BISTATIC CROSS-SECTIONS OF CONCAVE-CONVEX CONDUCTING CYLINDERS IN A CONTINUOUS RANDOM MEDIUM

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ABSTRACT

Bistatic radar cross-sections (BRCS) of concave-convex conducting cylinders in a continuous random medium are analyzed for E-wave and H-wave incidence, under the condition that the spatial coherence length of the incident waves around the cylinder is larger enough than the size of the cylinder. The numerical results show that the behavior of BRCS normalized to that in free space, which indicates the random medium effect on the BRCS, is independent of the polarization of incident waves and the body configuration. The results also show oscillation of the normalized BRCS in the neighborhood of the backward direction in case of weak backscattering enhancement.

INTRODUCTION

Imaging in random media is one of key subjects in the fields of radar engineering, medical engineering, astronomy, and so on. The Monostatic and Bistatic Radar Cross-Section (MRCS and BRCS) of a body in a random medium are a fundamental quantity in these fields. As well known, the statistical coupling of incident and scattered waves yields backscattering enhancement under a certain condition as one of representative phenomena in wave propagation in a random medium. In many practical cases a body is surrounded by a random medium and therefore it may happen that the RCS of the body is remarkably different from that in free space. However the analysis of wave scattering from a body in a random medium is not a problem to be solved by conventional methods because incident and scattered waves are all random.

We have presented an approach for analyzing the problem as a boundary value problem [1]. According to the approach, wave scattering from a conducting body may be treated as the following process. A wave radiated from a source propagates in a random medium, illuminates a body in a random medium and induces a surface current. A scattered wave produced by the surface current propagates in the random medium and a part of the scattered wave illuminates again on the body as a re-incident wave and induces the surface current, which produces another scattered wave. In this way scattered waves are recursively produced and we can get the scattered wave as the sum of all scattered waves produced by abovementioned method. Our approach is based on general results of both the independent studies on the surface current on a conducting body in free space and on the wave propagation and scattering in a random medium. A non-random operator, called current generator, is introduced to get the surface current from any incident wave. The operator depends only on the body surface and can be constructed by Yasuura's method. On the other hand, the wave propagation in a random medium is expressed by use of Green's function in the medium. Here, a representative form of the Green's function is not required but the moments are done for the analysis of average quantities concerning observed waves. It remains an unsettled question how to get the moments of Green's functions.

It is difficult to get exactly the fourth moment of Green's functions needed to analyze the RCS. When an incident wave propagates in a strong random medium and becomes sufficiently incoherent around a body, we can approximate the fourth moment as the sum of products of the second moments only for the backscattering and analyze the MRCS by using the moment and the current generator. The numerical results for circular and elliptic conducting cylinders show that these

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MRCS become nearly twice as large as those in free space under the condition that the coherence length of the incident waves around the cylinders is larger enough than the size of the cylinders [1, 2]. If the condition becomes invalid, they show that the MRCS is enhanced more than 2 times for some cases and is diminished for some other cases compared with that in free space [1, 2]; in particular, the remarkable behavior of MRCS different from that in free space is revealed for convex-concave conducting cylinders [3, 4].

When the average scattered intensity is enhanced in the backward direction, it is predicted to decrease in the neighborhood of the backward direction from the law of energy conservation and the statistical independency of scattered waves at points separated widely from each other [5]. To make clear numerically the prediction as well as the scattering characteristics for a practical body scattering, we try to analyze the BRCS of a conducting body in a random medium. We apply a two-scale asymptotic procedure [6, 7] to get a more general fourth moment of Green's functions and have a great success in analyzing the BRCS of conducting circular cylinders [8, 9]. The numerical results agree well with the law of energy conservation and show the prediction. We also find out that the BRCS oscillates as a function of the observation angle between the incidence and observation directions. When the intensity, scale-size and thickness of the random medium change, then the spatial coherence angle of scattered waves and the degree of multiple scattering by the random medium also change and as a result, the behavior of the BRCS becomes complicated.

As well known in wave scattering in free space, the surface curvature of a body has also a serious effect on the BRCS. Hence it is necessary to analyze the BRCS in a random medium for bodies of various surface curvatures. In this paper, the BRCS of a concave-convex conducting cylinder in a continuous random medium is numerically analyzed for both E-wave and H-wave incidences in the case that the spatial coherence length of the incident wave on the cylinder is larger enough than the size of the cylinder.

