DIFFRACTION FROM 2-D ARBITRARY STRUCTURES WITH CAVITIES OR EDGES: A RIGOROUS APPROACH

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ABSTRACT

The Method of Regularisation, adapted for arbitrarily shaped screens, is applied to solve two important diffraction problems: a) backscattering from a 2-D cavity having the typical form of an aircraft engine duct; b) analysis of a 2-D flanged parabolic reflector. These solutions provide efficient and rigorous full-wave analyses. The radar cross-section computed for duct cavities up to 150 wavelengths at normal and oblique incidence allows a detailed analysis of cavity resonance properties. Radiation patterns of the reflector antenna are computed for apertures up to 300 wavelengths so that the influence of adjoined flanges on its performance may be assessed.

INTRODUCTION

Accurate mathematical simulation of wave-scattering problems for scatterers with cavities and edges, especially electrically large cavities, is of great importance in many practical applications. It is desirable to develop methods that are uniformly valid in a wide frequency range for scatterers of arbitrary or general shape and are of guaranteed accuracy, especially for highly resonant structures (such as high Q-factor cavities).

One particularly difficult problem is the accurate prediction of the radar cross-section (RCS) of engine ducts and other large cavities. Presently there is a good deal of uncertainty about the reliability of such estimates when standard approaches such as the Method of Moments (MoM) or its equivalent are used. Attempts to improve accuracy (by increasing the discretisation parameter N) are either limited by computer memory availability or confounded by the onset of numerical instability. The aim of this work is to transform Maxwell's equations, so that when numerical techniques are applied, the effects of numerical instability are either eliminated or strongly controlled, permitting the reliable computation of a solution with guaranteed accuracy.

Another class of diffraction problems concerns reflector antennas. Various analytical/numerical techniques used to determine the characteristics of reflector antennas are reviewed in [1]. However, even with modern improvements, the moment-method technique fails to model large reflector antennas adequately (e.g. $D/\lambda > 100$ where D denote aperture size, and λ wavelength); the development of efficient and rigorous full-wave methods of analysis is obviously desirable.

One such approach is the so-called *Method of Regularisation* (MoR) [2]. The key idea behind this semi-analytical or analytical-numerical technique is the analytical transformation of an ill-conditioned first kind Fredholm integral equation, determining the unknown surface or line current densities, into a well-conditioned second kind Fredholm equation that is effectively solved numerically by a truncation method; this results in a reliable, stable, numerically accurate and efficient algorithm. It is therefore advantageous for electrically large structures with edges or cavities where MoM experiences difficulty. Thus it may be used to analyse cavities of 150λ length or reflectors with aperture size $D = 100\lambda - 500\lambda$ or more, which cannot be treated accurately by other methods.

Until recently the MoR was restricted to canonically shaped screens, such as spherical, spheroidal, circular cylindrical shells with apertures, and so on [2]. The next major step generalised this rigorous analysis to scattering from 2-D open screens of an arbitrary profile [3], retaining all the advantages such as guaranteed accuracy of computations, and stability in a wide frequency range; it thus provides a rigorous full-wave analysis for a wide class of 2-D scatterers. Various sources of excitation can be accommodated: line source, complex point source or plane wave excitation; front-fed or off-axis excitation. Near- and far-field radiation characteristics may be efficiently computed.

In this paper we employ the MoR to construct the solution algorithm to two important classes of diffraction problems: a) backscattering from a cavity having the typical form of an aircraft engine duct; b) analysis of 2-D parabolic reflector antenna with variable flanges. We consider two dimensional scattering from an infinitely long cylindrical surface with arbitrary cross-section that is independent of one axial direction, fixed to be the z-axis. The surface or screen is assumed to be infinitely thin, perfectly conducting, and is open, i.e., has an aperture. The cross-sectional profile should be sufficiently smooth for the Fourier series representation given below to converge.

REGULARISATION OF THE EFIE

Let L denote the contour of the 2-D reflector cross-section; it is regarded as part of a smooth closed contour S in which an aperture L' has been opened. S is parametrised in the form $S = \{p(\theta) = (x(\theta), y(\theta)) : \theta \in [-\pi, \pi]\}$, so that L corresponds to the subinterval $[-\theta_0, \theta_0]$. The single layer potential representation of the scattered field at a point q in terms of the surface current J_z is

$$E_z^s(q) = ikZ_0 \int_L G_2(k\,|p-q|)J_z(p)dl_p. \tag{1}$$

Here dl_p is the differential of arc length at $p \in L$, Z_0 denotes free space impedance, the free space Green's function $G_2(kR)$ equals $-\frac{i}{4}H_0^{(1)}(kR)$, and $R = |p-q|$. The boundary condition $E_z^s(p) = -E_z^{inc}(p)$ yields the usual Electric Field Integral Equation (EFIE) for the unknown current density. Once this is found, all relevant physical quantities such as surface current, near field and far field radiation characteristics are easily calculated.

