TIME DOMAIN DIFFRACTED FIELD BY AN ANISOTROPIC IMPEDANCE HALF-PLANE: UAPO EXPRESSION

C.Gennarelli (1), **G.Pelosi** (2), **G.Riccio** (3)

(1) D.I.I.E., University of Salerno, via Ponte Don Melillo, I-84084 Fisciano (SA), Italy
e-mail: gennar@diiie.unisa.it

(2) D.E.T., University of Florence, via C.Lombroso, I-50134 Florence, Italy
e-mail: pelosi@ingfi9.det.unifi.it

(3) As (1) above, but e-mail: riccio@diiie.unisa.it

ABSTRACT

A time domain solution for the field diffracted by a nonpenetrable half-plane characterized by an anisotropic impedance boundary condition is proposed in this paper. In the hypothesis that the impedance tensor components along the principal axes of anisotropy do not depend on the frequency, such a solution is obtained by determining the time domain version of the corresponding Uniform Asymptotic Physical Optics diffraction coefficients, whose frequency domain expression contains the transition function of the Uniform Geometrical Theory of Diffraction. The response to a unit step function plane wave incident on an anisotropic impedance half-plane is explicitly considered.

INTRODUCTION

In recent years, the interest to the research of time domain solutions for predicting the electromagnetic (EM) scattering by structures with known EM characteristics seems to be increased. This particular attention may be due to the great importance that such solutions have recently acquired in several strategic areas. In this context, a time domain version of the Uniform Theory of Diffraction (UTD) for a perfectly conducting wedge surrounded by an isotropic homogeneous medium was derived by Veruttipong in [1]. It was obtained by inversely transforming the high-frequency solution relevant to the considered canonical problem as reported in [2].

A Uniform Asymptotic Physical Optics (UAPO) approach has been recently proposed in [3] to solve the problem of the EM scattering by a nonpenetrable half-plane characterized by an anisotropic Impedance Boundary Condition (IBC) when illuminated at oblique incidence by an arbitrarily polarized plane wave. According to the anisotropic IBC, the tangential components of the electric and magnetic fields on the surface are coupled by means of a tensor having two principal axes of anisotropy, which are mutually orthogonal. The corresponding UAPO solution has been obtained by using a PO approximation of the surface current densities induced on the illuminated face and by performing a uniform asymptotic evaluation of the resulting radiation integral. Its expression contains the standard UTD transition function and furnishes the evaluation of the diffracted field by the edge in the considered structure without restrictions on the position of the principal axes of anisotropy. The UAPO solution is resulted to be simple to implement in an efficient computer code and, although approximate, accurate when compared with those in literature.

Aim of this work is to find the time domain UAPO solution for the field diffracted by an anisotropic impedance half-plane in the hypothesis that the impedance tensor components along the principal axes of anisotropy do not depend on the frequency, as already supposed in [4]. According to the Veruttipong's formulation, the evaluation of the time domain diffracted field at the observation point requires the integration with respect to the time of the matrix product between the inverse Laplace transform of the frequency domain diffraction matrix and the incident field at the diffraction point. Therefore the first step to solve the here considered problem consists in determining the time domain version of the UAPO diffraction coefficients previously derived in [3]. This is accomplished by taking into account that the UAPO solution is UTD-like with respect to the frequency. After this step, the response to a unit step function plane wave incident on an anisotropic half-plane is considered. This does not represent a limitation of the derived solution since the other responses can be obtained from the unit step response, f.i., the response to an impulse function plane wave can be easily determined by considering the derivative with respect to the time of the unit step response.

