# ANALYSIS OF CYLINDRICAL RECTANGULAR MICROSTRIP ANTENNA USING A NEW REPRESENTATION OF THE SPATIAL GREEN'S FUNCTION

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## ABSTRACT

An effective approach to the problem of a cylindrical rectangular patch placed on a cylindrical circular dielectric substrate surrounding a cylindrical circular metal rod is proposed. The main idea of this approach is to solve the problem in the space domain by Moment method using spatial Green's functions. The key point is that the singular part of the spatial Greens functions is extracted in an explicit form. An approximate model for the probe was introduced. The current amplitudes on the patch are found. Calculation of the far field in the frequency band allowed determining the five lowest resonances.

## INTRODUCTION

There are many applications in which conformal antennas are successfully used. Therefore developing rigorous theoretical approaches for their investigation is of a great interest. Most of the existing approaches, which allow analysing microstrip cylindrical antennas, are based on the moment method solution in the spectral domain [1]. In the present paper a new effective approach to the analysis of a cylindrical microstrip antenna (Fig.1) is proposed. First, the integral equation for the current is solved and all antenna characteristics are calculated in the spatial domain. Secondly, new representations for the spatial Green's functions (GFs) in a mixed potential formulation are used. We will go to the spectral domain to find the spectral GFs and will perform the Inverse Fourier Transformation (IFT) to determine the spatial GFs, which depend on the three spatial variables.

## A NEW REPRESENTATION OF THE SPATIAL GREEN'S FUNCTION

Note that the Inverse Fourier transform procedure in the cylindrical co-ordinate system consists of not only an integral over *h* (propagation constant for natural waves in the structure) but also a series over *n* (azymuthal functions)

$$G(r,\varphi-\varphi',z-z') = \frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty} e^{-\hat{i}n(\varphi-\varphi')} \int_{h=-\infty}^{\infty} g_n(r,h)\cdot e^{-\hat{i}h(z-z')}dh \qquad (1)$$

The principal problem is that the spatial GF (1) has a singularity at the origin. In (1) this singularity is hidden. Because of this fact the spectral GFs $g_n(r,h)$ decay slowly over *n,h* in (1) and the integral and sum converge extremely slowly. Also there are spectral GF singularities at the poles. All these kinds of ' bad behaviour' of the spectral GFs, namely, asymptotes, poles are determined and annihilated. The annihilating functions can be IFTransformed in an explicit form. Doing so we obtain a representation of the spatial GFs in which the singularity at the origin and the surface wave contribution are expressed in separate terms. Thus we get spatial GFs that consist of three parts. The first includes the source singularity in an explicit form, the second one is the surface wave contribution and the third part is a pure numerical one, which is the IFT of a smooth spectral function.

## MOMENT METHOD SOLUTION

The next step is to obtain an integral equation for the sheet electrical currents in the spatial domain. To get this form, in the beginning we start in the spectral domain where field components can be expressed in terms of the current in the mixed-potential formulation by introducing spectral GFs. Then the IFT is performed and the field components turn out to have been expressed as a result of currents in the spatial domain by means of integrals over the patch area in which the spatial GFs arise. The probe, which was used as an excitation element is modelled by a special line charge source. Satisfying the boundary conditions on the patch the mixed-potential integral equations are constructed

$$\vec{E}^{exc}(r_0,\varphi,z) = \int_{z'}\int_{\varphi'} \vec{J}(\varphi',z') \cdot \hat{G}^J(z,z',\varphi,\varphi') \cdot dS' + \vec{\nabla}_t \int_{z'}\int_{\varphi'} \vec{\nabla}'_t \vec{J}(\varphi',z') \cdot G^{\sigma}(z,z',\varphi,\varphi') \cdot dS' \qquad (2)$$

where $\vec{\nabla}'_t = \vec{i}_\varphi \cdot \frac{1}{r}\frac{d}{d\varphi} + \vec{i}_z \cdot \frac{d}{dz}$,

$J_{z,\varphi}(\varphi', z')$ are sheet electric current components on the patch, S' is the surface of the patch; the sign "^" means tensor; $\vec{i}_{(z,\varphi)}$ are unit vectors. The equations (2) are solved using the moment method. In order to apply the moment method the patch area is divided into rectangular cylindrical segments. A current distribution on the patch is expanded into a series of basis functions. As basis functions rooftop functions are used. The scheme of the moment method implies a number of steps. In the first step we need to calculate the fields, created by the rooftop current distributions. As the spatial GF can be split into two parts, namely, an analytical and a numerical, this field can also be discussed and calculated in the same manner. It is very important to point out that all integrals in the analytical part of the GFs can be calculated in an explicit form. Secondly, the field created by the basis functions is projected on test functions. Here we choose razor-blade test functions and will thus perform the integration over a grid, which is formed by lines, which connect centres of neighbouring segments. As a result we end up with a system of linear algebraic equations

$$Z \cdot I = V,$$

where *Z* is a coupling impedance matrix, *I* is the column matrix of the unknown rooftop current amplitudes, and *V* is the excitation column matrix.

