EIGENWAVES IN HARD SURFACE WAVEGUIDE FILLED WITH GYROTROPIC MEDIUM

Ari Viitanen(1)**, Tero Uusitupa**(1)

(1)**Electromagnetics Laboratory, Department of Electrical and Communications Engineering**
Helsinki University of Technology, P.O. Box 3000, FIN-02015 HUT, Finland
E-mail: ari.viitanen@hut.fi, tero.uusitupa@hut.fi

ABSTRACT

Eigenwaves in corrugated circular waveguide filled with gyrotropic material is considered. Fields are time harmonic and are propagating in z-direction with the real propagation factor. The depth of the corrugation is a quarter wave length and the special direction of the corrugation is in axial direction. The boundary condition of this kind of corrugated surface is equal to a hard surface boundary condition. Examples of gyrotropic materials are magnetoplasma and ferrite. The eigenvalue equation is formed and eigenfields for ferrite filled hard surface waveguide are considered.

INTRODUCTION

The tuned corrugated waveguides are often used in microwave techniques where special kind of properties for field propagation are needed, for example, in antenna horn feed for parabolic reflector antennas [1]. In corrugated waveguide there can propagate TE, TM and circularly polarized fields and in hard surface waveguide also TEM fields [2]. In this study the corrugation is in axial direction forming the boundary condition for hard surface waveguide. Time harmonic fields (depending on t as $e^{j\omega t}$) are considered and the propagating fields depend on z as $e^{-j\beta z}$, where the propagation factor β is a real number. Examples of gyrotropic materials are magnetoplasma and ferrite. The special direction of the gyrotropic material is also z direction, the gyrotropic material is biased with static magnetic field which is in z direction. In this study ferrite is considered in detail.
The electric and magnetic fields inside the waveguide are written in terms of axial and transverse components and the Maxwell equations with the constitutive relations for gyrotropic material are written in terms of axial and transverse parts. The transverse field components are eliminated which leads to a coupled Helmholtz equation for the axial electric and magnetic field components. The cut-off numbers and finally the axial field components are determined by the boundary condition for the hard surface; on the boundary of the waveguide the axial electric and magnetic field components should vanish. As a result the propagation constant for eigenwaves are obtained and the propagation factors are found to be different for the two eigenwaves. Finally, the transverse fields can be expressed in terms of the axial field components.

THEORY

Examples of gyrotropic materials are magnetoplasma and ferrite. The constitutive realations of magnetoplasma are

$$\mathbf{D} = [\epsilon_t \overline{\overline{I}}_t + \epsilon_z \mathbf{u}_z \mathbf{u}_z + j\epsilon_g \mathbf{u}_z \times \overline{\overline{I}}] \cdot \mathbf{E} \ , \quad \mathbf{B} = \mu_o \mathbf{H}. \tag{1}$$

The parameters are

$$\epsilon_t = \epsilon_o(1 + \frac{\omega_p^2}{\omega_g^2 - \omega^2}), \quad \epsilon_z = \epsilon_o(1 - \frac{\omega_p^2}{\omega^2}), \quad \epsilon_g = \epsilon_o \frac{\omega_p^2 \omega_g}{\omega(\omega_g^2 - \omega^2)} \tag{2}$$

where $\omega_p^2 = \frac{Ne^2}{m\epsilon_o}$ is the square of the plasma frequency and $\omega_g = \frac{|e|}{m} B_o$ is the gyrofrequency.
For ferrite the constitutive relations

$$\mathbf{D} = \epsilon \mathbf{E} \ , \quad \mathbf{B} = [\mu_t \overline{\overline{I}}_t + \mu_z \mathbf{u}_z \mathbf{u}_z + j\mu_g \mathbf{u}_z \times \overline{\overline{I}}] \cdot \mathbf{H}. \tag{3}$$

The material parameters are

$$\mu_t = \mu_o(1 + \frac{\omega_o \omega_m}{\omega_o^2 - \omega^2}), \quad \mu_z = \mu_o, \quad \mu_g = -\mu_o \frac{\omega \omega_m}{\omega_o^2 - \omega^2} \tag{4}$$

where $\omega_o = \gamma B_o$ is the Larmor precession frequency (B_o is the stength of the static magnetic flux density in z direction), $\omega_m = \gamma M_s$, γ is the gyromagnetic ratio and M_s is the saturation magnetization [3]. These two cases are quite similar and can be obtained by duality transformation from each other. Here, ferrite is studied in detail.

