3D AZIMUTH-ELEVATION-CARRIER ESTIMATION VIA VOLUME ARRAYS

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ABSTRACT

A new approach to the problem of joint azimuth-elevation-carrier (AEC) estimation of narrowband signals is introduced. The proposed approach makes it possible to solve the problem by using only spatial samples taken by volume antenna arrays. The sufficient conditions in terms of array geometry are formulated and proved. The simulation shows that the iterative Newton-based maximum likelihood (ML) AEC estimator has a performance close to the Cramer-Rao Bound (CRB). One major advantage of the proposed approach is its simplicity for practical implementation that could be attractive in many real world applications.

INTRODUCTION

The problem of joint AEC estimation arises in communication, radar, radio astronomy, sonar and many other applications. Conventional approaches to this problem, such as ML [1], space-time adaptive processing (STAP) [2], ESPRIT [3]-[6] and others [7] are based on utilising the large number of temporal samples for carrier estimation. The necessity to record, store and treat the temporal samples results in a considerably complicated implementation of DSP tools, especially in MW band, and increase in cost. Besides this, the STAP and ESPRIT-based methods are not statistically optimal. Computationally efficient ML-based methods [1] as well as ESPRIT-based algorithms [3]-[6] can be implemented only in arrays with periodic structure.

In this paper we show that the problem of joint AEC estimation can be solved by means of spatial samples collected in volume antenna arrays. The proposed approach does not require recording of a large number of array outputs, and the temporal averaging is necessary only to ensure a full rank of the covariance matrix of signal waveforms as well as to increase the SNR. Analog tools, however, can successfully carry out this procedure. Another important advantage of the proposed approach is that its implementation does not require arrays with periodic structure. The requirements regarding array geometry are quite weak, which is also attractive for many applications.

DATA MODEL AND PROBLEM FORMULATION

We assume that there are M point sources located in the far field that are emitting the unknown narrow-band complex deterministic signals $s_m(t),\ m=1...M$ in the direction of the measurement system. For every signal the parameters of interest are the azimuth β_m, the elevation ε_m and the carrier f_m. To describe the signal parameters let us introduce a vector $\boldsymbol{\mu}_m = f_m \boldsymbol{e}_m / f_{max} \in R^3$, $\boldsymbol{\mu}_m \in U_\mu = \{\boldsymbol{\mu} : \beta \in [0,\pi], \varepsilon \in [0,\pi], f \in [f_{min}, f_{max}]\}$, where $\boldsymbol{e}_m=[cos\beta_m cos\varepsilon_m,\ sin\beta_m cos\varepsilon_m,\ sin\varepsilon_m]^T$ is the unit vector in the Cartesian coordinate system and $(\cdot)^T$ denotes transpose. It is assumed that the measurement system consists of N omnidirectional point sensors. Then the model of antenna array output can be represented as follows:

$$\boldsymbol{z}(t) = \boldsymbol{x}(t) + \boldsymbol{n}(t) = \boldsymbol{A}(\gamma)\boldsymbol{s}(t) + \boldsymbol{n}(t),\ t = 1...L, \tag{1}$$

