i \tag{5}$$

$$n = n_i \approx n_e \approx n_0 \exp(e\phi/T_e) \tag{6}$$

where p is thermal pressure, ϕ is electric potential, $\mathbf{v}$ is the ion velocity and n_0 is constant.

It is convenient to introduce the following dimensionless quantities: $N = n/n_0$, $\tau = \Omega_i t$, $R = r/\rho_i$, $Z = z/\rho_i$, $\boldsymbol{V} = \mathbf{v}/c_s$, $\Phi = e\phi/T_e$, $T_i/T_e = a-1$, $M = v_p/c_s$, $v_p = \omega/k$. We look for solutions of Eq.(1)-(6) that depend on r, θ, z, and t through the variable $S = (k_r r + k_z z - \omega t)\Omega_i/\omega = (\alpha R + \gamma Z - M\tau)/M$, (here $\alpha = \sin\theta$, ω is the wave frequency, k_r and k_z are the r and z components of $\mathbf{k}$), then Eq.(6) becomes $N = \exp(\Phi)$. Considering that the waves propagate along the magnetic field (i.e., $\boldsymbol{k}$ parallel to $\boldsymbol{B}$), we obtain from Eq.(1)-(6)

$$\frac{1}{2}\left(\frac{dN}{dS}\right)^2 + \psi(N) = 0 \tag{7}$$

$$\psi(N) = \frac{\left[\left(N\sqrt{1-\frac{2a}{M^2}\ln N}\right)^2 - \left(\frac{a}{M^2-1}\right)^2 E_0^2\right]\left(N\sqrt{1-\frac{2a}{M^2}\ln N}\right)^2}{2\left[\frac{a}{M^2} - \left(1-\frac{2a}{M^2}\ln N\right)\right]^2} \tag{8}$$

Here $E_0 = -1/N \cdot dN/dS\,|_{N=0}$ is the normalized initial electric field. Eq.(7) is analogous to the energy integral of a classical particle in a 1-D "potential well". $\Psi(N)$ is called the "Sagdeev potential" [11].

III. NONLINEAR WAVES AND DISCUSSION

From Eq.(7) and (8), solutions for density waves can be obtained if the parameters lead to a "Sagdeev potential" $\Psi(N) < 0$. Our numerical solutions show that the waveform will be different if $\Psi(N)$ has different properties. Choosing S as the variable means that the wave is investigated in a coordinate system moving together with the wave.

In order to describe it in a more conventient way, we define a quantity

$$G_m = \sqrt{a}/M \exp\left[\left(1 - a/M^2\right)/\left(2a/M^2\right)\right] \tag{9}$$

1. Nonlinear Periodic Density Wave

When the plasma parameters satisfy one of the two conditions

$$0 < \left| \left(a/M^2 - 1\right)E_0 \right| < 1, \tag{10}$$

$$\left| \left(a/M^2 - 1\right)E_0 \right| > 1, \ a/M^2 > 1, \ \ G_m > 1 + \left| \left(a/M^2 - 1\right)E_0 \right|, \tag{11}$$

the Eq.(7) has a solution corresponding to a nonlinear periodic wave. So, each of the conditions (10) and (11) is sufficient for a nonlinear periodic density wave.

2. **Density Shock Wave**

When the plasma parameters satisfy the condition

$$\left| \left(a/M^2 - 1\right)E_0 \right| = 1, \quad G_m \le 2, \tag{12}$$

the Eq.(7) has a solution corresponding to a shock wave. So the condition (12) is sufficient for the existence of a density shock wave.

3. Soliton with a Density Dip

When the plasma parameters satisfy the condition

$$\left| \left(a/M^2 - 1\right)E_0 \right| = 1, \quad a/M^2 > 1, \quad G_m > 2, \tag{13}$$

the solution of Eq.(13) is the soliton with a dip. So, the condition (13) is consistent with the existence of the soliton with a density dip.

