MULTIPLE SCATTERING EFFECTS IN IONOSPHERIC RADIO SOUNDING: A NUMERICAL STUDY OF THE TRANSFER EQUATION IN THE SASIRC APPROXIMATION

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ABSTRACT

The problem of radio wave reflection from an optically thick, plane monotonic layer of ionospheric plasma with random density irregularities is considered. We investigate numerically for mid-latitude ionosphere conditions the influence of multiple scattering on the averaged intensity and angular distribution of the reflected signal using a special form of radiative transfer equation in invariant ray coordinates and its elaborated solution in the SASIRC approximation. Both vertical and slightly oblique sounding cases are treated. Conclusions from earlier versions of the theory showing a strong anomalous-attenuation effect are confirmed, with adjustments of the quantitative results.

INTRODUCTION

Intermediate-scale (100 m - 5 km) electron density irregularities influence HF signals propagating in the ionosphere in several ways. This is of importance both for radio communication and for experimental diagnostics of the ionospheric state by radio sounding techniques. It has recently been established that in vertical sounding of the ionosphere, the optical thickness for scattering by intermediate-scale irregularities is frequently considerably greater than unity [1]. This implies a multiplicity of scattering that can lead to a spatio-angular redistribution of the radio radiation flux. As a result, the mean intensity and arrival angles of the probing signal can be significantly altered, causing an "anomalous attenuation" effect near a sounder. For the case of total internal reflection of a signal from the ionospheric layer, such effects can be described by a special form of the radiative transfer equation represented in invariant ray coordinates [2], having an analytical solution for small-angle scattering which we call the SASIRC approximation. In our earlier work, numerical estimation of this solution included a further simplification using a one-term expansion of part of the integrand [2]. We now avoid this procedure, and study the full solution using special methods of adaptive numerical integration. As a result, we obtain smoother and more accurate intensity distributions of the reflected radiation, neatly representing the multiple scattering effects that occur in ionospheric vertical and slightly oblique sounding. The calculations have been done for a mid-latitude ionospheric plasma layer with field-aligned irregularities.

THEORY OF RADIATIVE TRANSFER IN A PLANE PLASMA LAYER WITH RANDOM IRREGULARITIES

We state briefly some results of the work [2] underlying our further calculations. Parameters that determine (in absence of random irregularities) the shape and position of a ray path in a plane plasma layer are called "invariant ray coordinates", and are here the polar and azimuth angles θ and φ and location $\vec{\rho}$ of point of arrival of a ray at the Earth's surface. The spatio-angular distribution of signal intensity upon reflection from a plane ionospheric layer with random irregularities is described by the radiation energy balance (REB) equation:

$$\frac{d}{dz}P(z,\vec{\rho},\theta,\varphi) = \int Q(z,\theta,\varphi,\theta',\varphi') \cdot \cdot \left\{ P\left[z,\vec{\rho}-\vec{\Phi}(z,\theta,\varphi,\theta',\varphi'),\theta',\varphi'\right] - P(z,\vec{\rho},\theta,\varphi)\right\} d\theta' d\varphi' , \tag{1}$$

where $P(z=0,\vec{\rho},\theta,\varphi)$ is the energy flux in the direction determined by the angles θ,φ through the point $\vec{\rho}=(x,y)$ at the ground,

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$$Q(z,\theta,\varphi,\theta',\varphi') = \sigma(\theta,\varphi,\theta',\varphi')C^{-1}(z,\theta,\varphi)\sin\theta'\left|\frac{d\Omega'_k}{d\Omega'}\right|, \quad (2)$$

$\sigma(\theta,\varphi,\theta',\varphi')$ is the differential scattering cross section, $C(z,\theta,\varphi)$ is the cosine of the inclination angle of a ray path to the z axis, corresponding to the invariant angles θ,φ ; $\left|\frac{d\Omega'_k}{d\Omega'}\right|$ is the Jacobian of the coordinate transformation from the current polar and azimuth angles of the wave vector to the invariant ray variables θ',φ'; the vector function $\vec{\Phi}(z,\theta,\varphi,\theta',\varphi')$ represents the displacement of the arrival point at the ground of a ray which has angular coordinates θ',φ' after scattering at the level z, relative to the arrival point of an incident ray with angular coordinates θ,φ .