where $\boldsymbol{x}(t) \in C^{N\times 1}$ is the vector of noiseless signals, $\boldsymbol{A}(\boldsymbol{\gamma})=[\boldsymbol{a}(\boldsymbol{\mu}_1) \ldots \boldsymbol{a}(\boldsymbol{\mu}_M)] \in C^{N\times M}$ is a matrix of steering vectors, $\boldsymbol{\gamma}=[\boldsymbol{\mu}_1^T \ldots \boldsymbol{\mu}_M^T]^T \in R^{3M\times 1}$ is a vector of parameters of M signals, $\boldsymbol{a}(\boldsymbol{\mu}_m)=[1,\ \exp\{j2\pi\ \boldsymbol{d}_2^T \boldsymbol{\mu}_m\},\ \ldots,\ \exp\{j2\pi\ \boldsymbol{d}_2^T \boldsymbol{\mu}_m\}]^T$ is a steering vector, $\boldsymbol{d}_n = d_n \boldsymbol{i}_n / \lambda_{min}$ is a vector of the nth sensor position ($\boldsymbol{d}_1$=$\boldsymbol{0}$), d_n is a distance to the nth sensor, $\boldsymbol{i}_n$ is a unit vector toward the nth sensor position, $\lambda_{min} = c/f_{max}$ is a minimum wavelength, c is the signal velocity in the medium, $\boldsymbol{D}$=[$\boldsymbol{d}_1$... $\boldsymbol{d}_N$]$\in R^{3\times N}$ is a matrix that specify antenna array geometry, $\boldsymbol{s}(t)\in C^{M\times 1}$ is a vector of the signal waveforms, L denotes the number of data snapshots available, $\boldsymbol{n}(t)\in \boldsymbol{C}^{N\times 1}$ is additive noise. We assume, that the sampled covariance matrix of signal waveforms $\hat{\boldsymbol{P}} = (L)^{-1}\Sigma_{t=1}^{L}\boldsymbol{s}(t)\boldsymbol{s}^H(t)\in C^{M\times M}$ has full rank, that is $rank(\hat{\boldsymbol{P}}) = M$, where $(\cdot)^H$ stands for Hermitian transpose. This means, that the number of temporal samples $L \geq M$. The noise is assumed to be stationary and ergodic complex Gaussian process with zero mean and covariance matrix $\sigma^2\boldsymbol{I}$, where σ^2 is an unknown scalar and $\boldsymbol{I}$ is the identity matrix. Another common-used assumption is that the number of signals M is known. If the number of signals is unknown the generalization of well-known approaches to the signal number estimation can be developed for the case considered here.

The first basic issue related with the problem of joint AEC estimation by 3-D arrays is the identifiability problem. This problem is usually formulated as the determination of the maximum number of harmonics that can be resolved for a given total sample size (see, e.g., [8]-[10] and references herein). To solve this problem, the assumption that the matrix $A(\boldsymbol{\gamma})$ has full rank is made. However, this assumption causes the next important question: under which conditions in terms of the number of array sensors N and their positions $\boldsymbol{D}$ the matrix $A(\boldsymbol{\gamma})$ has the full rank for any distinct $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U_\mu$? Therefore the problem to be solved in this paper can be formulated as follows: 1) To determine the basic requirements for the number N and positions $\boldsymbol{D}$ of array sensors, that guarantee the unique estimation of any distinct set $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U_\mu$ of signal parameters; 2) To derive the algorithm for finding the ML AEC estimates at the presence of sensor noise.

IDENTIFIABILITY OF AEC ESTIMATES

The necessary and sufficient conditions for identifiability of U_μ are [8]-[10] *NSC1:* the set of steering vectors $\boldsymbol{a}(\boldsymbol{\mu})$ is known; *NSC2:* for any $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U_\mu$ the matrix $A(\boldsymbol{\gamma})$ has full rank

$$rank[\boldsymbol{a}(\boldsymbol{\mu}_1)...\boldsymbol{a}(\boldsymbol{\mu}_M)] = min(N, M) = M < N; \tag{2}$$

NSC3: the number of sensors N, the rank R of matrix $\boldsymbol{P}$ and the number Q of parameters per signal satisfy to the following inequality $N \geq R + Q = M + 3$. The *NSC1* and *NSC3* hold if the antenna array is calibrated and its geometry $\boldsymbol{D}$ is known, so that the steering vector $\boldsymbol{a}(\boldsymbol{\mu})$ is a known function of its argument $\boldsymbol{\mu}$, and the maximum number of impinging signals $M \leq N - 3$. Therefore, our first problem can be reduced to the determination of the requirements for the sensor positions $\boldsymbol{D}$ that guarantee the implementation of the *NSC2*.

Consider a Vandermonde matrix $\boldsymbol{V} \in C^{N\times M}$ with entries of $[\boldsymbol{V}]_{n,m} = \alpha_m^{p_n}$, where $\alpha_1 \neq ... \neq \alpha_M$ are the generators and $p_n = n - 1$. An important property of the Vandermonde matrix is that any of its two rows and two columns has correspondingly $P_{row} \geq M - 1$ and $P_{col} \geq N - 1$ distinct elements. As a result any M rows and columns are linear independent and the Vandermonde matrix has full rank [11]. But it is evident that an arbitrary matrix $\boldsymbol{A} \in C^{N\times M}$ with entries of $[\boldsymbol{A}]_{n,m} = \alpha_m^{p_n}$, distinct generators and any $p_1 \neq ... \neq p_N$ has also full rank. Thus, we can state that the rank of such a matrix is defined as $rank(\boldsymbol{A}) = min(N - K_r, M - K_c)$, where K_r is the number of rows and K_c is the number of columns that are coincident to any other row or column respectively. The following Theorem formulates the sufficient conditions for identifiability of the set U_μ.

