A COMPREHENSIVE INVESTIGATION OF THE DISPERSION AND RADIATION PROPERTIES OF MICROSTRIP LEAKY-WAVE ARRAYS

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ABSTRACT

In this work a comprehensive investigation is presented on the features of microstrip leaky-wave arrays for microwave and millimetre-wave applications. A rigorous approach has been developed based on the method of moments in the spectral domain, which allows us to achieve an efficient description of such structures by means of a modal analysis in the unit-cell environment. In addition to a complete characterisation of the radiative properties as a function of the phase shift between adjacent elements, novel dispersion and radiation phenomena have been found, concerning in particular the end-of-scanning region and the occurrence of grating lobes. Numerical simulations on finite arrays confirm the theoretical predictions made on the basis of the dispersion properties of the involved leaky waves.

INTRODUCTION

Linear arrays of leaky-wave sources represent an interesting alternative to conventional two-dimensional phased arrays, due to the possibility of obtaining narrow scannable beams with only one-dimensional set of phase shifters [1,2]. In fact, by varying the frequency, the beam can be scanned in elevation, as is typical in leaky-wave radiators; by varying the phase shift between adjacent elements, the beam can be scanned in azimuth and, under ideal conditions, the direction of maximum radiation would describe a conical surface. Different implementations have been proposed and studied in the past, based either on metallic waveguiding structures [3], or, more recently, on planar radiating elements [4,5].

On this ground, based on a recently-proposed spectral-domain approach in the unit cell [6], the present study intends to analyse in depth various radiative features of linear arrays of microstrip leaky-wave antennas as a function of the phase shift between elements, by determining in a rigorous manner their scanning properties (see Figs. 1(a), (b), and (c)). We have performed parametric analyses of the dispersion behaviour of the structure as a function of the phase shift between elements and of the involved geometrical parameters: we have thus found novel modal characteristics, related to specific scanning properties, which have been verified through a full-wave numerical analysis of realistic microstrip arrays with a finite number of elements. These findings allow us to have a more complete understanding of the radiation mechanisms and distinctive features of this class of structures.

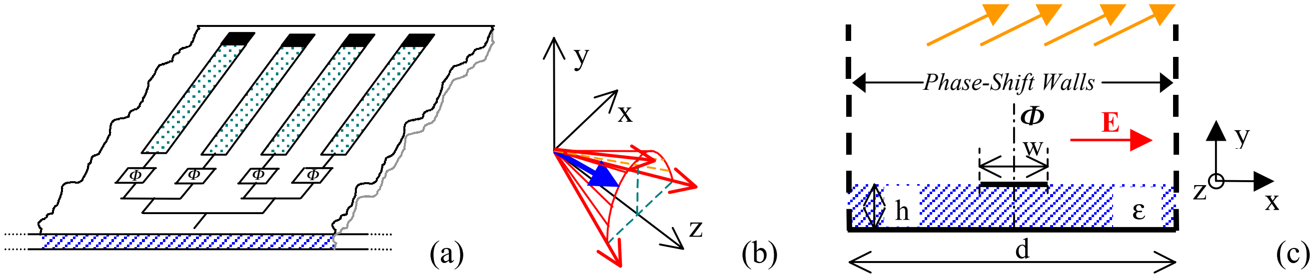


Fig. 1 – (a) Linear array of microstrip lines on a grounded dielectric slab, with phase shifters and matched loads; (b) conical beam scanning; (c) unit cell of the infinite array with phase-shift walls and relevant physical parameters.

ANALYSIS

The reference printed-circuit LWA array is represented by a microstrip structure, constituted by a linear equispaced array of metal strips etched on the surface of a grounded dielectric layer, as shown in Fig. 1(a). The spacing between elements is d, the width of each strip is w, and the background dielectric slab has relative permittivity ε_r and thickness h. In our analysis an infinite linear array is considered; the periodicity along the transverse x direction allows us to restrict the study to a single spatial period (unit cell) of the structure (see Fig. 1(c)). The full-wave analysis of the dispersion properties of the array has been carried out by means of the method of moments in the spectral domain, applied to the unit cell [6].