The Maxwell equations

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}, \quad \nabla \times \mathbf{H} = j\omega \mathbf{D} \tag{5}$$

are written in terms of axial and transverse components and $\nabla = \nabla_t - j\beta \mathbf{u}_z$ is denoted. The fields are written in terms of transverse and axial components

$$\mathbf{E} = \mathbf{e} + E_z \mathbf{u}_z, \quad \mathbf{H} = \mathbf{h} + H_z \mathbf{u}_z \tag{6}$$

and Maxwell equations for axial and transverse parts are written separately [3]. For ferrite that is

$$\nabla_t \times \mathbf{e} = -j\omega\mu_z H_z \mathbf{u}_z \tag{7}$$

$$\nabla_t E_z \times \mathbf{u}_z - j\beta \mathbf{u}_z \times \mathbf{e} = -j\omega\mu_t \mathbf{h} + \omega\mu_g \mathbf{u}_z \times \mathbf{h} \tag{8}$$

$$\nabla_t \times \mathbf{h} = j\omega\epsilon E_z \mathbf{u}_z \tag{9}$$

$$\nabla_t H_z \times \mathbf{u}_z - j\beta \mathbf{u}_z \times \mathbf{h} = j\omega\epsilon\, \mathbf{e} \tag{10}$$

The transverse fields $\mathbf{e}$ and $\mathbf{h}$ are eliminated, incerting $\mathbf{e}$ from (10) into (8) and solving $\mathbf{h}$ one obtains

$$\mathbf{h} = \left[\frac{j(\beta^2 - k_t^2)\overline{\overline{I}}_t - k_g^2 \mathbf{u}_z \times \overline{\overline{I}}}{(\beta^2 - k_t^2)^2 - k_g^4} \right] \cdot [\omega\epsilon \mathbf{u}_z \times \nabla_t E_z + \beta \nabla_t H_z] \tag{11}$$

and after that from (10) the transverse electric field is written as

$$\mathbf{e} = \left[\frac{j(\beta^2 - k_t^2)\overline{\overline{I}}_t - k_g^2 \mathbf{u}_z \times \overline{\overline{I}}}{(\beta^2 - k_t^2)^2 - k_g^4} \right] \cdot \left[\beta \nabla_t E_z - [k_t^2 \overline{\overline{I}}_t + j k_g^2 \mathbf{u}_z \overline{\overline{I}}] \cdot \frac{\mathbf{u}_z \times \nabla_t H_z}{\omega\epsilon} \right] \tag{12}$$

Incerting the transverse magnetic field into (9) and the transverse electric field into (7) the following second order coupled differential equations are obtained for axial field components

$$\frac{(\beta^2 - k_t^2)}{(\beta^2 - k_t^2)^2 - k_g^4} \nabla_t^2 E_z + \frac{\beta k_z \frac{\mu_g}{\mu_z}}{(\beta^2 - k_t^2)^2 - k_g^4} \nabla_t^2 (j\eta_z H_z) = E_z \tag{13}$$

$$\frac{\beta k_z \frac{\mu_g}{\mu_z}}{(\beta^2 - k_t^2)^2 - k_g^4} \nabla_t^2 E_z + \frac{1}{k_z^2} \frac{(\beta^2 - k_t^2)^2 k_t^2 + k_g^4}{(\beta^2 - k_t^2)^2 - k_g^4} \nabla_t^2 (j\eta_z H_z) = j\eta_z H_z \tag{14}$$

where it is denoted

$$k_t^2 = \omega^2 \mu_t \epsilon, \quad k_z^2 = \omega^2 \mu_z \epsilon, \quad k_g^2 = \omega^2 \mu_g \epsilon \tag{15}$$

The above equations are of the form

$$\begin{bmatrix} a & -b \\ -b & c \end{bmatrix} \begin{bmatrix} \nabla_t^2 E_z \\ \nabla_t^2 (j\eta_z H_z) \end{bmatrix} = \begin{bmatrix} E_z \\ j\eta_z H_z \end{bmatrix} \tag{16}$$

with the parameters

$$a = \frac{(\beta^2 - k_t^2)}{(\beta^2 - k_t^2)^2 - k_g^4}, \quad b = \frac{\beta k_z \frac{\mu_g}{\mu_z}}{(\beta^2 - k_t^2)^2 - k_g^4}, \quad c = \frac{(\beta^2 - k_t^2) k_t^2 + k_g^4}{k_z^2 [(\beta^2 - k_t^2)^2 - k_g^4]} \tag{17}$$

Taking new coefficients

$$A = -\frac{c}{ac - b^2} = k_t^2 - \beta^2 - k_t^2 \left(\frac{\mu_g}{\mu_t} \right)^2 \tag{18}$$

$$B = \frac{b}{ac - b^2} = k_z \beta \frac{\mu_g}{\mu_t} \tag{19}$$

$$C = -\frac{a}{ac - b^2} = (k_t^2 - \beta^2)\frac{\mu_z}{\mu_t} \tag{20}$$

the equation (16) can be written as

$$\nabla_t^2 \begin{bmatrix} E_z \\ j\eta_z H_z \end{bmatrix} + \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} E_z \\ j\eta_z H_z \end{bmatrix} = 0 \tag{21}$$