Theorem: For any $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U_\mu$ the matrix $A(\boldsymbol{\gamma})$ has full rank if: i) the array has a subarray with $K \leq N$ sensors, such that the number N_p of the parallel planes that can be passed through the points $\boldsymbol{d}_1,...,\boldsymbol{d}_K$ satisfy the conditions $N_p = K - 2 > M$, and ii) the maximum distance d_{max} between any nearest sensors of subarray is less then half of minimum wavelength $d_{max} \leq \lambda_{min}/2 = c/(2f_{max})$.
Proof: Let us find the roots $\boldsymbol{\mu}$ of the following set of equation

$$a_n(\boldsymbol{\mu}_1) = a_n(\boldsymbol{\mu})e^{j2\pi k}, \quad \boldsymbol{\mu}_1, \boldsymbol{\mu} \in U_\mu, \quad n = 2...K, \quad k = 0, \pm 1, ... \tag{3}$$

Condition ii) means, that the maximum sampling frequency in spatial domain is higher then the Nyquist rate, therefore (3) can be rewritten as follows

$$\boldsymbol{i}_n^T \boldsymbol{\mu} = c_n, \quad n = 2...K, \tag{4}$$

where $c_n = \boldsymbol{i}_n^T \boldsymbol{\mu}_1$. The vectors $\boldsymbol{\mu}^{(n)}$ that satisfy to the nth equation of (4) represent the set of points lying on a plane $\Xi(\boldsymbol{i}_n, c_n) \in R^2$ that is orthogonal to the vector $\boldsymbol{i}_n$ and located on the distances $|c_n| \le 1$ from the origin. Therefore the set U'_μ of roots of (4) are the intersection between U_μ and the planes $\Xi(\boldsymbol{i}_n, c_n)$:

$$U'_\mu(\boldsymbol{i}_2...\boldsymbol{i}_K) = U_\mu \bigcap_{n=2}^{K} \Xi(\boldsymbol{i}_n, c_n). \tag{5}$$

Observe, that the set U_μ is a space between two hemispheres of radiuses f_{min}/f_{max} and 1. Therefore, if the vectors $\boldsymbol{d}_1...\boldsymbol{d}_K$ do not lie on the same plane, i.e. $rank([\boldsymbol{d}_1...\boldsymbol{d}_K]) = 3$, then the set U'_μ is a point and only point $U'_\mu = \{\boldsymbol{\mu}_1\}$. It means, that if the subarray is a volume then for any $\boldsymbol{\mu}_1 \ne ... \ne \boldsymbol{\mu}_M \in U_\mu$ the inequality $\boldsymbol{a}(\boldsymbol{\mu}_1) \ne ... \ne \boldsymbol{a}(\boldsymbol{\mu}_M)$ holds true and $K_c = 0$. Observe also, that if $K_1 \le K$ sensors lie on a plane, then the set $U'_\mu(\boldsymbol{i}_2...\boldsymbol{i}_{K_1})$ is a section and consists of infinite number of elements. Hence, for any distinct $\boldsymbol{\mu}_1 \ne ... \ne \boldsymbol{\mu}_M \in U'_\mu(\boldsymbol{i}_2...\boldsymbol{i}_{K_1})$, K_1 equations of (3) hold and K_1 rows of the matrix $[\boldsymbol{a}(\boldsymbol{\mu}_1)...\boldsymbol{a}(\boldsymbol{\mu}_M)]$ are equal, i.e. $K_r = K_1$. The $K_r - 1$ rows can be rejected without loss of information. Therefore, to ensure the full rank of the matrix $\boldsymbol{A}(\boldsymbol{\gamma})$ the number of sensors lying in nonparallel planes has to satisfy the condition $N_p > M$. Since the plane can be passed through any three sensors, the minimum size of the subarray $K = N_p + 2$, so that we get the condition $N_p = K - 2 > M$.

