INFLUENCE OF REALISTIC SPECTRAL TRANSFER FUNCTION TO TIME DOMAIN MEASUREMENTS

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INTRODUCTION

It is often seen, that generating electromagnetic fields as very fast rising transients or ultra wide band signals leads to the fact, that the measured field pulse is different to the output signal of the generator. Within this presentation reasons for this effects are discussed. Therefore the behaviour of the test facility will be described in frequency as well as in time domain. The main goal is to show the links between them and to present relations error margins for time and frequency domain.

GENERAL DESCRIPTION OF FIELD GENERATION

Generally the influence of a test facility on a signal can be described as shown in fig. 1.

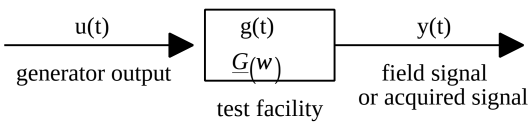


Figure 1. General signal transformation.

Using these model, the signal y(t) is given by the convolution of the generator output u(t) and the transfer function g(t) of the test facility

$$y(t)=\int_0^\infty u(t-t)g(t)dt = g(t)*u(t) \quad (1)$$

or in frequency domain as multiplication of the spectrums

$$\underline{Y}(w)=\underline{U}(w)\cdot\underline{G}(w). \qquad (2)$$

The transfer function g(t) or $\underline{G}(w)$ may describe the behaviour of an amplifier, a test site or a sensor system. Ideally the test facility should not change the shape of the signal u(t) that $y(t)$ should be only the time delayed generator signal u(t). Therefore the ideal transfer function of the test facility has to be a dead time term.

$$\underline{G}(w)_{T_T} = e^{-jwT_T} \quad (3) \qquad \text{with} \qquad \begin{aligned} \left|\underline{G}(w)\right| &= 1 \\ arc\left(\underline{G}(w)\right) &= w\cdot T \end{aligned} \qquad (4)$$

for the whole frequency range $(0Hz \le f \le \infty)$. Unfortunately this requirements cannot be guaranteed by real test facilities.

Most transient measurements suffer from one or more of the following limitations:

1. low pass effect
   This limitation is given by the upper frequency of the measurement equipment. Often it is called the frequency bandwidth.
2. high pass effect
   Some test facilities require a balun (balanced-unbalanced transformer). A balun is unable to transfer DC.
3. stochastic amplitude variation of frequency response
   Specially in field generators we can observe amplitude variation throughout the imperfections of the test site. This effect is well known and therefor not discussed within this paper.
4. phase shifting of frequency response
   This effect is well know a dispersion. Higher order field modes have a propagation velocity not equal to speed of light. Therefor we have dispersion effects in field generators.

Most of the effects are well defined in frequency domain but not in time domain. The following chapters shall link the frequency description to time domain.

LOW PASS EFFECT

Most of real measurement system are acting as a PT_2-system [1]. For simplicity we assume a PT_1-System as given by

$$\underline{G}_{PT_1} = \frac{1}{1+jwT} \quad (5)$$

Let's assume a step-function with a rise-time T_r as $u(t)$. Eq. 6 shows the step-function two ramp-function with one delayed by T_r. The goal of the measurement is to get $y(t)$ with the same rise-time T_r as $u(t)$.

$$u(t) = \frac{1}{T_r}\left(r(t) - r(t - T_r)\right) \quad (6)$$

$$F(jw) = \frac{j}{w} e^{-j\frac{wT_r}{2}} si\left(\frac{wT_r}{2}\right) \qquad (7)$$

The spectrum of the real step is given by eq. (7) and displayed in fig. 2. The frequency in fig. 2 is given in radians.

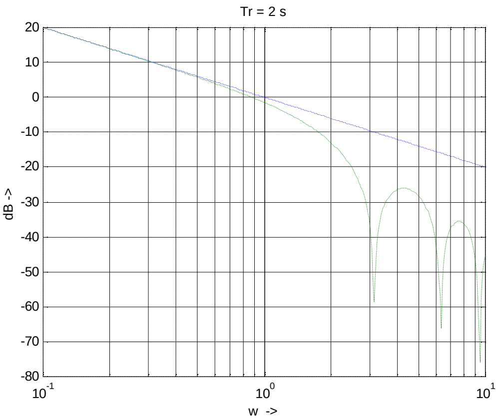


Figure 2. Spectrum of ideal and a real (T_r =2s) step-function

Our intention is to measure the real rise-time. Therefor we have to consider the frequency range up to the first zero-point of the si-function to get most of the differences between the two spectrums. The frequency is given by eq. (8). This gives the upper frequency limit of the test site. Better results can be got by extending this frequency.

$$\frac{wT_r}{2} \overset{!}{=} p \quad \Rightarrow \quad f\big|_{Hz} = \frac{1}{T_r} \qquad (8)$$

Explaining this a transfer function was used with a system rise-time of $T_{Syst.} = 5s$ as given below.