The REB equation (1) has an analytic solution in the SASIRC approximation, valid if the most probable distinction between angles θ',φ' and θ,φ in a scattering act is small. This situation is typical for irregularity spectra dominated by irregularities with scales larger than the sounding wavelength. A heuristic basis for the approximation ensues from analysis of the Poeverlein construction [2]. This solution has the form:

$$\widetilde{P}(z,\vec{\rho},\theta,\varphi) = \frac{1}{(2\pi)^2}\iint d^2q\, P_0(\vec{q},\theta,\varphi)\cdot$$
$$\cdot\exp\left\{i\vec{q}\vec{\rho} + \int_0^z dz'\iint d\theta' d\varphi' Q(z';\theta,\varphi,\theta',\varphi')\left[e^{i\vec{q}\vec{\Phi}(z';\theta,\varphi,\theta',\varphi')} - 1\right]\right\}, \quad (3)$$

where $P_0(\vec{q},\theta,\varphi)$ is the Fourier transform of distribution of radiation intensity on the Earth's surface in absence of irregularities $P_0(\vec{\rho},\theta,\varphi)$. In our previous treatment of the theory we used a first-order series expansion of the inner exponent. Now we avoid this unnecessary simplification.

TECHNIQUE OF NUMERICAL CALCULATION

Expression (3) for the ray intensity can be represented as a sum of two terms that correspond to coherent and incoherent parts of the reflected signal. For the multiple scattering regime the coherent part is usually negligible. Keeping only the incoherent term and explicitly extracting its real part we obtain finally:

$$\widetilde{P}_2(z,\vec{\rho},\theta,\varphi) = \frac{1}{(2\pi)^2}\left|\frac{\partial(x,y)}{\partial(\cos\theta,\varphi)}\right| P_0[\vec{\rho}_0(\theta,\varphi)]\iint d^2q\, F', \quad (4)$$

where

$$F' = \cos[\vec{q}\vec{\rho}_0(\theta,\varphi) - \vec{q}\vec{\rho} - I_s(\vec{q},\theta,\varphi)]\exp[I_c(\vec{q},\theta,\varphi)],$$
$$I_s(\vec{q},\theta,\varphi) = \int dz'\iint d\theta' d\varphi' \sin(\vec{q}\vec{\Phi})Q\,,\quad I_c = \int dz'\iint d\theta' d\varphi'[\cos(\vec{q}\vec{\Phi}) - 1]Q\,.$$

The total signal intensity P is obtained by integrating the ray intensity (4) over angles. To characterize the influence of irregularities on P, it is appropriate to introduce the quantity $L = P/P_0$, where P_0 is the total intensity at the same point in absence of scattering.

Our calculations have been carried out for isotropic plasma with a linear dependence of average electron concentration on height ($z = Hv + h_0$,where H is the layer thickness, h_0 is the distance between layer and Earth's surface, $v = \omega_e^2/\omega^2$, ω_e is the plasma frequency, $\omega = 2\pi f$, f is the radio frequency). The scattering cross section for isotropic plasma is

$$\sigma = \frac{1}{2}\pi k_0^4 v^2 F(\vec{\kappa}), \quad (5)$$

where $k_0 = 2\pi f/c$ and $F(\vec{\kappa})$ is the spatial spectrum of irregularities. We use a model of field-aligned irregularities, of the following spectral form:

$$F(\vec{\kappa}) = C_A (1 + \kappa_\perp^2 / \kappa_{0\perp}^2)^{-\nu/2} \, \delta(\kappa_\parallel) \quad , \tag{6}$$

where $\kappa_\perp$ and $\kappa_\parallel$ are the vector $\vec{\kappa}$ (irregularity spatial-harmonic) components orthogonal and parallel to the magnetic field lines, respectively, $\kappa_{\perp 0} = 2\pi / l_{0\perp}$, $l_{0\perp}$ is the "external" scale of the irregularity spectrum, ν is the spectrum index, $\delta(x)$ is the Dirac delta–function, C_A is a normalizing constant determined from the condition that the root-mean-square amplitude of the irregularities $\Delta N/N$ at a scale length R has the value δ_R.