ML ESTIMATES OF THE SIGNAL AEC

The ML estimates of the signal parameters $\hat{\boldsymbol{\gamma}} = [\hat{\boldsymbol{\mu}}_1^T, ...\hat{\boldsymbol{\mu}}_M^T]^T$ are obtained as a maximizer of the following criterion function (see, e.g., [1], [12]-[13]):

$$\hat{\boldsymbol{\gamma}}_{ML} = arg \max_{\gamma} \varphi(\boldsymbol{\gamma}), \qquad \varphi(\boldsymbol{\gamma}) = tr\boldsymbol{\Pi}(\boldsymbol{\gamma})\hat{\boldsymbol{R}}, \tag{6}$$

where tr is the trace of a matrix; $\hat{\boldsymbol{R}} = (L)^{-1}\Sigma_{t=1}^{L}\boldsymbol{z}(t)\boldsymbol{z}^H(t) \in C^{N\times N}$ is a sampled covariance matrix; $\boldsymbol{\Pi}(\boldsymbol{\gamma}) = \boldsymbol{A}(\boldsymbol{\gamma})\boldsymbol{A}^-(\boldsymbol{\gamma})$ is the projector onto the signal subspace; $\boldsymbol{A}^-(\boldsymbol{\gamma})$ is the Moore-Penrose pseudo-inverse of $\boldsymbol{A}(\boldsymbol{\gamma})$. Since direct maximization of $\varphi(\boldsymbol{\gamma})$ over $3M$-dimentional parameter space is a very expensive procedure the corresponding extension of the various techniques like MODE, simulated annealing, iterative, genetic, grid search approach and other can be developed.

SIMULATION RESULTS

In the simulations we assumed a 3-D conical array of four ring subarrays of radiuses λ_0; $0.5\lambda_0$; $0.25\lambda_0$; 0 and 12; 6; 3; 1 omnidirectional sensors in each, correspondingly, such that N=22. The subarrays were placed on four planes parallel to the ground surface. The distance between planes was $d_z = \lambda_0/2$, where λ_0 is a central frequency. We consider scenario of L=32 snapshots and $M = 2$ equally powered narrowband sources with the DOA's $\varepsilon_1 = 10^0, \beta_1 = 10^0, \varepsilon_2 = 15^0, \beta_2 = 15^0$ and relative carriers $\nu_1 = 0.9, \nu_2 = 1.3$, where $\nu_m = f_m / f_0 = \lambda_0 / \lambda_m$. Fig. 1 displays the empirical RMSE's of (a) DOA and (b) carrier estimation versus the SNR for iterative Newton-based AEC ML estimator [13]. In this figure, the CRB [12] is also shown.

As it follows from this figure, the joint azimuth, elevation and carrier estimates have well-known properties of ML estimates [12]. For instance, the RMSE like the CRB monotonically decreases with increasing SNR. But RMSE does

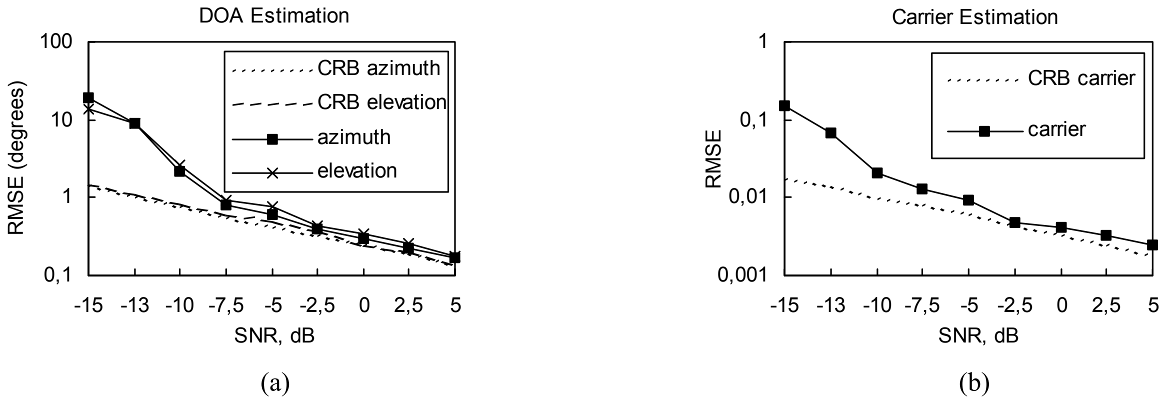

Fig.1. Empirical RMSE's of (a) DOA and (b) carrier estimation versus the SNR for the iterative AEC ML estimator.

