CALCULATING THE USABLE FREQUENCY RANGE OF TEM-WAVEGUIDES

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ABSTRACT

In principle all TEM waveguides and TEM cells have no frequency limitation for propagation of the Transverse Electromagnetic (TEM) mode. The well known problems applying TEM cells occur from the unwanted generation, propagation and resonances of higher order modes in such a waveguide. The resonances can be seen as the most disturbing effect. In general the presence of the first resonance of a higher order mode represents the upper frequency limit of a TEM waveguide. Therefor the goal of this paper is to present a closed equation to calculate this significant parameter. After some introductionary notes the general method to handle the field propagation as a series expansion of higher order field modes will be repeated. Each mode propagation can be seen as a signal propagation along a specific transmission line defined by the characteristic impedances and the propagation velocity. Resonances on transmission lines can be observed, if both sides of the line are mismatched. In the next step it will be shown how the "mode transmission line parameters" are converted in an equation describing the first resonant frequency for a given TEM cell.

INTRODUCTION

TEM-waveguides have been established as a very effective tool for electromagnetic field generation and measurement. They provide the TransversalElectroMagnetic (TEM)-mode as a fundamental field mode starting at DC. This TEM-mode is also known as antenna-farfield.

Nevertheless for high frequencies the TEM-mode is disturbed by the generation of higher order field modes. All these higher order modes have a certain cut-off frequency and may show resonating effects. First of all it is necessary to differentiate between the cut-off frequency and the resonating effects. The cut-off frequency can be seen as an intrinsic property of the cross-section of the TEM-cell. It specifies the fact that a certain mode is able to propagate in this cross-section. To build up resonating effects field mode propagation can be seen as wave propagation along a transmission line. Such a line shows resonances if both ends are mismatched. This mismatch is given by a specific cross-section with the cut-off frequency equal to the used frequency. In principle only a two-port TEM-cell shows resonances because the one-port cells like GTEM-cells have a mismatch only at one end. The other end is matched by resistors and absorbers. Therefor two-port TEM-cells will be considered in this presentation. Specially around resonant frequencies the TEM-mode will be suppressed by the higher order modes. To use the TEM-cell as a pure TEM-wave generator the first resonant frequency has to be determined.

Within this presentation an analytical calculation, which was presented by Groh in her Ph.D.-thesis [3], will be shown to get this first resonance frequency. Therefore a two-port TEM-cell with a rectangular cross-section will be considered. The wave-propagation of higher order modes can be seen as mode voltage/current propagation along a mismatched transmission line with the characteristic impedance and propagation constant of the higher order modes. For eyery frequency the length of this mode-transmission line is given by the points for which cross-section the mode is below cut-off. Under this assumption the wave-number and the characteristic impedance of the mode have to be calculated. All this results in a closed equation to predict the resonant frequency and the upper limit of the usable frequency range of such a TEM-cell.

HIGHER ORDER MODE PROPAGATION

The TEM mode can be used to generate an approximate free space plane wave. TEM wave propagation is possible in waveguides consisting of at least two separate conductors. The transverse field distribution of the TEM mode is identical to the transverse electrostatic field between the two conductors. In order to approximate a linearly polarized plane wave as would exist in ideal free space, two finite parallel plates give the desired field structure. To avoid radiated emissions and ambient noise enclosed TEM lines were developed. They are known as TEM cells.

All these waveguides have in common that the intentional TEM wave interacts by coupling with unwanted modes in nonuniform regions of the waveguide. All modes propagate independently from another in the adjoining uniform waveguide regions. A powerful method to analyze wave propagation and coupling in TEM waveguides is based on the concept of generalized telegraphist's equations, which were first proposed and studied by Schelkunoff [6]. In [4] an extensive report is given of the application to TEM cells.

