MULTISCALE ANALYSIS OF LARGE COMPLEX ARRAYS

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ABSTRACT

A Multi-Resolution (MR) approach to the EFIE analysis is presented, that keeps the different scales of the problem directly into the basis functions representation. The basis is constructed via the "dual-isoscalar" technique introduced by the authors, and possesses both spectral and spatial resolution, that can be shown to be crucial in handling the condition problem: the obtained MoM matrix is stable against clipping sparsification, and the use of iterative solvers convenient. The presented technique works for both rectangular and triangular meshes: in this contribution test problems meshed with a triangular grid will be considered.

INTRODUCTION

The design-oriented numerical simulation of large and complex arrays involve structures that are electrically large yet with fine geometrical details that require much-smaller than wavelength discretizations. These are among the most challenging issues in Computational ElectroMagnetics (CEM); the EFIE-MoM approach is largely used in the simulation of these structures: it is well known that matrix size and condition number involved in the problems of interest are its key limitations. In these problems, the structure of the solution exhibits very different scales of variation. For examples, local interactions in sub-wavelength details, and at edges and discontinuities, generate small-scale details - with high spatial frequency- in the solution. On the other hand, distant interactions - as well as resonant lengths - are responsible for the low-(spatial)frequency, slow spatial variations. A number of technique have been presented in the past years to overcome the above difficulties, and most notably the Fast Multipole Method [1] that has proven very successful especially in RCS prediction problems. Here, we focus on keeping the different scales directly into the formulation and solution process, a typical Multi-Resolution (MR) instance; this leads to a good control of the conditioning of the matrix, a key in the convergence of iterative solvers, as well as the gateway to sparsification of the matrix.

The other way of addressing MR or multi-level issue is to operate on the MoM matrix- rather than embedding multi-scale constructs into the basis functions themselves; works of this type range from "blind" matrix transformations related to wavelet transforms - that do not use direct information on the geometry - to the work [2], that instead has the advantage of incorporating spatial information in to the algorithm, in a manner somewhat alike to FMM. These approaches do not take directly into consideration the conditioning problem, which is instead central in the present work.

Finally, another multiscale approach has been pursued by these authors through the concept of Synthetic basis Function eXpansion (SFX) [4, and ref. therein]; in it, global basis functions are generated from the solution on portions of the structure (e.g. radiators in an array)), and then used to reduce the size of the problem; multiple grids are employed to reduce the filling time [5].

THE DUAL-ISOSCALAR APPROACH

The intrinsic difficulties of generating and employing vector MR ("wavelet") functions in 3D or 2.5D problems is overcome by the "dual-isoscalar" approach recently introduced by the authors [3]that will only be briefly summarized here.

The generation of the MR basis is approached dividing the unknown surface current into its solenoidal (TE) and quasi-irrotational (quasi-TM, qTM) components, that can be mapped to scalar quantities that posses the same degree of regularity in both (local) spatial directions, and on which the introduction of wavelet-like constructs is easier. The qTM part is related to the charge density, and thus defined over the cells of the mesh, and the TE part is derived from a "solenoidal potential", defined over the (inner) nodes of the mesh; this duality is present at all stages of the generation of the MR functions.

The MR construction begins by defining a "starting", coarse mesh of the structure, called level $j = -1$, as done with any mesher from the CAD. Subsequent nested domain-decomposition steps are then performed, that are equivalent to the generation of hierarchically nested families of grids of increasing detail, similarly to mesh "h-refinement"; each of these meshes is identified as level-j; the finest such mesh, with $j = L$ is called the "pixel" mesh. On all meshes of level $j \geq 1$, the MR functions are defined via the corresponding scalar quantities: the charge (for qTM) refers to cells of the mesh, the solenoidal potential (for TE) to the nodes. On the scalar quantities, MR is obtained applying "templates" that employ the wavelet constructs; finally, each scalar (TE or qTM) MR scalar function is mapped onto the corresponding vector function. The above is trivial for the TE (it only involves a gradient), and for the qTM it employs the results in [6].

The $j = -1$ mesh is treated differently: on it, "connecting functions" are defined that link the various sub-domains. In the simplest instance, connection functions are simply the loops (TE) and stars (qTM) of that mesh. The choice of the connecting functions on the coarsest grid impacts on the overall numerical efficiency, yet not in a dramatic manner. Considerations on this subject are offered in [7].

The generation scheme in [3] was described for a rectangular mesh for the sake of simplicity, but the DIS scheme works for triangular meshes as well, since its scalar "inner nature" does not require separable current components (like cartesian components are in rectangular meshes). Application to triangular grids can be obtained almost directly by application of the described algorithms; in this, we note that the passage from scalar to vector functions relies on the results in [6], that apply indifferently to Rao-Wilton-Glisson (RWG) functions on triangular or rectangular cells.

SAMPLE RESULTS

In the following, we concentrate on sparsification, and consider the relative error either on the current, or on the *inverse* matrix MoM matrix, i.e. $\epsilon = \| Z_{spars}^{-1} - Z^{-1} \| / \| Z^{-1} \|$, both for the employed MR- and standard RWG bases. An example of real-life array with a rectangular mesh is presented in Fig.1; the array has eight stacked patches, on a three-layer dielectric structure with two levels of metallization, meshed with a rectangular grid with 2735 unknowns, that includes also the beam-forming network. For this structure, the error-vs.-sparsification curve is shown in Fig. 2; there, the effect of the choice of various connecting functions at the coarsest level is also shown (a discussion on this specific aspect is in [7]).