not achieve the CRB even when SNR tend to infinity since ML estimates are not statistically efficient if N and $L < \infty$. But at high SNR the high accuracy of the carrier estimation can be reached by means of the proposed method. This conclusion is important from practical point of view because in practice the measurement systems have acceptable performances only when they work at high SNR.

CONCLUSION

The problem of joint AEC estimation can be solved by means spatial samples recorded by volume antenna array. We have formulated and proved the sufficient conditions in terms of array geometry that guarantee uniqueness of AEC estimation. For the case of sensor noise the iterative ML estimator having high computationally efficiency can be used. Statistical simulation confirmed that the RMSE like the CRB monotonically decreases with increasing SNR. Therefore at high SNR the high accuracy of joint AEC estimation can be reached by the proposed method. The possible applications could include the wide-range spectrum monitoring direction finders, Doppler radar systems, wireless communication systems and other.

REFERENCES

[1] L. Swindlehurst, P. Stoica, "Maximum likelihood methods in radar array signal processing," *Proceedings of IEEE*, vol 86, 1998, pp. 421-441.

[2] T. K. Sarkar, H. Wang, S. Park, R. Adve, J. Koh, K. Kim et al., "A deterministic least-squares approach to space-time adaptive processing (STAP)," *IEEE Trans. Antennas and Propagation,* vol. 49, pp. 91 –103, January 2001.

[3] A.N. Lemma, A.-J. Van der Veen and E.F. Deprette, "Joint angle-frequency estimation using multi-resolution ESPRIT," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Proces.*, Seattle, Washington, May 1998, pp. 1957-1960.

[4] M.D. Zoltowski and C.P. Mathews, "Real-time frequency and 2-D angle estimation with sub-Nyquist spatio-temporal sampling," *IEEE Trans. Signal Processing*, vol. 42, pp. 2781-2794, October 1994.

[5] M. Haardt and J.A. Nossek, "3-D unitary ESPRIT for joint angle and carrier estimation," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Proces.*, Munich, Germany, April 1997, pp. 255-258.

[6] P. Strobach, "Total least squares phased averaging and 3-D ESPRIT for azimuth-elevation-carrier estimation," *IEEE Trans. Signal Processing*, vol. 49, pp. 54-62, January 2001.

[7] M. Viberg and P. Stoica, "A computationally efficient method for joint direction finding and frequency estimation in colored noise," *Proc. of 32 ASILOMAR. Conf. on Signals, Systems and Computers*, Pacific Grove, CA, 1998, pp. 1547-1551.

[8] B. Hochwald and A. Nehorai, "Identifiability in array processing models with vector-sensor applications," *IEEE Trans. Signal Processing*, vol. 44, pp. 83-95, January 1996.

[9] J. Tao, N.D. Sidiropoulos, and J.M.F. ten Berge, "Almost-sure identifiability of multidimensional harmonic retrieval," *IEEE Trans. Signal Processing*, vol. 49, pp. 1849 -1859, September 2001.

[10] K.-C. Tan, S.S. Goh and E.-C. Tan, "A study of the rank-ambiguity issues in direction-of-arrival estimation," *IEEE Trans. Signal Processing*, vol. 44, pp. 880-887, April 1996.

[11] P. Stoica and R. Moses, *Introduction to spectral analysis*. Upper Saddle River, NJ: Prentice Hall, 1997.

[12] P. Stoica and A. Nehorai, "MUSIC, Maximum Likelihood and Cramer-Rao Bound". *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. 37, pp. 721-741, May 1989.

[13] D. Starer and A. Nehorai, "Newton Algorithms for Conditional and Unconditional Maximum Likelihood Estimation of the Parameters of Exponential Signals in Noise," *IEEE Trans. Signal Processing*, vol. 40, pp.1528-1534, June 1992.