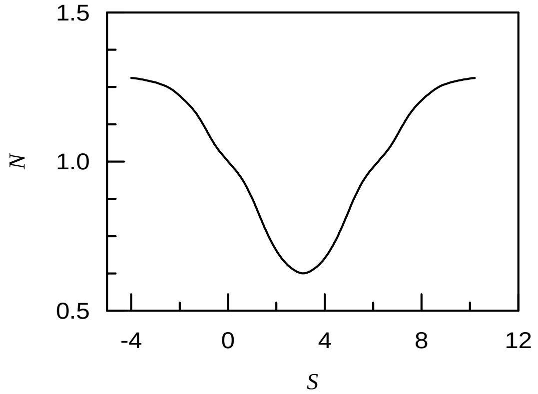

Fig.1 Soliton with a density dip for $a/M^2 = 10$ and $|E_0| = 1/9$

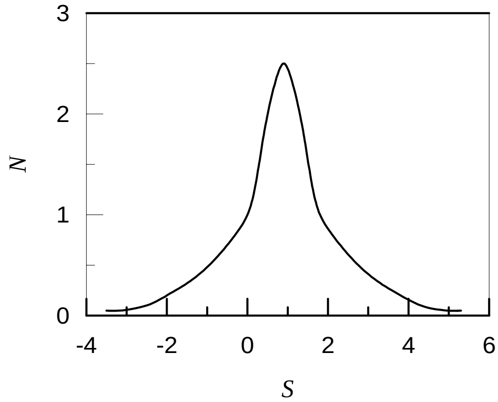

Fig.2 Soliton with a density hump for $a/M^2 = 0.2$ and $|E_0| = 1.25$

Fig. 1 gives the numerical solution of a solition with a density dip for $a/M^2 = 10$ and $|E_0| = 1/9$. The normalized density N in Fig.1 has a maximum of about 1.3 and a minimum of about 0.52.

4. Solitons with a Density Hump

When the plasma parameters satisfy one of the three conditions

$$\left|\left(a/M^2 - 1\right)E_0\right| = 1,\ a/M^2 < 1,\ G_m > 2, \tag{14}$$

$$\left|\left(a/M^2 - 1\right)E_0\right| > 1,\quad \frac{a}{M^2} < 1, \tag{15}$$

$$\left|\left(a/M^2 - 1\right)E_0\right| > 1,\qquad a/M^2 > 1,\quad G_m \le 1 + \left|\left(a/M^2 - 1\right)E_0\right|, \tag{16}$$

the soliton solution with a density hump can be obtained from Eq.(7). So, each of the conditions (14), (15) and (16) is consistent with the existence of a soliton with a density hump.

Fig. 2 demonstrates the numerical solution of the soliton with a density hump for $a/M^2 = 0.2$ and $|E_0| = 1.25$. The normalized density in Fig.2 has a maximum of about 2.5 and a minimum of about 0.1.

The other solution of the Eq.(7) is $N \equiv 1$ under the condition $a/M^2 = 1$ or $E_0 = 0$. In this case, the density will remain constant at n_0, and there will be no density wave.

IV. DISCUSSION

So far, we have described all nonlinear solutions of Eq.(7) and came to the conclusion that all nonlinear density waves mentioned above exist in low-β plasma with cylindrical symmetry. The density dip and hump solitons were observed by the Freja scientific satellite. *Wu et. al.*[14,15] considered the solitons as SKAWs because they are associated with electromagnetic spikes. In this paper, we give an electrostatic model for the nonlinear waves. Nevertheless, we also can compare our results with observations on board Freja. The densities of the dip and hump solitons observed by Freja can vary between 30 and 50%, sometimes even as high as 80%[15]. Our result shows that the densities in the dip or hump solitons change more than 20% (see Fig. 1), and sometimes even by more than a factor of 2 (see Fig. 2). Especially, the waveform of the dip soliton is very similar to that observed by the Freja satellite on June 13, 1993 [16].

According to Equation (6) we can calculate the electric potential and the electric field. So, there are also some electric disturbances accompanying the density solitons in our electrostatic model. The amplitudes of the magnetic disturbances associated with the density solitons observed by the Freja satellite are only several 10^{-2} nT or less. Therefore, the magnetic disturbances are very weak and can be neglected for the ambient magnetic field B_0=0.2G in Freja's orbit region. So, our electrostatic model can also be a candidate for density solitons observed by the Freja satellite.

V. CONCLUSION

An electrostatic model for the solitons observed by Freja scientific satellite in upper ionosphere is established. the MHD equations are used in a cylindrical coordinate system and the "Sagdeev potential" is derived for to study the existence of nonlinear plasma density waves propagating along the magnetic field in a low-β plasma with cylindrical symmetry. The result shows the existence of soliton with a density hump and soliton with a density dip, periodic density wave and density shock wave. The theoretic amplitudes of the solitons are consistent with the Freja's observation, some waveforms of the solitons are very similar to that observed by the Freja, too. So, our electrostatic model can be a candidate for density solitons observed by the Freja satellite.

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