$$F(jw) = \frac{1}{1+jw5} \qquad (9)$$

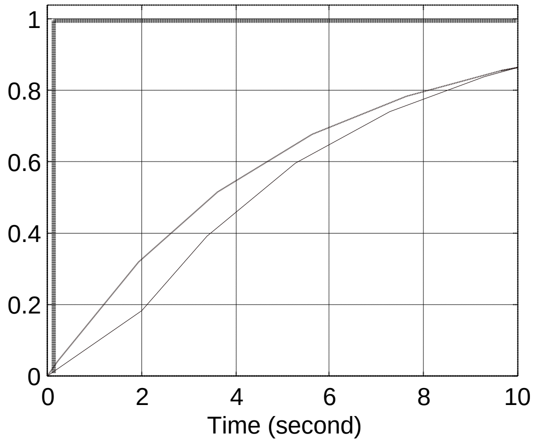


Figure 3: System response to an ideal and a real step

Figure 3 displays the system response to an ideal step (solid line) and a real step with $T_r = 2s$ (dotted line). It is interesting to see that the system responses with the same function.

HIGH PASS EFFECTS

Similar consideration are done for high-pass systems like baluns. For example the transfer function of a balun is given by eq. 10. It may represent a High-Pass-Filter with the lower frequency limit of f_g.

$$\underline{G}_{BALUN} = \frac{jw}{w_g + jw}$$
$$with: \; f_g = \frac{w_g}{2p} = \frac{1}{2pT_g} \qquad (10)$$

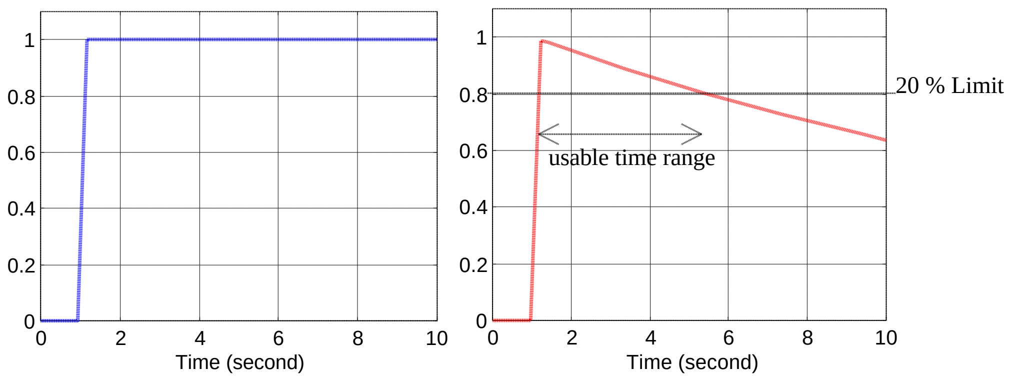
Feeding such a balun with a ideal step-function we get eq. 11 as output signal of the system.

$$y(t) = e^{-\frac{t}{T_g}} \qquad (11)$$

$$y_{\Delta t}(\Delta t) = e^{-\frac{\Delta t}{T_g}}$$
$$T_g = -\frac{\Delta t}{\ln y_{\Delta t}} \Rightarrow f_g\big|_{Hz} = -\frac{\ln y_{\Delta t}}{2p\Delta t} \qquad (12)$$

$$F(jw) = \frac{jw20}{1+jw20} \qquad (13)$$

To ensure a constant signal like the top of a rectangular pulse for a certain time T_{DC} we assume a system as given in eq. 10. Now we can calculate the lower frequency limit by taking the normalised step-response and taking the amplitude at time Δt. T_G has to be smaller than the pre-defined time T_{DC}. Now a transfer function was used as given in eq. 13. Figure 4 displays the system response to an ideal step (solid line).



Let us assume that the DC-component may not fall below 20 %. The usable time span can be calculated by eq. 14. This value fits best to time span as depicted from figure 4.

Figure 4: System response to an ideal step at $t = 1s$

$$
\begin{aligned}
y_{\Delta t}(\Delta t) &= e^{-\frac{\Delta t}{T_g}} \\
\frac{y_{\Delta t}(\Delta t)}{y_0} &= 1 - Limit \Rightarrow 0.8 \\
(1 - Limit) &= e^{-\frac{\Delta t}{T_g}} \\
-T_g \cdot \ln(1 - Limit) &= \Delta t \Rightarrow \Delta t = -\ln 0.8 \cdot 20 \approx 4.46s
\end{aligned}
\qquad (14)
$$

PHASE SHIFTING OF FREQUENCY RESPONSE

Only time delay (see eq. 4) is accepted for an ideal transfer function. In this chapter the effects of a non perfect phase response is discussed. For Impulse Radiating Antennas such effects occur as dispersion of a pulse. The dispersion effects are generated by higher order mode propagation. The propagation velocity of the TEM-mode is equal to the speed of light c_0 but higher order modes are propagating with a specific but lower velocity. This leads to the fact that the spectrum of a pulse field is propagated very frequency dependently. Again we tested the system with an double-exponential pulse (T_r=5ns, T_f=100ns). The amplitude of the frequency response was set to 1. The phase of $\underline{G}(w)$ was randomly varied within the limits of 5° to 40°. $\underline{G}_{real}(w) = 1 \cdot e^{j \pm error \cdot rand(0,1)}$ (15)

The results for 10 calculations of the pulse response are displayed in fig. 5. The curves $y_{\min}(t) = u(t)(1 - 10\%)$ and $y_{\max}(t) = u(t)(1 + 10\%)$ are displayed, too.