FORMULATION

Consider the problem of electromagnetic wave scattering from a perfectly conducting cylinder embedded in a continuous random medium, as shown in Fig.1. Here L is the thickness of the random medium surrounding the cylinder and is assumed to be larger enough than the size of the cylinder cross-section. The random medium is assumed to be described by the dielectric constant ε, the magnetic permeability μ and the electric conductivity σ, which are expressed as

$$\varepsilon = \varepsilon_0[1 + \delta\varepsilon(\boldsymbol{r})] , \ \ \mu = \mu_0 , \ \ \sigma = 0 , \tag{1}$$

where ε_0, μ_0 are constant and $\delta\varepsilon(\boldsymbol{r})$ is a random function with

$$\langle \delta\varepsilon(\boldsymbol{r}) \rangle = 0 , \ \ \langle \delta\varepsilon(\boldsymbol{r}_1) \cdot \delta\varepsilon(\boldsymbol{r}_2) \rangle = B(\boldsymbol{r}_1 - \boldsymbol{r}_2) . \tag{2}$$

Here the angular brackets denote the ensemble average and $B(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ is a correlation function of the random function. For many cases, B can be approximated as

$$B(\boldsymbol{r}_1 - \boldsymbol{r}_2) = B_0 \exp\left[-\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{l^2} \right] , \tag{3}$$

$$B_0 \ll 1 , \ \ kl \gg 1 , \tag{4}$$

where B_0, l are the intensity and scale-size of the random medium fluctuation, respectively, and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Under the condition (4), depolarization of electromagnetic waves due to the medium fluctuation can be neglected; and the scalar approximation is valid. In addition, the small scattering-angle approximation is also valid; and re-icident waves are negligible at the first stage of analysis.

Suppose that the current source is a line source, located at $\boldsymbol{r}_{\mathrm{T}}$, far from and parallel to the cylinder. Then an average intensity of scattered waves u_{s} is given as follows [1]: for E-wave incidence,

$$\langle |u_{\mathrm{s}}|^2 \rangle = \int_S \mathrm{d}\boldsymbol{r}_1 \int_S \mathrm{d}\boldsymbol{r}_2 \int_S \mathrm{d}\boldsymbol{r}'_1 \int_S \mathrm{d}\boldsymbol{r}'_2 [Y_{\mathrm{E}}(\boldsymbol{r}_1; \boldsymbol{r}'_1) Y^*_{\mathrm{E}}(\boldsymbol{r}_2; \boldsymbol{r}'_2) \langle G(\boldsymbol{r}; \boldsymbol{r}_1) G(\boldsymbol{r}'_1; \boldsymbol{r}_{\mathrm{T}}) G^*(\boldsymbol{r}; \boldsymbol{r}_2) G^*(\boldsymbol{r}'_2; \boldsymbol{r}_{\mathrm{T}}) \rangle] , \tag{5}$$

and for H-wave incidence,

$$\begin{aligned} \langle |u_{\mathrm{s}}|^2 \rangle \ = \ & \int_S \mathrm{d}\boldsymbol{r}_1 \int_S \mathrm{d}\boldsymbol{r}_2 \int_S \mathrm{d}\boldsymbol{r}'_1 \int_S \mathrm{d}\boldsymbol{r}'_2 \Big\{ Y_{\mathrm{H}}(\boldsymbol{r}_1; \boldsymbol{r}'_1) Y^*_{\mathrm{H}}(\boldsymbol{r}_2; \boldsymbol{r}'_2) \\ & \cdot \frac{\partial}{\partial n_1} \frac{\partial}{\partial n_2} \langle G(\boldsymbol{r}; \boldsymbol{r}_1) G(\boldsymbol{r}'_1; \boldsymbol{r}_{\mathrm{T}}) G^*(\boldsymbol{r}; \boldsymbol{r}_2) G^*(\boldsymbol{r}'_2; \boldsymbol{r}_{\mathrm{T}}) \rangle \Big\} , \end{aligned} \tag{6}$$

where S and G denote the cylinder surface and Green's function in a random medium respectively, $\partial/\partial n$ denotes the outward normal derivative at $\boldsymbol{r}$ on S, the asterisk denotes the complex conjugate, and $Y_{\rm E}$ and $Y_{\rm H}$ are current generators for E-wave and H-wave incidence respectively:

$$Y_{\rm E}(\boldsymbol{r};\boldsymbol{r}') \simeq \boldsymbol{\phi}_M^*(\boldsymbol{r})A_{\rm E}^{-1} \ll \boldsymbol{\phi}_M^{\rm T}(\boldsymbol{r}')\ , \quad Y_{\rm H}(\boldsymbol{r};\boldsymbol{r}') \simeq \frac{\partial \boldsymbol{\phi}_M^*(\boldsymbol{r})}{\partial n}A_{\rm H}^{-1} \ll \boldsymbol{\phi}_M^{\rm T}(\boldsymbol{r}')\ . \tag{7}$$