Any field in a uniform waveguide can be described in terms of eigenfields of normal modes. In the uniform coaxial guide these modes propagate independently from each other in both axial directions. The fields of nonuniform waveguides are represented in terms of modes of waveguides, which correspond to the local cross section of the nonuniform guide. The fields of these coaxial waveguide modes are derived from vector potentials, which only have a longitudinal component. In inhomogeneous waveguides the transverse fields are a function of longitudinal coordinate *z*, the direction of propagation. Transverse fields for every cross section are expressed by the following set of orthogonal transverse vector functions.

$$\vec{E}_{tr}(x,y,z)=\sum_{p=1}^{\infty}V^{(p)}(z)\cdot\vec{e}_{tr}^{(p)}(x,y,z) \qquad V^{(p)}=\int_A \vec{E}_{tr}\cdot\vec{e}_{tr}^{(p)}\cdot dA$$
$$\vec{H}_{tr}(x,y,z)=\sum_{p=1}^{\infty}I^{(p)}(z)\cdot\vec{h}_{tr}^{(p)}(x,y,z) \quad (1) \qquad I^{(p)}=\int_A \vec{H}_{tr}\cdot\vec{h}_{tr}^{(p)}\cdot dA \quad (2)$$

The index *p* addresses either the TEM mode ($p = T$), an E mode ($p = E_1 \ldots E_¥$) or a H mode ($p = H_1 \ldots H_¥$). The voltage and current coefficients $V^{(p)}$ and $I^{(p)}$) in (1) are determined to (2).

With Maxwell's equations for the time harmonic an infinite system of coupled ordinary differential equations (3) known as *generalized telegraphist's equations* is obtained.

$$\frac{dV^{(p)}}{dz}=-g^{(p)}(z)\cdot Z_W^{(p)}(z)\cdot I^{(p)}(z)+\sum_{q=1}^{\infty}C_{pq}(z)\cdot V^{(q)}(z)$$
$$\frac{dI^{(p)}}{dz}=-\frac{g^{(p)}(z)}{Z_W^{(p)}(z)}\cdot V^{(p)}(z)-\sum_{q=1}^{\infty}C_{qp}(z)\cdot I^{(q)}(z) \quad (3)$$

Eq. (3) consists of two different parts. The first part only describes the interaction between the current and the voltage component of the same mode p. There the parameters $g^{(p)}(z)$ and $Z_W^{(p)}(z)$ are termed propagation constant and wave impedance as known from transmission lines. The infinite sums of the second part represent for $p \neq q$ the coupling between different field modes. This part describes the excitation of the higher order modes. Figure 1 displays this interactions.

In case of wave propagation exclusively in the TEM mode the physical voltage $V = V^{(T)}\cdot ü$ and current $I = \frac{I^{(T)}}{ü}$ can be expressed using the transformation ratio ü and the voltage and current coefficients of the TEM mode.

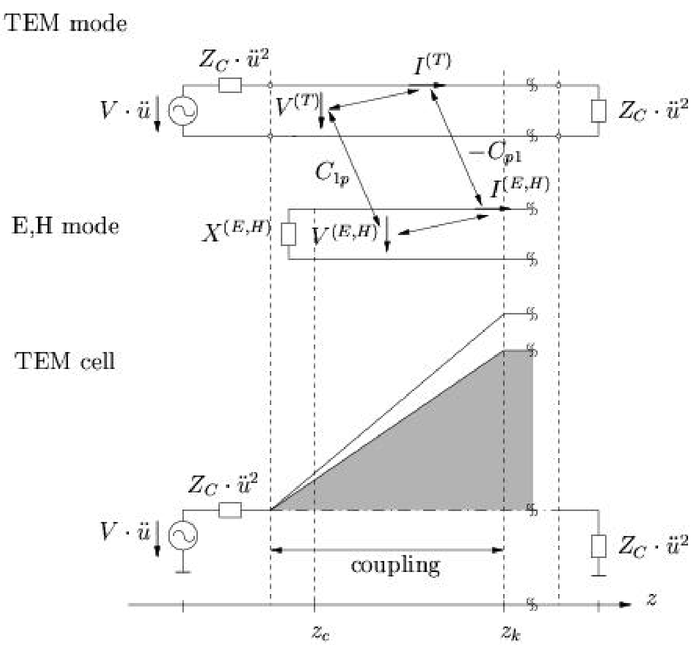

Fig. 1: Field propagation represented by coupled transmission lines

$$ü^2=\sqrt{\frac{m}{e}}\cdot\frac{1}{Z_C}$$

with Z_C as the characteristic impedance of the transmission line.