With a suitable choice of the physical parameters, each microstrip supports a perturbed EH_1 mode in a regime of spatial leakage [6]. We recall here that a crucial point in the determination of the modal spectrum of the array under analysis is the spectral character of the spatial harmonics, which constitute the (transverse) Floquet representation of the sought fields and currents: such harmonics can be either *proper* (they are attenuated at infinity in the free-space region above the array plane, i.e., in the *y* direction) or *improper* (they exponentially diverge at infinity in the *y* direction). The choice of the proper or improper determination for each spatial harmonic determines the Riemann sheet where the numerical root search of the dispersion equation has to be performed: it is seen that a modal solution will be *physical* if the real part (β) of its propagation constant k_z is less than the real parts of all the branch points associated to the *improper* spatial harmonics, if any [6]; otherwise, the solution will be *nonphysical*.

As is known, the determination of the normalized phase (β/k_0) and leakage (α/k_0) constants of the leaky mode on which the array operation is based allows us to predict its main radiative features as a function of the involved physical parameters: in particular, the main-beam pointing angle is related to the phase constant, while the beam width is related to the attenuation constant for a fixed antenna efficiency [2,3]. The propagation constant k_{yn} of the n^{th} spatial harmonic in the *y* direction is given by $k_{yn} = (k_0^2 - k_{xn}^2 - k_z^2)^{1/2}$. The wavenumber k_{xn} of the n^{th} spatial harmonic in the *x* direction is purely real, being related to the imposed phase shift Φ between elements: $k_{xn} = \beta_{xn} = \Phi/d + 2\pi\, n/d$. The wavenumber $k_z = \beta - j\alpha$ in the longitudinal *z* direction is in general complex. With reference to the spherical angular coordinates (φ, θ) (measured from the *x* and *y* axis, respectively), for the case of a phased array supporting a leaky mode in a physical regime, the angular location (φ_m, θ_m) of the main radiated beams can be estimated by means of the following approximate relations (see, e.g., [3]):

$$\varphi_m = \tan^{-1}(\beta_{xn}/\beta), \quad \theta_m = \sin^{-1}\left(\sqrt{(\beta/k_0)^2 + (\beta_{xn}/k_0)^2}\right) \tag{1}$$

where the index *n* corresponds to the improper spatial harmonics which constitute the leaky-mode field. It can be observed that a physical leaky mode with the $n = 0$ improper harmonic will radiate one main beam only (absence of grating lobes), while a physical leaky mode with more than one improper spatial harmonic (e.g., the $n = 0$ and then $n = -1$) will give rise to a corresponding number of radiated beams.

NUMERICAL RESULTS

In order to explore the scanning features of the array shown in Fig. 1, we have determined the variation of the normalized phase (β/k_0) and leakage (α/k_0) constants as a function of the phase shift Φ between elements.