## RESULTS

As soon as the rooftop amplitudes are determined we are able to calculate the field in any point of space. Using a special evaluation of the Inverse Fourier Transform in the cylindrical co-ordinate system for the far zone [2] the far field components are calculated and analysed in a wide frequency band. In Fig.2 normalized far field components $E_\theta$ and $E_\varphi$ are calculated in fixed direction when the frequency varies. This figure shows that resonances are occuring. Note that all parameters are the same as in the conference paper [3]. There is a good agreement between the position of the resonance frequency of the resonance type (1,0) in this paper and in [3]. The type of resonance is given in brackets by the number of field variations along z- and φ- co-ordinates, respectively. Also in Fig.2 the locations of the next four resonances are shown when frequency varies. The current distributions calculated at the resonances (1,0) and (1,2) are plotted in Fig.3 and Fig.4, respectively.

## CONCLUSIONS

An effective approach to the problem of a cylindrical rectangular patch placed on a cylindrical circular dielectric substrate surrounding a cylindrical circular metal rod is proposed. The problem is solved in the spatial domain. The integral equations for the electrical current on the patch are obtained in the mixed potential formulation using new representations of the spatial GFs. The spatial GFs are derived in terms of three parts. The first is an analytical one, which includes the singularity at the origin in an explicit form. The second is the surface wave contribution and the third is a pure numerical one, which is the IFT of a smooth spectral function. An efficient algorithm for the numerical part of the spatial GF was built. The cylindrical probe was modelled by a special line charge source. The moment method is applied and the current distribution on the patch is investigated. Far fields are calculated in a wide frequency band. It is shown that resonances are occurring.

## REFERENCES


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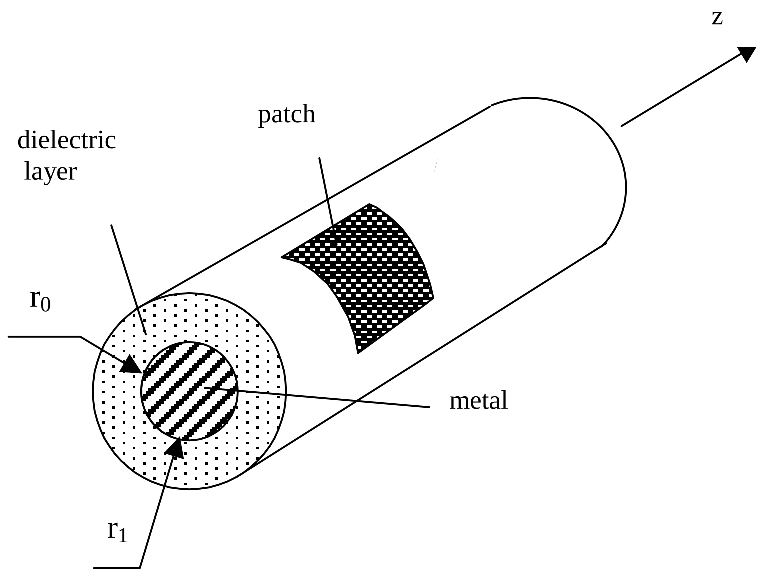


Fig.1. Cylindrical microstrip antenna.