This matrix equation is written in terms of eigenvalues and eigenvectors. The eigenvalues of the matrix and the corresponding eigenvectors are solved. The eigenvalues of the matrix are

$$\lambda_+ = \frac{A + C + \sqrt{(A - C)^2 + 4B^2}}{2} \tag{22}$$

and

$$\lambda_- = \frac{A + C - \sqrt{(A - C)^2 + 4B^2}}{2} \tag{23}$$

The corresponding eigenvectors are

$$E_{z+} = E_z - j\frac{A - C + \sqrt{(A - C)^2 + 4B^2}}{2B}\eta_z H_z \tag{24}$$

and

$$E_{z-} = E_z - j\frac{A - C - \sqrt{(A - C)^2 + 4B^2}}{2B}\eta_z H_z \tag{25}$$

By using this eigenfield decomposition the original matrix equation reduces to two separate scalar Helmholtz equations

$$\nabla_t^2 E_{z\pm} + \lambda_\pm E_{z\pm} = 0 \tag{26}$$

the solutions of which are, since $\lambda_\pm > 0$

$$E_{z\pm} = A_\pm J_n(\sqrt{\lambda_\pm}\rho)e^{jn\varphi} \tag{27}$$

The boundary condition of hard surface at $\rho = a$: $E_z = 0,\ H_z = 0$ leads to the eigenvalue equation

$$J_n(\sqrt{\lambda_+}a)J_n(\sqrt{\lambda_-}a) = 0 \tag{28}$$

from which the eigenvalues are

$$\lambda_\pm = \left(\frac{p_{ns}}{a}\right)^2 = k_c^2 \tag{29}$$

where p_{ns} are the zeros of the Bessel function and a is the radius of the waveguide. Writing the eigenvalues with the material parameters and the propagation factor, finally, the propagation factors for the eigenfields are obtained

$$\beta_\pm^2 = k_t^2 - \left[\left(\frac{1 + \frac{\mu_t}{\mu_z}}{2}\right)k_c^2 \pm \sqrt{\left(\frac{1 - \frac{\mu_t}{\mu_z}}{2}\right)^2 k_c^4 + k_z^2(k_z^2 - k_c^2)\left(\frac{\mu_g}{\mu_z}\right)^2}\right] \tag{30}$$

Also, the eigenvectors are written in terms of material parameters and propagation factors as

$$E_{z\pm} = E_z - jg_\pm \eta_z H_z \tag{31}$$

where it is denoted

$$g_\pm = \frac{k_c^2 - k_t^2 + \beta_\pm^2 + k_t^2\left(\frac{\mu_g}{\mu_t}\right)^2}{k_z \beta_\pm \frac{\mu_g}{\mu_t}} \tag{32}$$

In a special case, if $\omega \to \infty$, the relative permeability $\frac{\mu_t}{\mu_z} \to 1$, and the gyrotropy parameter $\frac{\mu_g}{\mu_t} \to 0$, the parameter $g_\pm \to \mp 1$ and the eigenfields reduce to the wave-fields [4]. After that the axial field components are

$$E_z = -\frac{g_-}{g_+ - g_-}E_{z+} + \frac{g_+}{g_+ - g_-}E_{z-} \tag{33}$$

and

$$\eta_z H_z = \frac{j}{g_+ - g_-} E_{z+} - \frac{j}{g_+ - g_-} E_{z-} \tag{34}$$

The transverse field components are written with these axial field components by using (11) and (12).

DISCUSSION

The elimination of transverse field components in the Maxwells equations leads to the coupled Helmholtz equation for axial electric and magnetic field components which is diagonalized. The solution of this diagonalized equation is a certain combination of the axial electric and magnetic fields. The eigenfields are combinations of TE and TM fields in contrary to the fields in hard surface waveguide filled with anisotropic medium where the eigenfields are exactly TE and TM fields [5]. The total field is a combination of the two eigenfields. Since the eigenwaves are propagating with different propagation factor, the polarization of the total field is changed. This effect can be used for mode transformation between TE and TM fields. As an application, the section of gyrotropic hard surface waveguide of a proper length can be used as a mode transformer. Similar kind of mode transformation effect was considered in [6] by using chiral medium. The material parameters of gyrotropic medium can be changed by external magnetic field, so, it is possible to obtain the optimal mode transformation properties by tuning. Because of mode transforming property the gyrotropic hard surface waveguide can be used as a matching element for waveguides or between waveguides and antennas.

References

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