Introduce the new unknown function $z(\tau)$ defined by $z(\tau) = ikZ_0J_z(p(\tau))\frac{dl}{d\tau}$ when $\tau \in [-\theta_0, \theta_0]$; outside this interval z is defined to be zero. The EFIE becomes

$$\int_{-\pi}^{\pi} G_2(kR(\theta,\tau))z(\tau)d\tau = -E_z^{inc}(p(\theta)),\ \theta \in [-\theta_0, \theta_0] \tag{2}$$

where $R(\theta,\tau) = R((x(\theta), y(\theta)), (x(\tau), y(\tau)))$. When the excitation is a plane wave incident at angle α to the x-axis, $E_z^{inc}(p(\theta)) = \exp(ik[x(\theta)\cos\alpha + y(\theta)\sin\alpha])$; on the other hand, if a complex line source is located at the complex point (x_s, y_s), then $E_z^{inc}(p(\theta)) = -\frac{i}{4}H_0^{(1)}(kR(\theta, s))$ where $R(\theta, s) = R((x(\theta), y(\theta)), (x_s, y_s))$. Equation (2) together with the vanishing requirement on z is completely equivalent to the EFIE.

Following the technique of [3], split the kernel of integral equation (2) into singular and regular parts,

$$H_0^{(1)}(kR(\theta,\tau)) = \frac{2i}{\pi}\ln\left|2\sin\frac{\theta-\tau}{2}\right| + H(\theta,\tau). \tag{3}$$

Expand H in a double Fourier series, and the incident field and the solution in Fourier series,

$$H(\theta,\tau) = \sum_{p=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h_{np}e^{i(n\theta+p\tau)},\quad 2E_z^{inc}(p(\theta)) = \sum_{n=-\infty}^{\infty} g_n e^{in\theta},\quad z(\tau) = \sum_{n=-\infty}^{\infty} \varsigma_n e^{in\tau}, \tag{4}$$

where $\theta, \tau \in [-\pi, \pi]$. We thus obtain *dual series equations* with exponential kernel,

$$\sum_{n=-\infty}^{\infty}{}' |n|^{-1}\varsigma_n e^{in\theta} - 2\sum_{n=-\infty}^{\infty} e^{in\theta}\sum_{p=-\infty}^{\infty} h_{n,-p}\varsigma_p = \sum_{n=-\infty}^{\infty} g_n e^{in\theta},\quad \theta \in [-\theta_0, \theta_0], \tag{5}$$

$$\sum_{n=-\infty}^{\infty} \varsigma_n e^{in\theta} = 0, \qquad \theta \notin [-\theta_0, \theta_0]. \tag{6}$$

for which the unknowns $\{\varsigma_n\}_{n=-\infty}^{\infty}$ are to be found. (The prime signifies omission of the term $n = 0$.)

An analytical process of regularisation, based on Abel's integral equation technique (see [2]), transforms the dual series equations to an infinite system of linear algebraic equations of the form

$$Z_m + \sum_{n=1}^{\infty} Z_n H_{nm} = C_m. \tag{7}$$

This regularised system is a Fredholm matrix equation of the second kind. Its mathematical properties ensure that truncation to a finite system produces a set of linear equations whose solution is guaranteed to converge to the exact solution as the truncation order N_{tr} increases. In general the correct choice of N_{tr} depends upon electric size of the scatterer; for the calculations described below, an accuracy of 3-4 decimal places is guaranteed by taking $N_{tr} > 2kl+20$ for the 'duct' problem (l is length of the cavity, see below) and $N_{tr} \geq 4\pi(D/\lambda)(F/D)+20$ for the 'antenna' problem, where F and D denote the focal length and aperture width of the reflector. Amongst other tests, convergence after truncation is verified by the near vanishing of the current outside the interval $[-\theta_0, \theta_0]$. The computer implementation thus has high numerical stability in a wide frequency range and is efficient in time and memory resources. Also it should be noted that the matrix elements H_{nm} are simple functions of the Fourier coefficients h_{nm}, not requiring further numerical integration.