Here, $\boldsymbol{\phi}_M = [\phi_1, \phi_2, \cdots, \phi_M]$, in which $\phi_m = H_m^{(1)}(kr)e^{im\theta}$; $m = 0, \pm 1, \cdots, \pm N$ and $M = 2N + 1$, $\boldsymbol{\phi}_M^{\rm T}$ denotes the transposed vector of $\boldsymbol{\phi}_M$, $A_{\rm E}$ and $A_{\rm H}$ are positive definite Hermitian matrices of $M \times M$

$$A_{\rm E} = \begin{bmatrix} (\phi_1,\phi_1) & \cdots & (\phi_1,\phi_M) \\ \vdots & \cdots & \vdots \\ (\phi_M,\phi_1) & \cdots & (\phi_M,\phi_M) \end{bmatrix}\ , \quad A_{\rm H} = \begin{bmatrix} (\partial\phi_1/\partial n,\partial\phi_1/\partial n) & \cdots & (\partial\phi_1/\partial n,\partial\phi_M/\partial n) \\ \vdots & \cdots & \vdots \\ (\partial\phi_M/\partial n,\partial\phi_1/\partial n) & \cdots & (\partial\phi_M/\partial n,\partial\phi_M/\partial n) \end{bmatrix}\ , \tag{8}$$

$$(\phi_m, \phi_n) \quad \equiv \quad \int_{\rm S} \phi_m(\boldsymbol{r})\phi_n^*(\boldsymbol{r}){\rm d}\boldsymbol{r}\ . \tag{9}$$

And $\ll \boldsymbol{\phi}_M^{\rm T}$, denotes the following operation of each element of $\boldsymbol{\phi}_M^{\rm T}$ and the incident wave $u_{\rm in}$ to the right of the $\boldsymbol{\phi}_M^{\rm T}$:

$$\ll \phi_m(\boldsymbol{r}), u_{\rm in}(\boldsymbol{r}) \gg \equiv \phi_m(\boldsymbol{r})\frac{\partial u_{\rm in}(\boldsymbol{r})}{\partial n} - \frac{\partial \phi_m(\boldsymbol{r})}{\partial n}u_{\rm in}(\boldsymbol{r})\ . \tag{10}$$

The calculations of (5) and (6) require the Fourth moment of Green's functions, which can be written as

$$\langle G(\boldsymbol{r};\boldsymbol{r}_1)G(\boldsymbol{r}_1';\boldsymbol{r}_{\rm T})G^*(\boldsymbol{r};\boldsymbol{r}_2)G^*(\boldsymbol{r}_2';\boldsymbol{r}_{\rm T})\rangle = G_0(\boldsymbol{r};\boldsymbol{r}_1)G_0^*(\boldsymbol{r};\boldsymbol{r}_2)G_0(\boldsymbol{r}_1';\boldsymbol{r}_{1\rm T})G_0^*(\boldsymbol{r}_2';\boldsymbol{r}_{2\rm T}) \times m_{\rm s}\ , \tag{11}$$

where G_0 is Green's function in free space. The $m_{\rm s}$ includes multiple-scattering effects of random medium and can be obtained by two-scale method [6–9].

NUMERICAL RESULTS

As an example, parameters of the random medium are assumed to satisfy $B_0k^2Ll = \pi^2$. The target is assumed to be a conducting cylinder whose cross section is expressed by $r = a[1 - \delta\cos 3(\theta - \phi)]$. Here we restrict the shape and size to $\delta = 0.2$ and $ka = 3$. We calculate BRCS of the cylinder σ in the cases of $\phi = 0$ and $\phi = \pi$ for E-wave and H-wave incidence, respectively, and normalize them to those in free space σ_0. When we plot the normalized BRCS as functions of β, then all the results for different ϕ and polarization coincide with each other, as shown in Fig.2. There is a backscattering enhancement peak. A twin depression appears at both sides of the peak; and the depression value is less than one. The coincidence of the results means that the behavior of the normalized BRCS is independent of the body configuration and the polarization of incident waves. We observe that the normalized BRCS tends to one as increasing the angle between incidence and observation directions. The integration value of the normalized BRCS is almost one, which fact shows that the results agree with the law of energy conservation.