On the above assumptions, the expression for L acquires a form convenient for numerical calculations:

$$L = \frac{(2H + h_0)^2}{\pi^2} \iint dx_t dy_t \frac{\tan\sqrt{x_t^2 + y_t^2}}{\sqrt{x_t^2 + y_t^2}} \iint d^2 q F' \ , \tag{8}$$

where $x_t = \theta \cos\varphi$, $y_t = \theta \sin\varphi$. The problem is complicated by the necessity to calculate 7-fold integrals with sufficient accuracy. We evaluate the integrals over q using a global adaptive algorithm stated in [3].

RESULTS OF CALCULATIONS

Calculations were carried out for the following set of parameters: h_0 = 150 km; H = 100 km; ν = 3; $l_{0\perp}$ = 10 km; R = 1 km; radio frequency f = 5 MHz, inclination of geomagnetic field γ = 25°. Figure 1 shows the distribution (in angular coordinates x_t, y_t overhead) of brightness of the scattering region for the case of vertical sounding (coordinates of receiver on the Earth's surface, x = 0, y = 0, coincide with coordinates of transmitter), calculated for a relatively low irregularity amplitude $\delta_R = 0.002$. It is seen that multiple scattering leads to diffuse reflection from the ionosphere, with a specific structure in the angular spectrum: three distinct maxima are present. A sharp central peak is oriented along the magnetic meridian plane. Two wider but lower peaks are located symmetrically relative to the magnetic meridian. The effect may be called "astigmatism of the ionospheric mirror." The overall angular width of the received beam in the first example is ~3 deg. For higher irregularity amplitude (see Fig. 2 where $\delta_R = 0.006$) the side maxima are yet lower and wider, although they still dominate the total energy flux.

For $\delta_R = 0.01$ the total angular width of the scattered beam is ~20 deg. This significant sky area indicates the active role of intermediate-scale irregularities in the ionogram phenomenon of spread F (see [4]). Dependence of the integrated signal attenuation caused by multiple scattering is shown in Fig. 2. For this very typical irregularity amplitude range (compare with Fig. 1 from [4]) we obtain attenuation

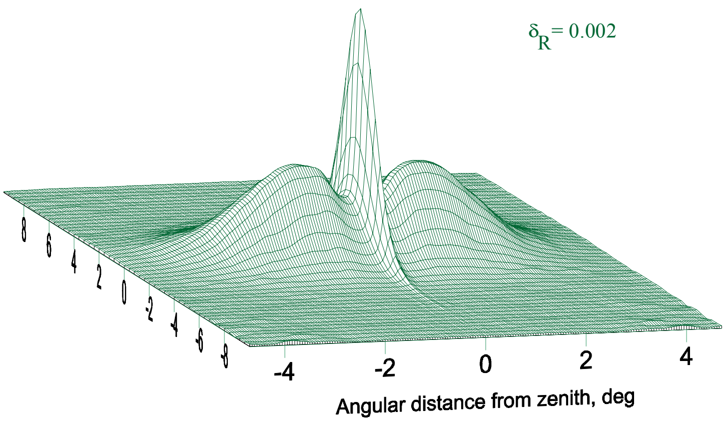

Figure 1. Angular distribution of the beam intensity of received signal overhead, for relatively low irregularity amplitude.

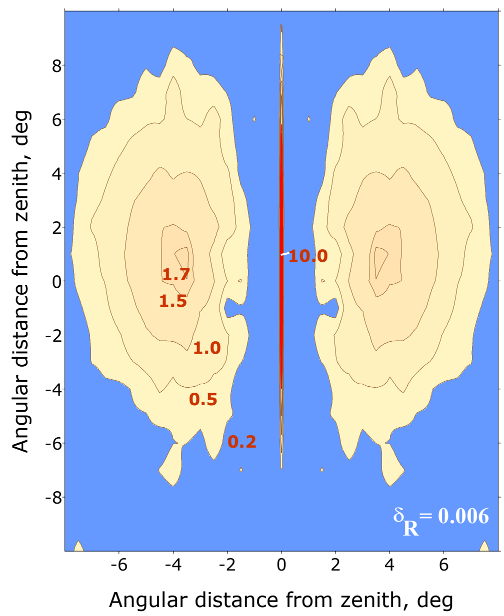

Figure 2. Same as in Fig. 1, but as a contour plot, and for higher irregularity amplitude.

values quite consistent with averaged experimental estimates of anomalous attenuation (see Fig. 3 from [4]). Thus our theory successfully reconciles experimental results acquired by two entirely independent methods, based on echo phase and averaged amplitude measurements.