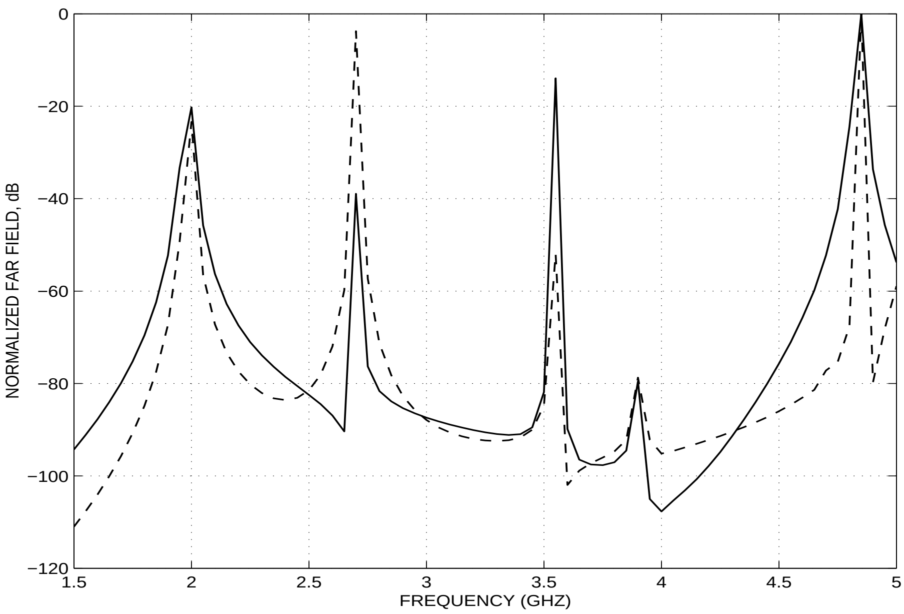


Fig.2. Normalized field components: $E_\theta$ (solid line) and $E_\varphi$ (dashed line) in the far zone are plotted when the frequency varies. ($r_0/r_1$=1.01524, ε=2.2, patch size: $\Delta_z$=0.05 (m), $\Delta_\varphi$=0.05 (m), $r_0$=0.05 (m)). The observation point is θ=π/8, φ= π/8. The frequency step is 0.05 GHz. The maxima correspond to the types of resonances: f=2 GHZ for types (1,0) and (0,1); f=2.7 GHZ for (1,1); f=3.55 GHZ for (2,0) and (0,2); f=3.9 GHZ for (1,2) and (2,1); f=4.85 GHZ for (3,0) and (0,3).

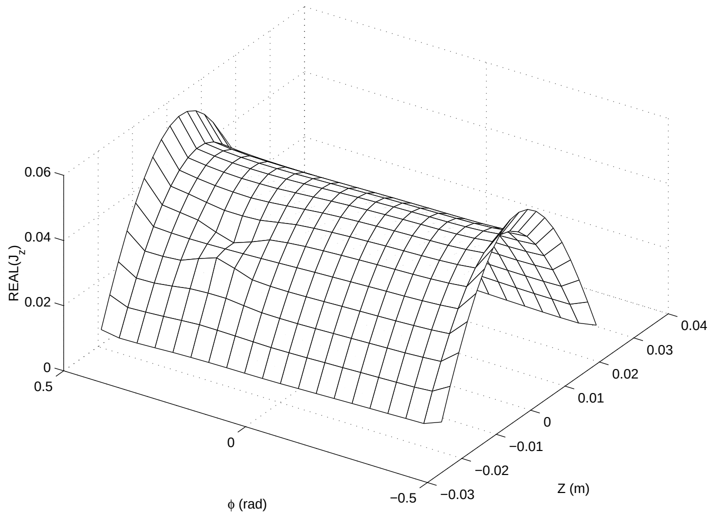


Fig.3. The distribution of the real part of the current component $J_z$ at the resonance frequency f=2 GHz for the resonance type (1,0). All geometrical and material parameters are the same as in Fig.2. The lines of equal amplitudes are slightly distorted in the region where the source is located.

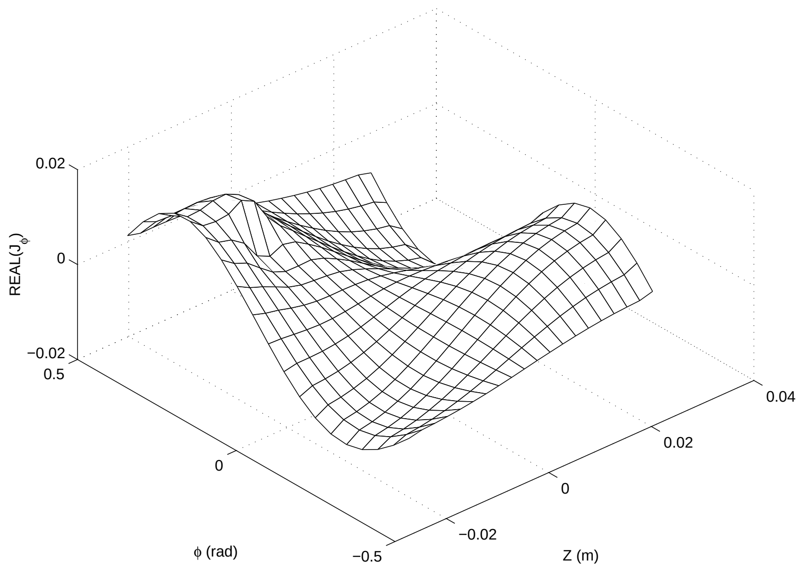


Fig.4. The distribution of the real part of the current component $J_\varphi$ at the resonance frequency f=3.9 GHz for the resonance type (1,2). All geometrical and material parameters are the same as in Fig.2.