A NEW TREATMENT OF THE PROPAGATION AND REFLECTION OF ELECTROMAGNETIC SIGNALS

Orsolya E. Ferencz, Csaba Ferencz

Eötvös University, Geophysical Dept., Space Research Group, H-1518 Budapest, Pf. 32, Hungary
spacerg@sas.elte.hu

ABSTRACT

The overwiev of the nature of the earlier (approximative) phenomenological descriptions of the propagation and reflection of signals in inhomogeneous media demonstrated that the earlier descriptions (models) - except the non-coupled W.K.B. method in slowly varying media - have an inherent misunderstanding, i.e. the starting physical concept is wrong. In the new model the physical concept contains the fact that the "forward" and "backward" propagating parts of the electromagnetic signals form a unique mode during the whole derivation process of the Maxwell's equations. The new solution in strictly monochromatic case and the derivation of the arbitrary shaped solutions are presented.

INTRODUCTION

"Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concept with which the theory operates" [1]. In the case of electromagnetic wave propagation in (linear, time invarying) inhomogeneous media the "objective reality" is the fact that the original (e.g. "forward" propagating or "source-") signal will attenuate during the propagation through the inhomogeneity, and simultaneously generaties another (reflected or scattered) signal part propagating in other diretion(s) in every point of the space in which the inhomogeneity exists. The "physical concept" is the description mode of this phenomenon, i.e. the derivation method of the solutions of Maxwell's equations and the assumtions used during this derivation process. As it is well known the whole process of derivatrion a real (full wave) solution of Maxwell's equations is a self consistent field (SCF) process, e.g. [2]. During this process we must make two crucial decisions. One is the supposition of the starting form of the solution (of the propagating signal), e.g. a single time harmonic [3-7, etc.] or an arbitrary shaped signal [8]. The second is the description of the medium-signal interaction for the given case and for the supposed form of the solution, e.g. [2, 5, 8]. Besides these an assumption was used generally during the earlier derivation processes (e.g. in the linear media): the different signal-parts appearing during the propagation in the inhomogeneous medium as a direct consequence of the propagation of the source-signal will propagate independently each other on such a manner that the coupling between them is managable by additional calculations if it is necessary. However, this assumption is wrong.

THE PROBLEM ITSELF

Demonstrating the problem let us investigate the propagation of a strictly monochromatic e.m. plane wave in a linear, time-invarying, lossless, non-dispersive, simple medium and let the inhomogeneity of the medium be one dimensional, i.e. the permittivity is definiable and changing only the *x*-direction and could be descibe as a scalar function of *x*

$$\varepsilon = \varepsilon(x) \quad \text{and} \quad \mu_0$$

is the permeability, it is constant and identical to the permeability of the vacuum. If we suppose that the solution of the Maxwell's equation could be a ***single*** monochromatic signal than the electric field strength is $\overline{E} = \overline{E}_0(\overline{r})e^{j[\omega t - \varphi(\overline{r})]}$, the magnetic field strength is $\overline{H} = \overline{H}_0(\overline{r})e^{j[\omega t - \varphi(\overline{r})]}$ and the starting form of the Maxwell's equations is well known

$$\begin{aligned} \overline{\nabla} \times \overline{H} &= j\omega \cdot \varepsilon(x) \cdot \overline{E}, \qquad & \overline{\nabla} \cdot \overline{H} &= 0, \\ \overline{\nabla} \times \overline{E} &= -j\omega\mu_0 \cdot \overline{H}, \qquad & \overline{\nabla} \cdot [\varepsilon(x)\overline{E}] &= 0. \end{aligned} \tag{1}$$

After some common steps

$$\left(\overline{\nabla}\times\overline{H}_0\right) - j\,\overline{\nabla}\varphi\times\overline{H}_0 = j\omega\varepsilon\,\overline{E}_0\,,$$
$$\left(\overline{\nabla}\times\overline{E}_0\right) - j\,\overline{\nabla}\varphi\times\overline{E}_0 = -j\mu_0\overline{H}_0\,. \tag{2}$$

It is a common method to separate the "real" and "imaginarial" parts of these equations - see e.g. the eikonal solution, the W.K.B. approximation [2, 5] - to determine the propagating signals. From the "imaginarial" part of (2) it is possible to derive the j function, however, from the "real" part in every case the result is

$$E_0 = \text{constant} \qquad \text{and} \qquad H_0 = \text{constant}\,,$$

i.e. the amplitude of the signal can not change during the propagation. This result is trivially wrong. The result is the same using every earlier derivation methods, except the non-coupled W.K.B. approximation in quasi-homogeneous cases and the description of signal-scattering. (A more detailed analysis of this character of the older known methods is presented in "unpublished" [9].) If we compare the character of the amplitude changing of a signal (solution) propagating in a homogeneous, but lossy medium to the solution investigated above, we must realise that in the homogeneous, lossy case the "amplitude" term of the solution remain constant, the cause of the attenuation is the phase function, which has an imaginarial part in this case.