NUMERICAL RESULTS AND DISCUSSION

To illustrate the method, we consider a contour which approximates a duct. With a scale parameter a and a width parameter $q(< 1)$, its parametrisation is $x = a\cos\theta$, $y = a\left[\arctan\left(\frac{3}{2}\cos\theta\right) + q\sin\theta\right]$; recall that the parameter θ_0 determines the metal and the gap regions. The electric length of the duct, fully closed, is approximately $l = 2\sqrt{2}ka$ (see Fig. 1, $a = 1, q = 0.5, \theta_0 = 120^0$). The RCS is computed for the open duct with the wave directly incident on the aperture (the angle of incidence $\alpha = 0$, see Fig. 2). The matrix equation (7) is well-conditioned even near whose frequencies corresponding to quasi-eigenvalues of the cavity as demonstrated by the condition number plotted in Fig. 3. A typical surface current is shown in Fig. 4, as a function of the parameter θ (the cavity occupies the region $[-\theta_0, \theta_0]$). The accuracy of the new method has been checked by comparison with solutions generated by the MoM; for the same accuracy the number of equations required by MoR is typically one half to one quarter smaller than that of MoM.

In the second problem the contour is formed by a parabolic reflector of focal length F and aperture width D, to which semi-circular flanges of radius w have been added (see Fig. 5). It is extended to a smooth closed contour S by smoothly joining another parabola L' (a rear-side 'aperture') so that the tangent varies continuously at the four points where semi-circular sections join parabolic sections. By way of illustration, the radiation pattern for the structure with $D = 50\lambda$ is presented in Fig. 6, $F/D = 0.96$. The complex line source, used to simulate a 2-D Gaussian beam [4], is located on the x-axis at $(d, 0)$ with corresponding complex coordinates $(x_s, y_s) = (d + ib, 0)$; our choice of $kb = -9.06$ tapers the level of illumination at the reflector edges by $-10dB$. The flanges are chosen here for purpose of illustration, and are not necessarily optimized. Improvements to the basic design of reflector antennas and benefits of addition of flanges are discussed in [5].

CONCLUSION

The method provides a full-wave analysis for a wide class of 2-D scatterers, so that different shapes of the screen, the effect and presence of subreflectors, specific shape of rims, and related features, can be examined rigorously. The resonance phenomena of the cavity can be studied accurately. The MoR is uniformly valid for both electrically small and large scatterers.

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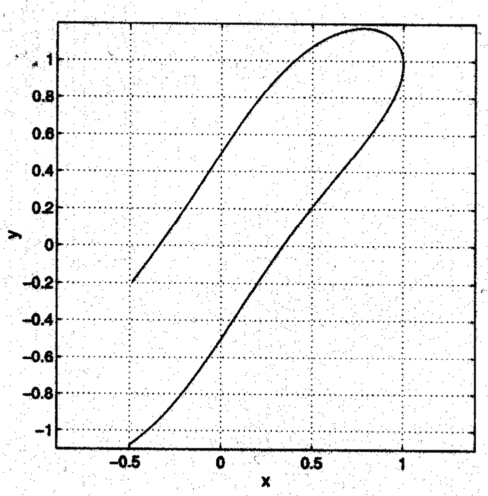


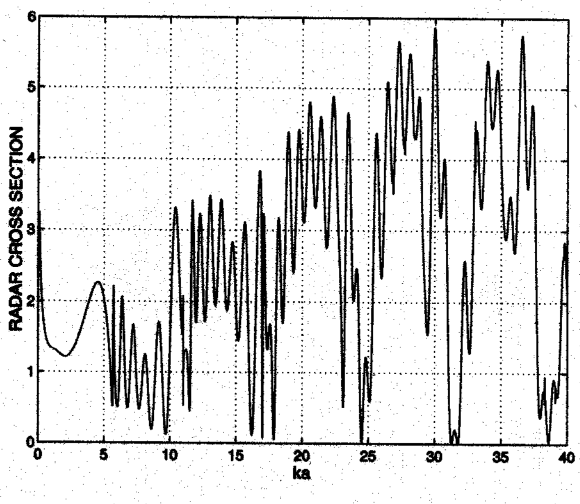
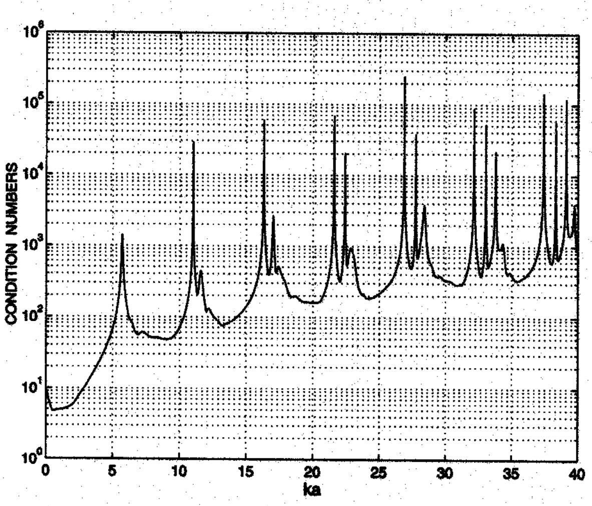