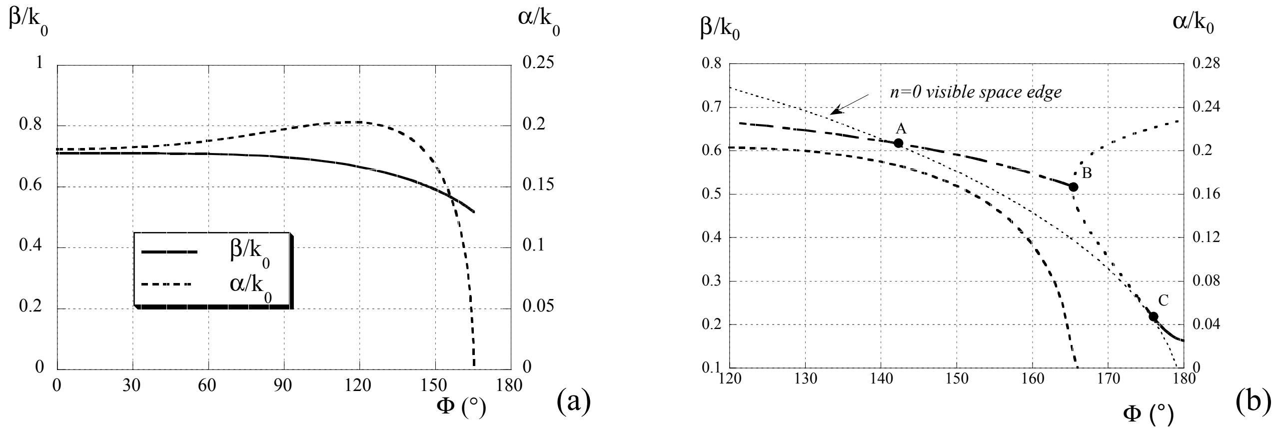


Fig. 2 – (a) Normalised phase constant β/k_0 (*solid line*) and normalised attenuation constant α/k_0 (*dashed line*) as a function of the phase shift Φ between elements; (b) detail of the transition region between leaky and surface-wave regimes; legend for β/k_0: *solid line*: real proper; *dotted line*: real improper; *dashed-dotted line*: complex improper.

A typical case is presented in Fig. 2(a) for a structure with $\varepsilon_r = 10.2$, $h = 0.635$ mm, $w = 3.3$ mm, and $d = 12.3$ mm (see [4]); the analysis has been performed at the fixed frequency $f = 12.2$ GHz, for which the operating mode is in a spatial leakage regime in the absence of space shift between elements. The phase-constant curve can be seen to remain quite flat as the phase shift is increased, giving rise to the expected closely-conical beam scanning, while the attenuation constant decreases monotonically. Such behaviour is typical of many LWA arrays (see, e.g., [3]); the higher values of phase shift correspond to the end-of-scanning region, in which the attenuation constant of the leaky mode decreases rapidly and finally becomes zero. This is usually explained in physical terms as due to the radiated beam direction which progressively lowers and finally hits the array plane. However, it is not clear if the whole curve represented in Fig. 2(a) is physical, and also what happens by further increasing the phase shift, above the value for which the attenuation constant becomes zero.

In order to clarify these points, we have investigated the modal properties of the involved mode over the *full* range of phase-shift values, and we have represented the relevant curves together with the edges of visible spaces for the n = 0 spatial harmonic [7]. The visible space for the n^{th} harmonic is the set of values of the normalised phase constant and phase shift for which that harmonic propagates with an essentially real wavenumber in the transverse y direction. The dotted curves in Fig. 2(b) represent the edge of *visible space* for the spatial harmonics n = 0. The leaky-wave complex improper solution (with the n = 0 harmonic improper) can be seen to exit the visible space of the n = 0 harmonic for $\Phi \cong 141°$, thus entering a region where it is *nonphysical*, i.e., it does not directly contribute to the field representation [1].

As it can be observed in Fig. 2(b), the attenuation constant is different from zero at the phase-shift value where the phase-constant curve intersects the edge of the visible space for the n = 0 spatial harmonic (point A in Fig. 2(b)). This means that, in the free-space region above the array plane, the n = 0 spatial harmonic corresponds to a plane wave. By further increasing the phase shift, the attenuation constant becomes zero, and the complex solution splits into *two real improper* branches, which are also nonphysical (point B in Fig. 2(b)): they correspond to improper surface waves that give rise to a field which grows exponentially at infinity in the y direction. The lowest real improper branch becomes tangent to the n = 0 visible-space edge, and at the tangency point (point C in Fig. 2(b)) it changes its spectral nature, because the n = 0 harmonic becomes proper. The solution then evolves into a *real proper* solution, which is *physical* and represents a *surface wave* of the grating, propagating and carrying power in a guided-wave fashion along the xz plane. The modal behaviour shown in Fig. 2(b) resembles the *spectral gap* which is found to occur between the leaky-wave and bound-wave ranges in the modal dispersion diagrams of many open waveguides (see, e.g., [2]). Here the transition does not occur by varying the *frequency*, but rather by varying the *phase shift* imposed between adjacent elements of the infinite array. It can be noticed that, in the case examined in Fig. 2, the spacing between array elements is equal to $\lambda_0/2$ (λ_0 being the free-space wavelength), so that the visible-space edges for the n = 0 and n = –1 spatial harmonics are tangent and grating lobes do not occur.

A topic worthy of further theoretical investigation for leaky-wave arrays is the possible occurrence of grating lobes in the scanning process. In particular, we have studied the dispersion behaviour of the EH$_1$ mode of microstrip arrays with increasing spatial periods.