In waveguides which contain homogeneous, isotropic and lossless matter, the wave number is $k = w\sqrt{me}$. With the cutoff wave number $k_c^{(p)}(z)$ the propagation constant for each mode is given by $g^{(p)}=\sqrt{k_c^{(p)2}-k^2}$. For $k^2 > k_c^{(p)2}$ the mode is above cutoff and propagates with

$$g^{(p)} = jb^{(p)} = jk\sqrt{1-\left(\frac{k_c^{(p)}}{k}\right)^2} \qquad (5)$$

The field wave impedance of E and H modes is given by

$$Z_W^{(E)}(z) = \frac{1}{we}\sqrt{k^2 - k_c^{(E)2}(z)}$$
$$Z_W^{(H)}(z) = wm\sqrt{\frac{1}{k^2 - k_c^{(H)2}(z)}} \qquad (6)$$

It has to be noted that field wave impedances of the different modes are different. Only the TEM mode is coupled via eq. (4) to realistic voltages and currents along the line. Following this only the TEM can be influenced by terminating the transmission line. Furthermore it has to be noted, that the wave impedances as well as the propagation constants of the different modes are dependent on the z-coordinate.

RESONANCES ON TRANSMISSION LINES

This article addresses the resonances on transmission line systems. Chapter 2 has described the propagation of higher order modes. Now the fundamentals for resonances on transmission lines are taken into account. It is well known, that the following requirements have to be fulfilled for resonances.

1. The transmission line has to be mismatched at both ends.
2. The length of the line has to be $\frac{l}{4}$ or $\frac{l}{2}$ for the first resonance.

The relation of these two points to field calculation in TEM cells shall be considered in this section. Koch [5], Kärst [4] and other have shown, that always the feeding section of a TEM waveguide represents a mismatch for the higher order modes. Two port TEM cells like the well known Crawford cell [2] have two feeding section. Therefor this cell type has an intrinsic mismatch at both ends. They show resonances.

One port cells like the GTEM cell try to avoid the second mismatch. They place broadband termination consisting of resistors (matching TEM mode) and absorbers (matching higher order E, H modes). The observed real world problems with one port TEM waveguides result in the imperfect behavior of the absorbers. Normally they show perfect working for a limited frequency span. As the propagation wavelength of inhomogeneous waveguides depends not only on the frequency but also on the geometry (see eq. 11), a simple evaluation of the second point is not possible in case of a GTEM-cell.

RESONANCES IN TEM-WAVEGUIDES

This chapter will show the derivation of an analytical description for the resonances of a two-port TEM cell. As an example the classical Crawford cell from [2] will be considered. Groh has shown in [3] that this procedure is generally applicable.

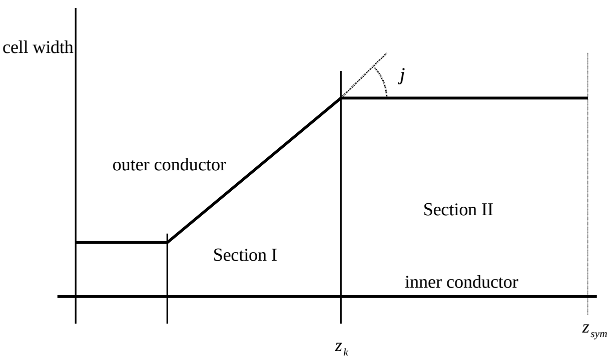

Fig. 2: Side view of a two-port TEM cell

Disregarding coupling eq. (3) yields

$$\frac{dV^{(p)}}{dz} = -g^{(p)}(z)\cdot Z_W^{(p)}(z)\cdot I^{(p)}(z)$$
$$\frac{dI^{(p)}}{dz} = -\frac{g^{(p)}(z)}{Z_W^{(p)}(z)}\cdot V^{(p)}(z) \qquad (7)$$