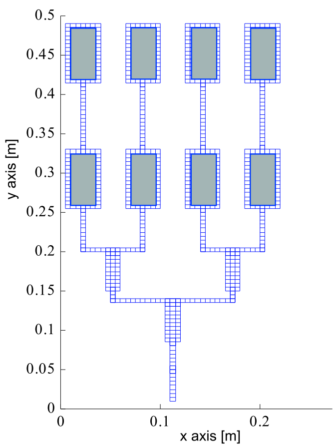

Figure 1: Geometry of the array antenna, rectangular mesh.

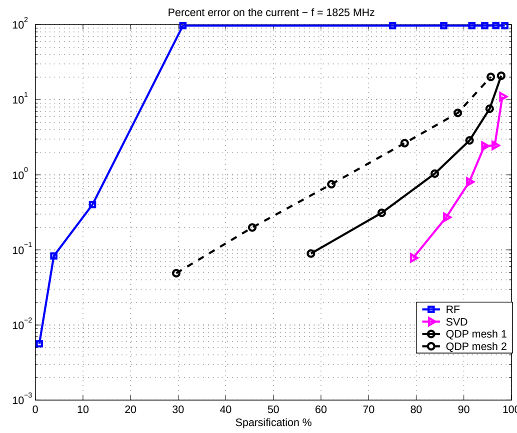

Figure 2: Percent error on the current vs. percent level of sparsification (upper end of band)

Next, we show the efficiency of the scheme on triangular meshes, employing as prototypical test case the single H-shaped patch (probe fed) shown in Figs. 3 and 4, with 304 unknowns; in the MR basis, there are $L = 2$ levels, and no TE functions at the coarsest level $j = -1$. The first important observation is that the MoM matrix in the RWG basis has a condition number of about 700, that drops to about 60 in the MR basis after diagonal preconditioning. This is reflected in the possibility of sparsifying the matrix by clipping entries below a threshold, as seen in Fig. 5; note that while in the RWG basis the error explodes for moderate sparsifications, for the MR basis it remains below 1% up to a sparsity of 75% (25% non-zero entries), which is remarkable for a structure dominated by near-field terms.

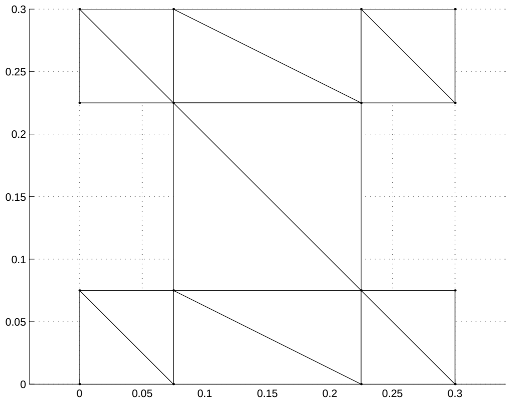

Figure 3: H-shaped patch, coarsest-level mesh.

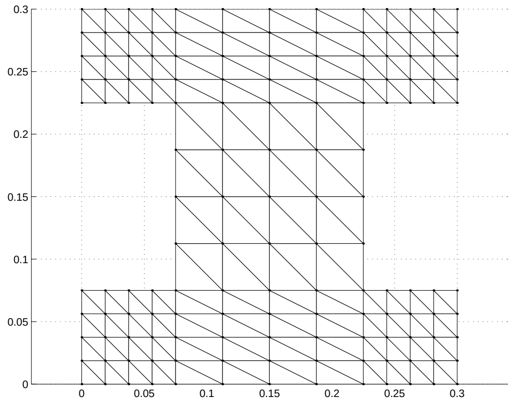

Figure 4: H-shaped patch, pixel-level mesh.

Next, Fig. 6 shows the convergence history of the systems obtained with the RWG and MR bases: the advantage in converegence speed is apparent.

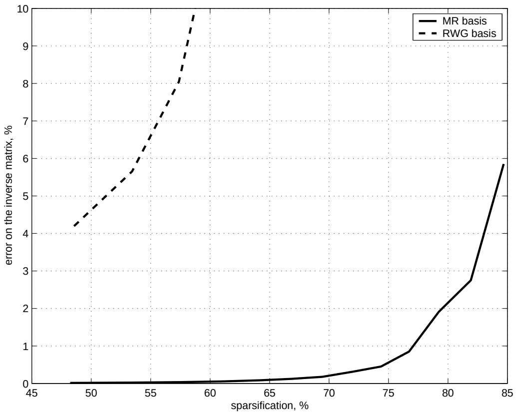

Figure 5: Percent error on the inverse matrix vs. percent sparsification.

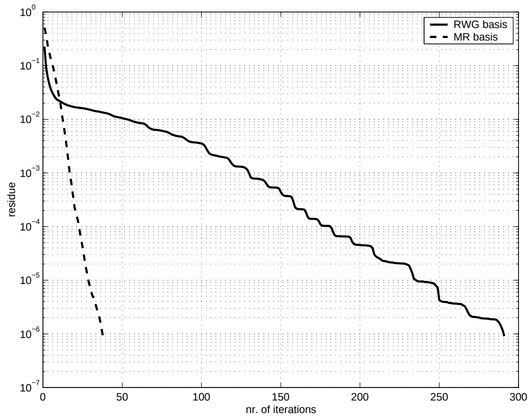

Figure 6: CG Convergence history.

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