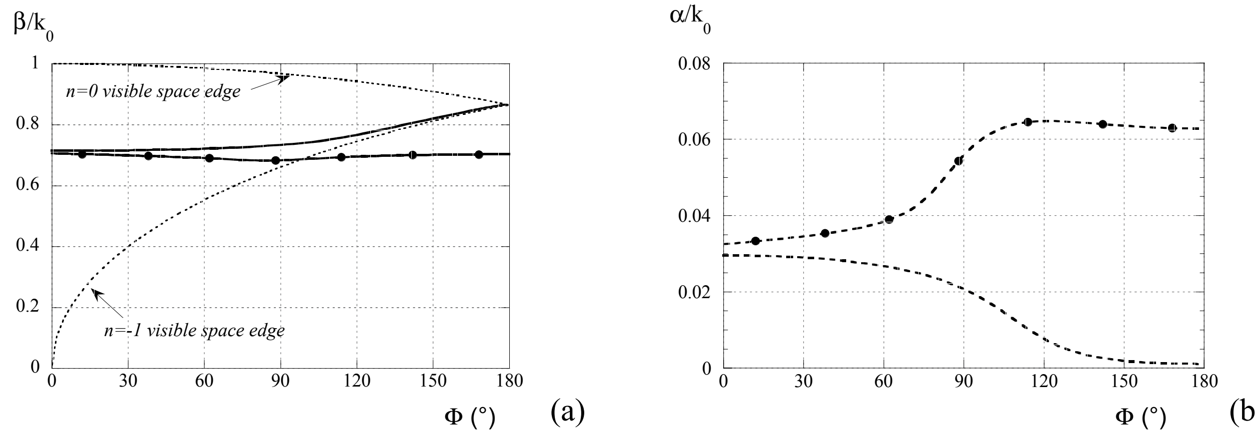
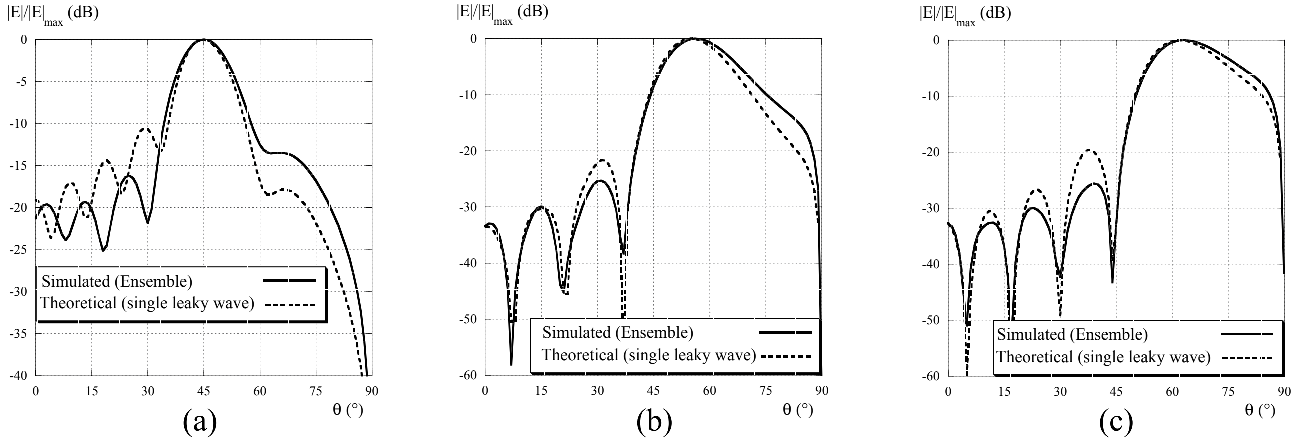


Fig. 3 – (a) Normalised phase constants β/k_0 (*solid lines*) and (b) normalised attenuation constants α/k_0 (*dashed lines*) as a function of the phase shift Φ between elements for both the perturbed EH$_1$ mode (*lines without dots*) and the new complex improper mode (*lines with dots*).

