CONVECTIVE PROPERTIES OF LOW-FREQUENCY GRADIENT-DRIFT WAVES IN THE AURORAL E REGION

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ABSTRACT

At small growth rates, the E region gradient-drift instability switches from absolute to convective. We present a modified WKB method to model convective wave propagation and growth at high latitudes. We will show that the convective character causes the wavevector component along the magnetic field to increase with time, resulting in a wave stabilization process. At the same time, the instability has a large upward group velocity which limits the time spent in the unstable region of the density gradient. We will show the results of wave evolution calculations and compare them with recent radar observations of gradient-drift waves.

INTRODUCTION

Ionospheric irregularities have long been studied with radars by examining the characteristics of the backscatter. In particular, the SuperDARN chain of radars is routinely measuring irregularities on scales of the order of 10 meters. One example is the recent observations of “SLERPS”, slow long-lived E region plasma structures[2], which are thought to be gradient-drift waves since they appear in regions where we expect large gradients due to precipitation. However, their expected growth rate is so small that we must consider a wave convective theory to fully understand them and explain the radar observations.

Traditionally, gradient-drift have been modeled with a plane waves decomposition. Both frequency ω and wave number $\mathbf{k} = (k_\perp, k_\|)$ are assumed to be free parameters to the problem, with the frequency being given by $\omega = (\mathbf{k} \cdot \mathbf{v}_e + \Psi \mathbf{k} \cdot \mathbf{v}_i)/(1 + \Psi)$, where $\mathbf{v}_e$ and $\mathbf{v}_i$ are the electron and ion velocities, respectively, and $\Psi = (\nu_e \nu_i)/(\Omega_e \Omega_i) + \theta^2 (\nu_e \Omega_e)/(\nu_i \Omega_i)$ depends on the electron-neutral and ion-neutral collision frequencies ν_e and ν_i as well as their cyclotron frequencies Ω_e and Ω_i. The angle $\theta \approx k_\|/k$ is called the aspect angle, meaning the angle off perpendicularity to the magnetic field.

The collision frequencies are exponential functions of height through the exponential decay of the neutral density. Therefore, the frequency itself is a strong function of altitude between 80 and 110 km. This means that the assumption of a plane wave decomposition is strictly inconsistent with the description of the problem. When waves traverse a significant range of altitudes, it becomes necessary to couple waves with differing frequencies, as the wave trains move through the ionosphere and their frequency changes[3]. Alternatively, one can drop the plane wave decomposition and model propagation in a way that is more compatible with the frequency being a function of height. We propose here a modified WKB approach.

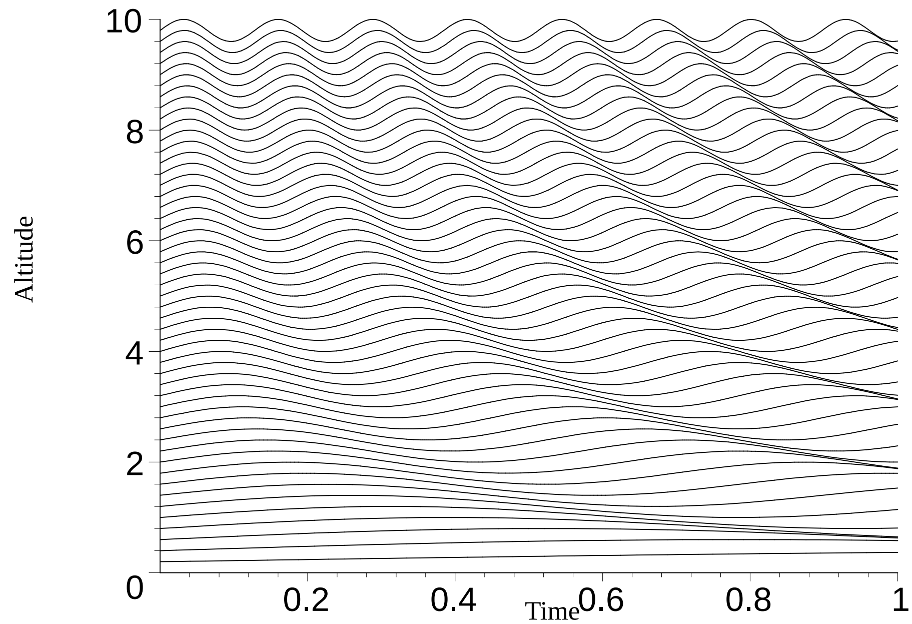

Figure 1: Cartoon of a wave train with a frequency that varies with altitude. Initially, all altitudes were chosen to be in phase, but as a result of the frequency mismatch, the wave develops a vertical structure as well as vertical phase and group velocities.

THEORETICAL DESCRIPTION

As shown in Figure 1, a wave train will develop vertical structure as a result of the frequency being a function of height. In our model we account for this by the following WKB-like description:

$$\frac{\delta n}{n_0}(z,t) = A(z,t)\exp[iS(z,t)] \tag{1}$$

where, in contrast with standard WKB, we assume both amplitude A and phase S to be real. This allows us to model wave propagation through changes in the phase, and wave growth or decay through changes in the amplitude, with the assumption that changes in the amplitude are over larger spatial and temporal scales than those in the phase. We can then define a local frequency and parallel wave number using

$$\omega(z,t) = -\frac{\partial S(z,t)}{\partial t} \tag{2}$$

$$k_{\|}(z,t) = \frac{\partial S(z,t)}{\partial z} \tag{3}$$

Note that ω and $k_{\|}$ would have been constant and uniform had we assumed a plane wave description. Here instead, they are allowed to change in time and space.