This means that in the "physical concept" is not enough to know that the signal amplitude must be change, however, the "physical concept" must contain the general structure of the solution in connection to the real physical process behind the changings as a part of the supposed form of the solution at the start of derivation.

THE REAL FULL WAVE SOLUTION IN MONOCHROMATIC CASE

Let us continue the investigation of the propagation in an inhomogeneous, linear medium with isotropic permittivity, see e.g. (1); however, during the supposition of the form of the monochromatic solution let us use the Method of Inhomogeneous Basic Modes (MIBM) [2, 7, 8]. Therefore

$$\overline{E} = \sum_{i=1}^{n} \overline{E}_{0i}(\overline{r})\, e^{j[\omega t - \varphi_i(\overline{r})]} \quad \text{etc.} \quad \text{and} \quad \overline{k}_i(\overline{r}) \mathrel{\hat{=}} \overline{\nabla}\,\varphi(\overline{r}) \tag{3}$$

The Maxwell's equations are after a simplification by exp(*jwt*)

$$\sum_i \left(\overline{\nabla}\times\overline{H}_{0i}\right) e^{-j\int \overline{k}_i\,d\overline{r}} - \sum_i j\overline{k}_i\times\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = \sum_i j\omega\,\varepsilon(x)\,\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}}\,,$$
$$\sum_i \left(\overline{\nabla}\times\overline{E}_{0i}\right) e^{-j\int \overline{k}_i\,d\overline{r}} - \sum_i j\overline{k}_i\times\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = -\sum_i j\omega\,\mu_0\,\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}}\,,$$
$$\sum_i \mu_0\left(\overline{\nabla}\cdot\overline{H}_{0i}\right) e^{-j\int \overline{k}_i\,d\overline{r}} - \sum_i j\,\mu_0\,\overline{k}_i\cdot\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = 0\,, \tag{4}$$
$$\sum_i \left[\overline{\nabla}\,\varepsilon(x)\right]\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} + \sum_i \varepsilon(x)\left(\overline{\nabla}\cdot\overline{E}_{0i}\right) e^{-j\int \overline{k}_i\,d\overline{r}} - \sum_i j\overline{k}_i\cdot\varepsilon(x)\,\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = 0.$$

Let us separate (4) as it was defined by MIBM - see e.g. in [8] - and than it follows:

- the equation-system defining the inhomogeneous basic modes are for each value of *i* independently

$$\overline{k}_i\times\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = -\omega\,\varepsilon(x)\,\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}}\,, \qquad \overline{k}_i\cdot\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = 0\,,$$
$$\overline{k}_i\times\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = \omega\,\mu_0\,\overline{H}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}}\,, \qquad \overline{k}_i\cdot\varepsilon(x)\,\overline{E}_{0i}\,e^{-j\int \overline{k}_i\,d\overline{r}} = 0; \tag{5}$$

- the "coupling"-equations are

$$\sum_i \left(\overline{\nabla}\times\overline{H}_{0i}\right)e^{-j\int \overline{k}_i\, d\overline{r}} = 0,\quad \sum_i \left(\overline{\nabla}\cdot\overline{H}_{0i}\right)e^{-j\int \overline{k}_i\, d\overline{r}} = 0,$$

$$\sum_i \left(\overline{\nabla}\times\overline{E}_{0i}\right)e^{-j\int \overline{k}_i\, d\overline{r}} = 0,\quad \sum_i \left[\overline{\nabla}\,\varepsilon(x)\right]\overline{E}_{0i}\, e^{-j\int \overline{k}_i\, d\overline{r}} + \varepsilon(x)\sum_i \left(\overline{\nabla}\cdot\overline{E}_{0i}\right)e^{-j\int \overline{k}_i\, d\overline{r}} = 0. \qquad (6)$$

From (5) we can derive the inhomogeneous basic modes, and using these modes in (6) it is possible to determine the real spce-time finctions of the propagating and reflected signals. Solving (5) and supposing that the source-signal propagates parallel to *x*-axis it follows from (5) that *i*=1 and 2 where

$$\overline{k}_1 = -\overline{k}_2\ ,\qquad \left|\overline{k}_i\right| = k \qquad \text{and} \qquad Z_0(x) = \sqrt{\mu_0/\varepsilon(x)}$$

and these gradients are parallel to *x*-axis, i.e. to the gradient of the permittivity. For the source-signal *i*=1 and for the reflected signal part generated by the source-signal *i*=2. Using these results in (6) the final form of the "coupling"-equations is

$$\frac{dE_{01y}}{dx} = \frac{1}{Z_0}\frac{dZ_0}{dx}\left(E_{01y} - E_{02y}\, e^{j2\int k\,dx}\right),$$

$$\frac{dE_{02y}}{dx} = \frac{1}{Z_0}\frac{dZ_0}{dx}\left(E_{02y} - E_{01y}\, e^{-j2\int k\,dx}\right). \qquad (7)$$