In Fig. 3 the case is presented of a microstrip array with ε_r =2.2, h =0.508mm, w = 8.5 mm, and d = 24 mm (see [5]). The simulation is performed at the fixed frequency f = 12.5 GHz, for which d/λ_0 = 1. A valid prediction of the possible grating-lobe occurrence came from the discovery of an *additional*, *previously unknown* complex improper solution, obtained with the n =0, -1 harmonics improper, shown as a line with black dots in Fig. 3(a). The perturbed EH$_1$ mode and the new leaky mode have almost the same phase constant near Φ =0° (see Fig. 3(a)); by increasing the phase shift, the phase-constant curve of the former gradually bends upwards, remaining above the n = -1 visible-space edge, while the phase constant of the new mode is rather flat and becomes physical at Φ = 100°. The attenuation constants of the two modes are shown in Fig. 3(b). It can be seen that they are very similar for Φ =0°; then, by increasing the phase shift, the attenuation constant of the perturbed EH$_1$ mode decreases monotonically, while that of the new mode considerably increases. However, it remains sufficiently low to give rise to appreciable radiation and grating-lobe phenomena, with possible degradations of the overall radiation pattern.

In order to verify the radiative properties of microstrip leaky-wave arrays predicted on the basis of the above-described dispersion analysis, full-wave numerical simulations of realistic arrays with a finite number of elements have been performed by means of a commercial software (Ensemble^{TM}) based on the method of moments in the spatial domain. In particular, we have considered a structure with parameters as in Fig. 3, with N =8 elements of finite length L = 150 mm along z. To assess the effect of the new leaky mode in its physical and nonphysical ranges, different cases of excitation phasing were simulated at the fixed frequency f =12.5 GHz, such that L/λ_0 =6.25. In Fig. 4(a) the normalized radiation pattern of the above-described finite array is shown as a function of θ in the φ =0° elevation plane, for the case of in-phase feeding (Φ =0°). The pattern obtained on the basis of our modal approach is calculated by assuming that the current on each strip is the modal current of the perturbed EH_1 mode. In both the radiation patterns calculated with our theory ('theoretical' pattern) and obtained with Ensemble ('simulated' pattern), the direction of maximum radiation lies in the φ =0° plane. It can be seen that the theoretical and simulated results are in good agreement as regards the main lobe features, i.e., pointing angle and beam width. The overall comparison between the theoretical and simulated results shows that the EH_1 leaky-mode currents accurately represent the continuous-spectrum currents excited along the microstrip lines, while the additional new leaky mode is nonphysical for Φ =0°.



(a) (b) (c)

Fig. 4 – Normalised radiation patterns as a function of the elevation angle θ for a finite array with N = 8 aperture-coupled microstrip elements, for different phase shifts: (a) Φ =0°, (b) Φ = 160° (main beam), (c) Φ = 160° (grating lobe).

In Figs. 4(b) e 4(c) the normalized radiation pattern of the above-described finite array is shown for the case Φ =160°. In Fig. 4(b) the simulated and theoretical radiation patterns in the main-beam plane φ_m =32° are shown. The pointing angle is θ_m = 56° in both radiation patterns, and it is very well predicted by (1). In Fig. 4(c) the simulated and theoretical radiation patterns in the grating-lobe plane φ_m = -38° are shown. The pointing angle is θ_m = 63° in both radiation patterns. The very good agreement observed between the simulated and theoretical radiation patterns confirms that the main contribution to the current on the strips comes from the new leaky mode.

CONCLUSION

In the present study, a number of original results have been found as regards important radiation properties for phased arrays of printed leaky-wave antennas. First, we have furnished a consistent interpretation of the behaviour of the beam scanning in terms of the spectral properties of the harmonics supported by the periodic structure: in particular, by examining what happens when the phase shift is increased, an original transition region has been discovered, which links continuously the leaky wave, responsible for the radiation, with a surface wave supported by the planar structure. In addition to this, the scanning properties have been investigated also as a function of the distance between the radiating elements, with the aim to make clear what possible degradations of radiation may derive. We have shown that, as the spatial period is increased, strong modifications occur in the patterns of the leaky-wave phase and attenuation constants, and our analysis explains why these odd behaviours occur. Moreover, a complete study has emphasised that previously-disregarded 'physical' modes can furnish additional contributions to radiation, thus sensitively altering the predicted performance of these structures.

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