Taking the second derivatives of (2) and (3), we can relate the frequency and parallel wave number through the so-called "Whitam relation"

$$\frac{\partial k_{\|}}{\partial t} = -\frac{\partial \omega}{\partial z} \tag{4}$$

This equation describes the evolution of vertical structure seen in Figure 1, and clearly shows that a non-uniform frequency must necessarily imply a change in $k_{\|}$. A change in $k_{\|}$, on the other hand, implies a change in the frequency through the factor Ψ, which means that neither frequency nor parallel wave number can be assumed to be free parameters to the problem. Both are true functions of time and space according to (4).

The vertical motion of gradient-drift and Farley-Buneman waves is given by the vertical (parallel) group velocity

$$v_{g\parallel} = -2\theta \frac{\hat{\mathbf{k}} \cdot (\mathbf{v}_e - \mathbf{v}_i)}{1+\Psi} \cdot \frac{\nu_e \Omega_e}{\nu_i \Omega_i} \tag{5}$$

In this expression, the last factor is of the order of 10^4 to 10^6, which means we can get vertical group velocities of 20 km/s even for small aspect angles of 0.5 degrees. As a result, all waves with growth rates less than 10 s^{-1} must be modeled with a non-uniform frequency, because they will travel through a large part of the ionosphere, while undergoing growth. Consequently, only very fast growing modes can truly be described as plane waves, because their amplitude grows quickly enough for non-linear processes to take over the evolution before the vertical motion can have any effect on their evolution.

Finding an expression for the amplitude of the waves can be done in two ways, both yielding the same result. The first one is to apply the usual Gradient-Drift perturbation theory to find an equation for the evolution of the perturbed density, $\delta n/n_0$, and then use a "brute-force" multi-timing multi-scaling approach to determine the long time scale evolution in the amplitude. A more elegant approach is to use the principle of conservation of wave action[1]. This principle is a generalization of the conservation of wave energy in cases for which the frequency changes with position. In the present case, wave growth or decay occurs in the ion frame of reference, which varies with altitude: as the ions become less collisional and more magnetized with altitude, their speed increases with height. As long as these changes are slow compared to the wave period and wavelength, conservation of wave action applies. However, we must also allow for local growth when dealing with convective instabilities in general. In the end, we obtain

$$\frac{\partial}{\partial t}\left(\frac{U}{\omega'}\right) + \nabla \cdot \left(\frac{\mathbf{v}_g' U}{\omega'}\right) = S \tag{6}$$

Here, U is the wave energy density, and the primed quantities ω' and $\mathbf{v}_g'$ are given in the ion frame of reference, i. e.

$$\omega' = \frac{\mathbf{k} \cdot \mathbf{v}_D}{1+\Psi} \qquad \text{and} \qquad \mathbf{v}_g' = \mathbf{v}_g - \mathbf{v}_i \tag{7}$$

where $\mathbf{v}_D = \mathbf{v}_e - \mathbf{v}_i$ is the electron-ion drift. The term S on the right hand side is a source/sink term that accounts for local gains or losses in wave action through internal processes in the wave, e. g. the Farley-Buneman or Gradient-Drift growth mechanisms.

Finding an equation for the amplitude is now fairly easy by substituting wave energy and frequency into (6), which gives

$$\frac{\partial \ln A}{\partial t} + v_{g\parallel} \frac{\partial \ln A}{\partial z} = \gamma + \gamma_{\text{conv}} \tag{8}$$

with

$$\gamma_{\text{conv}} = -\frac{1}{2} v_{g\parallel} \left[\frac{\partial \ln k_\parallel}{\partial z} - 2\frac{\partial \ln(1+\Psi)}{\partial z} + (2+\Psi)\frac{\partial \ln(\mathbf{k} \cdot \mathbf{v}_D)}{\partial z} + \frac{\partial \ln n_0}{\partial z} \right] \tag{9}$$

Here, γ is the local growth rate from the processes included in the source term S and γ_{conv} is the convective growth rate coming from the inhomogeneity of the medium. Under typical conditions, convective growth is of the order of 1–5 s^{-1}, which can make waves convectively unstable, even though they might be absolutely stable. Essentially this means that unless waves have a large local growth rate, the convective effects are going to modify or even dominate the growth and decay of the wave. Of particular interest here is the last term in the expression for γ_{conv}, $\frac{1}{2} v_{g\parallel} \partial \ln n_0 / \partial z$, which can have a large impact on decametric waves. For these waves, just with a typical vertical gradient induced by energetic particle precipitation and a normal vertical group velocity of tens of kilometers per second this term can easily play a dominant role in their evolution.

In this paper, we present the results obtained from numerically solving the non-linear PDE (4) for the propagation characteristics of the waves, as well as the linear PDE (8). We also show how the non-linearity of (4) leads to the creation of discontinuities in the wave: parts of a wave train moving up from lower altitudes run into slower parts generated from higher altitudes, causing the wave to crash. This represents a new nonlinear mechanism limiting wave amplitude.

Finally, we show the wave amplitude as a function of frequency and aspect angle, which allows us to compare the results with radar observations.

SUMMARY

Our main conclusion in the end is that the wave amplitude can have very little to do with the local growth rate, under typical gradient drift conditions at HF frequencies. Instead, the amplitude is often determined by convective effects. In this regard, we will also present the spectral characteristics of the waves, in the form of plots of the amplitude as a function of frequency and aspect angle, which we will then compare to radar observations. In particular, we will use our model to explore the properties of the "SLERPS" mentioned at the beginning.

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