Fig.3 shows the effect of the random medium intensity on the normalized BRCS by changing $B_0 = 1.0\times10^{-7}$, 2.5×10^{-7}, and 5.0×10^{-7} at $k^2Ll = 4\pi^2\times10^6$. As B_0 decreases, a small twin peak appears in the case of $B_0 = 1.0\times10^{-7}$ just outside the depression. The oscillation of BRCS is considered to be caused by an interference of incident and scattered waves. To study the interference, we analyze the degree of spatial coherence Γ between $P_{\rm in}$ and $P_{\rm s}$ shown in Fig.1:

$$\Gamma(L,\beta) = \langle G(L,\beta/2;0)G^*(L,-\beta/2;0)\rangle / \langle |G(L,0;0)|^2\rangle\ . \tag{12}$$

Fig.4 shows Γ for the three cases of B_0. The Γ for $B_0 = 1.0\times10^{-7}$ has high value for small β, and the interference becomes therefore wider spatially but with reduced backscattering ehancement.

CONCLUSION

The bistatic radar cross-sections (BRCS) of concave-convex conducting cylinders in a continuous random medium have been analyzed numerically. The results agree well with the law of energy conservation, and show that the behavior of the BRCS normalized to those in free space, which indicates the random medium effect, is independent of the body configuration and the polarization of incident waves. When the intensity of the random medium changes, then the spatial coherence angle also changes; as a result, we observe the complicated behavior of BRCS. We should note that the above results are under the condition that the spatial coherence length of incident waves on the body is larger enough than the body size. If the condition is not satisfied, then it is expected that BRCS may change more complicatedly, depending on the shape and size of the body and the polarization of incident waves. The analysis is an important subject to be discussed in the near future.

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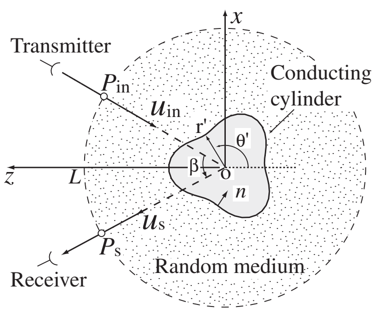

Fig.1: Geometry of the scattering problem from a conducting cylinder surrounded by a random medium.

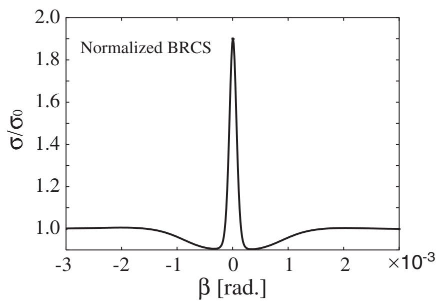

Fig.2: BRCS normalized to that in free space.

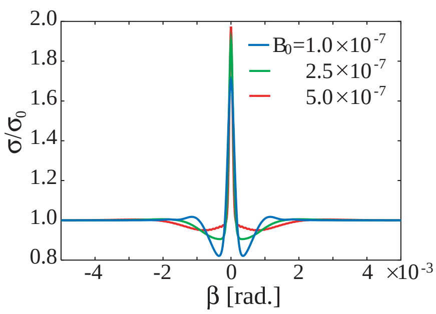

Fig.3: The effect of the random medium intensity on the BRCS.

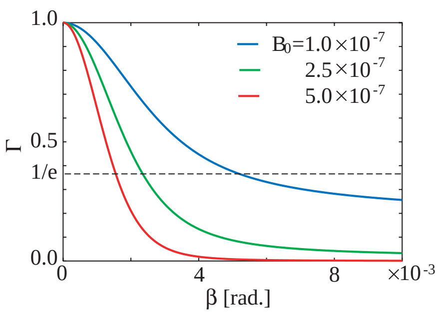

Fig.4: The degree of spatial coherence for different intensities of the random medium.