Fig. 1 (left). The duct geometry.
Fig. 2 (right). Radar cross section of the cavity with $\theta_0 = 120°$, $q = 0.5$, $\alpha = 0$ and $N_{tr} = 128$.



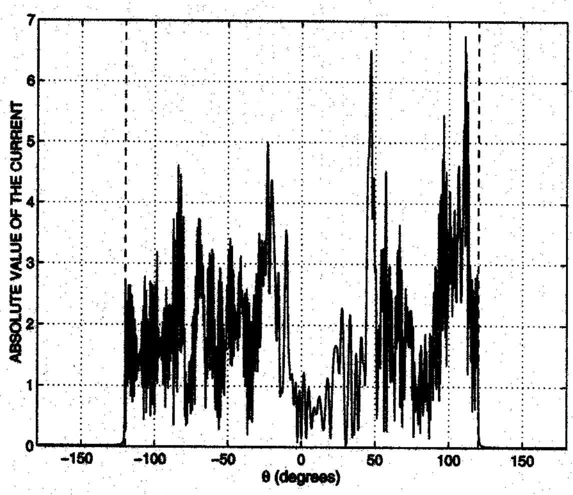
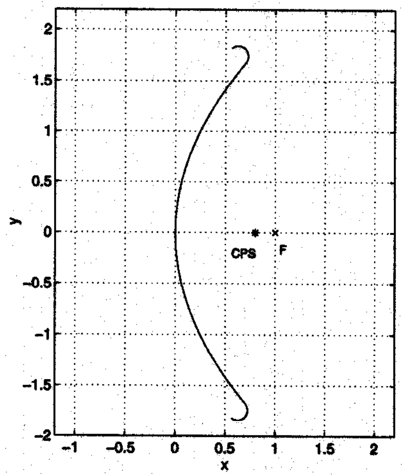


Fig. 3 (left). Condition number for the cavity calculation of Fig. 2.
Fig. 4 (right). Surface current for a duct of length approximately 75λ: $\theta_0 = 120°$, $q = 0.5$, $ka = 200$, $N_{tr} = 1024$.



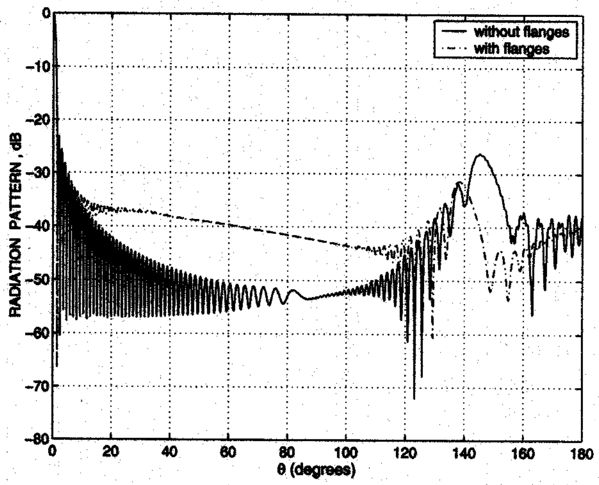


Fig. 5 (left). Contour L of the parabolic reflector antenna with semi-circular flanges: focal point F, line source location CPS.
Fig. 6 (right). Far field pattern for the parabolic reflector antenna: $D = 50\lambda$, $F/D = 0.96$, source location at $(F, 0)$ (see text). The flanges are quarter circles of radius $w = 0.2F$. $N_{tr} = 512$.