Figures 4 and 5 present results of similar calculations for slightly oblique radio sounding. It is seen that positioning of the receiver some distance apart from the transmitter's meridian plane leads to violation of the symmetry observed in Figures 1 and 2. With growth of the distance one of the side maxima, which location is closer to the undisturbed angle of arrival, gradually becomes dominant, but the former central peak continues to play a noticeable role. Characteristic two-maxima structure of the obliquely reflected signal suggests something like double refraction. This effect is not of magnetoionic nature, it is caused by the scattering and may be a counterpart of other irregularity-related phenomena (see, for example, [5]). Present calculations confirm our earlier conclusion that the intensity of the received signal in the vicinity of the sounder ($x \lesssim$ 70 km for $\delta_R = 0.003$) is significantly reduced by scattering from an irregular ionospheric layer.

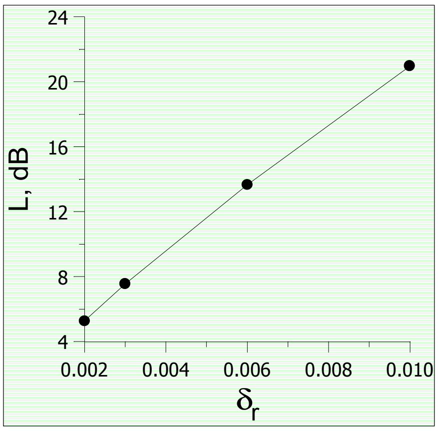

Figure 3. Anomalous attenuation vs. root-mean-square irregularity amplitude.

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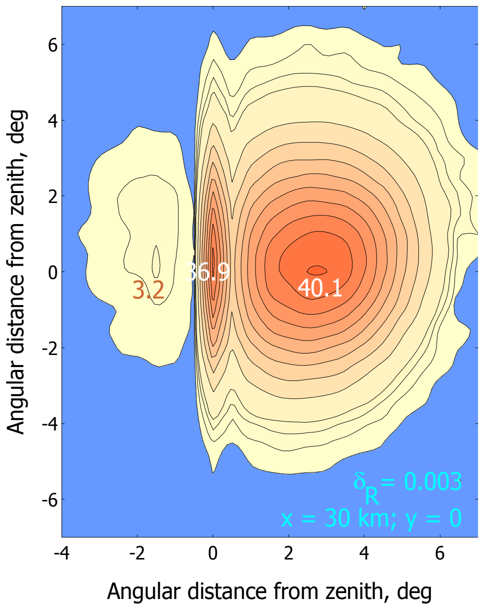

Figure 4. Angular structure of received signal for the receiver position shifted from the transmitter's magnetic meridian plane: x = 30 km, y = 0.

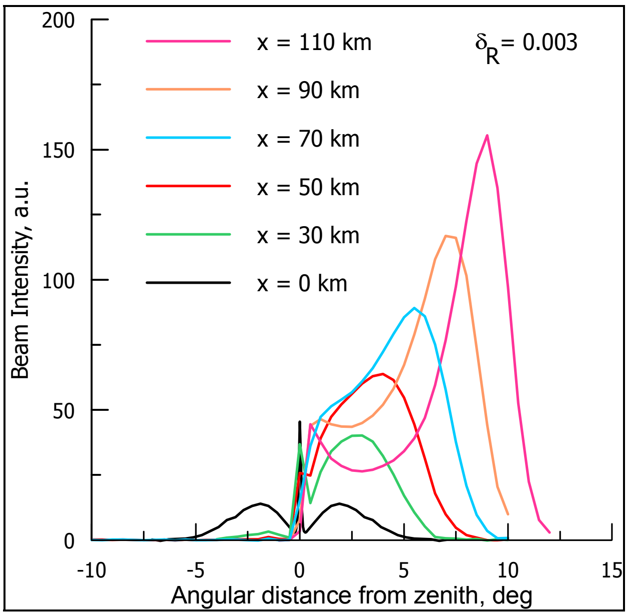

Figure 5. Cross-section of angular structure of the received radio signal along a line orthogonal to the meridian plane, for several positions of the receiver.