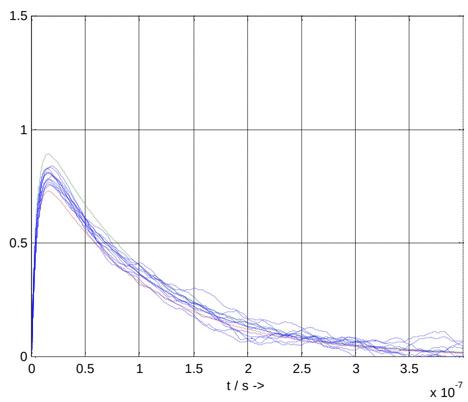


Figure 5. Pulse response for a phase variation of +/- 10° of the frequency response

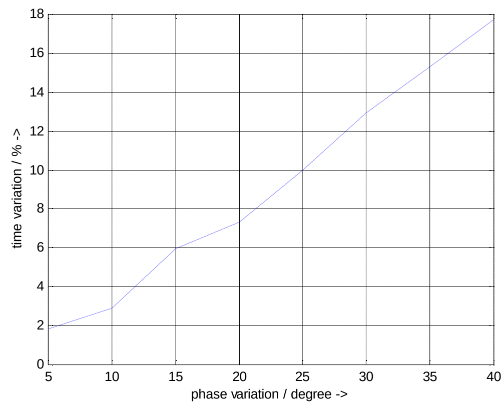


Figure 6. Variation in time domain vs. phase

The dependence between the phase variation in degree and the variation in time domain in percentage is given in fig. 6. It can be seen that the 20°-variation gives only 8%-variation in time domain. From this figure one is able to estimate the expected error in the other domain.

Phase shifting in frequency domain means time delay in the time domain. A constant phase shifting leads to a stronger influence to low frequencies than to high frequencies. On the other hand some test systems introduce a frequency dependent time delay. To study this effect the time delay of $\underline{G}(w)$ was randomly varied within the limits of 5ns to 30ns as written in eq. 15.

$$\underline{G}_{real}(w) = 1 \cdot e^{-jw \cdot T_T (\pm \cdot rand(0,1))} \qquad (15)$$

The results for 30 calculations and a maximum time delay of 5ns of the pulse response are displayed in fig. 7. Comparing figure 7 and figure 5 the late time effects in figure 5 are much more stronger than on figure 7. This effect results from the arbitrary assumption of a random phase shifting. The time delay assumption seems to be more realistic.

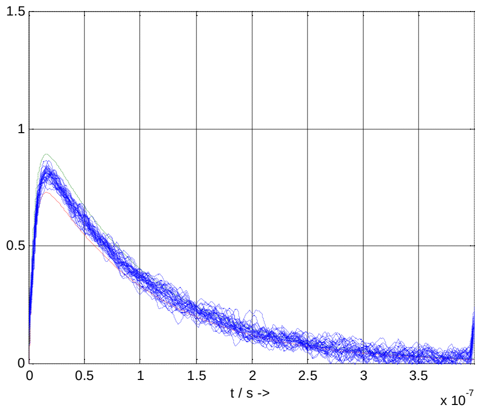


Figure 7. Pulse variation for a maximum time delay of 5ns

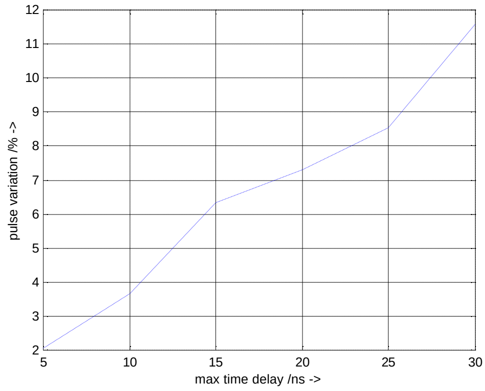


Figure 8. Pulse variation vs. max. time delay

As depicted in fig. 8 a dependence between the maximum time delay and the pulse variation in percentage is given. A time delay of 25ns leads to a pulse variation of 8%. It has to be taken into account that a double-exponential pulse with a rise time of T_r=5ns and a fall time of T_f=100ns is used. Under this aspect fig. 8 can be normalised with the rise time of 5ns. The ratio $T_d / T_r = 4$ leads to a pulse variation of $\approx 8\%$.

CONCLUSION

Values have been shown to predict the time domain behaviour from given frequency data. To ensure a shape inherent pules transition the cut-off frequency has to be chosen greater than $f_{high} \geq \frac{1}{T_r}$. A variation of less than $\pm 3dB$ in amplitude or 20° in phase results in a variation of less than 10% for the pulse. A variation in frequency-dependent time delay for a pulse with the rise time T_r of less than $T_d / T_r = 4$ leads to a pulse variation of $\approx 8\%$.

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