The boundary conditions of the solutions of (7) are defined out of the inhomogeneity and they are the followings: the medium is inhomogeneous from *x*=0 to $x=x_M$ and it is homogeneous out of this range; the source of the signal exists somewhere in the *x*<0 halfspace defining the starting values of the source-signal amplitudes, i.e.

$$E_{01y}(x<0) = \text{constant} = E_{10} \qquad \text{and} \qquad H_{01y}(x<0) = \text{constant}\,;$$

the amplitudes of the reflected (backward propagating) signal part are in the halfspace $x>x_M$

$$E_{02y}(x>x_M) \equiv 0 \qquad \text{and} \qquad H_{02y}(x>x_M) = 0\,.$$

The final differential equations deriving from (7) are of non-Riccati type and have closed form solutions, which are in this case

$$E_{1y}(x,t) \cong E_{10}Z_0\left\{1 - \int_0^x \frac{d(\ln Z_0)}{du}\, e^{j2\int_0^u k\,dv}\left[\int_u^{x_M}\frac{d(\ln Z_0)}{dw}\, e^{-j2\int_0^w k\,dv}\, dw\right]du\right\} e^{j\left(\omega t - \int_0^x k\,dv\right)},$$

$$E_{2y}(x,t) \cong E_{10}Z_0\left\{\int_x^{x_M}\frac{d(\ln Z_0)}{du}\, e^{-j2\int_0^u k\,dv}\, du\right\} e^{j\left(\omega t + \int_0^x k\,dv\right)}\ . \qquad (8)$$

The high frequency assymptote of this solution (8) is the correct one, i.e.

$$E_{1y}(x,t) = E_{10}Z_0\, e^{j\left(\omega t - \int_0^x k\,dv\right)},\quad E_{2y}(x,t) = 0\ . \qquad (9)$$

SOLUTION FOR ARBITRARY SHAPED SIGNALS

In the case of arbitrary shaped signals a new type of problem exists. This new problem is the mathematical mixing of two phenomena which have different physical origin (background). One of these phenomena is the arbitrary character, the arbitrary changing in space and time of the shape-function of the propagating signals. The other phenomenon is the effect of the inhomogeneous medium point-by-point in space to the shape of the signals propagating trough the inhomogeneity. These two phenomena must separate each other, however, this speparation is not so simple as it was seen in the case of monochromatic waves, see equations (4), (5) and (6). Therefore it is necessary to apply to original form of solutions using in MIBM which was defined in [2, 7] originally for monochromatic waves. Therefore let the form of solutions be

$$\overline{E} \triangleq \sum_i a_i(x)\overline{E}_i(x,t) \quad \text{and} \quad \overline{H} \triangleq \sum_i a_i(x)\overline{H}_i(x,t) \tag{10}$$

Let us investigate the propagation in an inhomogeneous magnetised (anisotrpic) plasma as it was made in [8]. Applying (10) in the Maxwell's equtations and the MIBM philosophy the inhomogeneous bacis modes are defined by the following equations independently for each *i* value:

$$\begin{aligned} &\overline{\nabla}\times\overline{H}_i = \overline{J}_i + \varepsilon_0 \frac{\partial \overline{E}_i}{\partial t}, \qquad \overline{\nabla}\cdot\overline{H}_i = 0 \ , \\ &\overline{\nabla}\times\overline{E}_i = -\mu_0 \frac{\partial \overline{H}_i}{\partial t} \ , \qquad \overline{\nabla}\cdot\overline{E}_i = \frac{\rho_i}{\varepsilon_0} \ . \end{aligned} \tag{11}$$

The solutions of (11), i.e. the inhomogeneous basic modes in the following could be derive for example on such a way which was used in [8] for weak and strong inhomogeneities. Using the solutions of (11) in the remaining part of the Maxwell's equations, i.e. in the "coupling"-equations we can derive the real full wave solution of the propagating signal. The "coupling"-equations are

$$\begin{aligned} &\sum_i \overline{\nabla} a_i \times \overline{H}_i = 0 \ , \qquad \sum_i \overline{\nabla} a_i \cdot \overline{H}_i = 0 \ , \\ &\sum_i \overline{\nabla} a_i \times \overline{E}_i = 0 \ , \qquad \sum_i \overline{\nabla} a_i \cdot \overline{E}_i = 0 \ . \end{aligned} \tag{12}$$

In the most simple cases *i*=1 and 2, i.e. a source-signal and a reflected signal exist.

CONCLUSIONS

It was presented that the earlier phenomenological descriptions of the propagation in inhomogeneous media - except the mirror-type and scattering-typoe descriptions and the non-coupled W.K.B. approximation - have an inherent misunderstanding and therefore these methods are wrong. Using a better "physical concept" of the structure of propagating signals and the Method of Inhomogeneous Basic Modes it was possoble to find correct full-wave solutions for propagation of e.m. signals in inhomogeneous media. The final form of the differential equations to be solved is non-Riccatian. The main cause of the misunderstanding is the fact, that the source and reflected parts of the signal form a unique mode.

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