Decoupling these equations and substituting the characteristic parameters $g^{(p)}(z)$ and $Z_W^{(p)}(z)$ by (5) and (6) gives

$$\frac{d^2V^{(H)}}{dz^2} = \left(k_c^{(H)^2}(z) - k^2\right)\cdot V^{(H)}(z)$$
$$\frac{d^2I^{(E)}}{dz^2} = \left(k_c^{(E)^2}(z) - k^2\right)\cdot I^{(E)} \qquad (8)$$

Eq. (8) is valid for all TEM waveguides. The following calculation are done for a waveguide like figure 2. Two sections can be determined from fig. 2. Section I represents the feeding section with an varying cell width. Section II shows the so called homogeneous section.

In section I the eq. (8) is solved by

$$K_1 \sqrt{a' z} \cdot J_n(kz) + K_2 \sqrt{a' z} \cdot J_{-n}(kz) \tag{9a}$$

with the Bessel function $J_n(kz)$ and $n = \sqrt{\frac{1}{4} + \left(\frac{k_{c,norm}}{2a'}\right)^2}$, $k_{c,norm} = k_c^{(E,H)}(z) \cdot 2a(z)$ and $a' = \tan j$. For further details see [3].

For section II eq. (8) is solved by

$$K_3 \cdot \cos\left(z\sqrt{k^2 - k_c^2(z)}\right) + K_4 \sin\left(z\sqrt{k^2 - k_c^2(z)}\right) \tag{9b}$$

The constants K_1 to K_4 may be solved analytically. K_1 is chosen to 1. Therefor K_2 leads to 0. Combining eq. (9a) and (9b) K_3 and K_4 can be written as:

$$\begin{pmatrix} K_3 \\ K_4 \end{pmatrix} = \frac{1}{\sqrt{k^2 - k_c^2(z_k)}} \cdot \begin{pmatrix} \frac{\sqrt{k^2 - k_c^2(z_k)}}{1} \cos\left(z_k \sqrt{k^2 - k_c^2(z_k)}\right) & -\sin\left(z_k \sqrt{k^2 - k_c^2(z_k)}\right) \\ -\frac{\sqrt{k^2 - k_c^2(z_k)}}{1} \sin\left(z_k \sqrt{k^2 - k_c^2(z_k)}\right) & \cos\left(z_k \sqrt{k^2 - k_c^2(z_k)}\right) \end{pmatrix} \cdot \begin{pmatrix} \sqrt{a' z_k} \cdot J_n(kz_k) \\ \frac{a' - 2na'}{2\sqrt{a' z_k}} \cdot J_n(kz_k) + \sqrt{a' z_k} \cdot J_{n-1}(kz_k) \end{pmatrix} \tag{10}$$

Concluding this calculations finally we get the first resonance as equation (11).

$$f_{res} = \frac{1}{2p\sqrt{em}} \sqrt{\left(\frac{\frac{1}{2}p - \arctan\left(\frac{K_3}{K_4}\right)}{z_{sym}}\right)^2 + k_c^2(zsym)} \tag{11}$$

This equation (11) is valid for all symmetric TEM waveguides of inherent shape.

CONCLUSION

Within this presentation the analytical calculation of the first resonance of a two-port TEM waveguide is demonstrated. Now it is possible to predict the usable frequency range of TEM waveguides. In the past only formulas based on the assumption of an empty waveguide acting as a cavity resonator are available. As practical experience has shown this assumption is not valid. This paper has validated this experience.

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