TIME DOMAIN VERSION OF THE UAPO DIFFRACTED FIELD

Let us consider the diffraction of a transient plane wave impinging on the edge of a nonpenetrable half-plane surrounded by free-space and characterized by an anisotropic IBC, which is represented by the impedance tensor $\overline{\overline{Z}} = Z_{x'}\,\hat{x}'\hat{x}' + Z_{z'}\,\hat{z}'\hat{z}'$, x' and z' being the two mutually orthogonal principal axes of anisotropy. As shown in Fig. 1, the z'-axis is tilted of an angle ξ with respect to the z-axis. Note that the surface impedance values $Z_{x'}$ and $Z_{z'}$ are supposed to be not dependent on the frequency. They are equal in the case of an isotropically loaded face and reduce to zero in the perfectly conducting case. The incidence direction is determined by the angles β' and ϕ'. In particular, β' is a measure of the incidence direction skewness with respect to the edge ($\beta' = \pi/2$ corresponds to the normal incidence). The observation point P at distance r from the point Q, at which the diffracted field is excited, is on the Keller's diffraction cone and is specified by the angles $\beta = \beta'$ and ϕ. According to the Veruttipong's formulation, the evaluation of the time domain diffracted electric field $\underline{\mathcal{E}}^d(r,t)$ at P is given by the following integration [1]:

$$\underline{\mathcal{E}}^d(r,t) = A(r) \int_{t_0}^{t-(r/c)} \underline{\underline{\mathcal{D}}}\left(t - \frac{r}{c} - \tau\right) \cdot \underline{\mathcal{E}}^i(Q,\tau)\, d\tau \ , \qquad t - \frac{r}{c} > t_0 \ , \tag{1}$$

where $A(r)$ is the spreading factor equal to $1/\sqrt{r}$ in the case of incident plane wave, t_0 is the time when the forcing function at Q is turned on, c is the velocity of the light in the free-space, $\underline{\mathcal{E}}^i(Q,t)$ is the incident electric field at Q, and $\underline{\underline{\mathcal{D}}}(t)$ is the time domain diffraction matrix. Its elements are obtained by Laplace's inversion of the elements of its frequency domain counterpart [1].

In accordance with the results obtained in [3], the frequency domain UAPO solution for the diffraction matrix related to an anisotropic impedance half-plane is:

$$\underline{\underline{D}} = -\underline{\underline{M}}\, \frac{-\exp(-j\pi/4)}{2\sqrt{2\pi k}}\, \frac{F\left(2k\,r \sin^2\beta' \cos^2\left((\phi \pm \phi')/2\right)\right)}{\sin^2\beta' \left(\cos\phi + \cos\phi'\right)} \ , \tag{2}$$

in which k is the free-space wavenumber, which depends on the frequency, and $F(\cdot)$ is the UTD transition function [2]. The upper/lower sign applies for $y \gtrless 0$. In the accepted hypothesis, the elements of the 2×2 matrix $\underline{\underline{M}}$ do not depend on the frequency. They are explicitly reported in [3] and account for the expressions of the electric and magnetic PO surface current densities $\underline{J}_s$ and $\underline{J}_{ms}$ induced on the illuminated face:

$$\zeta \underline{J}_s = \left\{\left[(1-R_{22})E_\perp^i - R_{21}E_{||}^i\right]\cos\delta^i\, \hat{e}_\perp + \left[R_{12}E_\perp^i + (1+R_{11})E_{||}^i\right]\hat{t}\right\} \ , \tag{3}$$

$$\underline{J}_{ms} = \left\{\left[(1-R_{11})E_{||}^i - R_{12}E_\perp^i\right]\cos\delta^i\, \hat{e}_\perp - \left[(1+R_{22})E_\perp^i + R_{21}E_{||}^i\right]\hat{t}\right\} \ , \tag{4}$$

wherein ζ is the free-space impedance, $E_{||}^i$ and $E_\perp^i$ are the incident electric field components parallel and perpendicular to the ordinary plane of incidence, δ^i is the incidence angle, $\hat{e}_\perp = (\hat{s}^i \times \hat{n})/|\hat{s}^i \times \hat{n}|$, $\hat{e}_{||}^i = \hat{e}_\perp \times \hat{s}^i$, and $\hat{t} = \hat{n} \times \hat{e}_\perp$, $\hat{s}^i$ being the unit vector of the incidence direction and $\hat{n} = \hat{y}$ the unit vector normal to the surface.

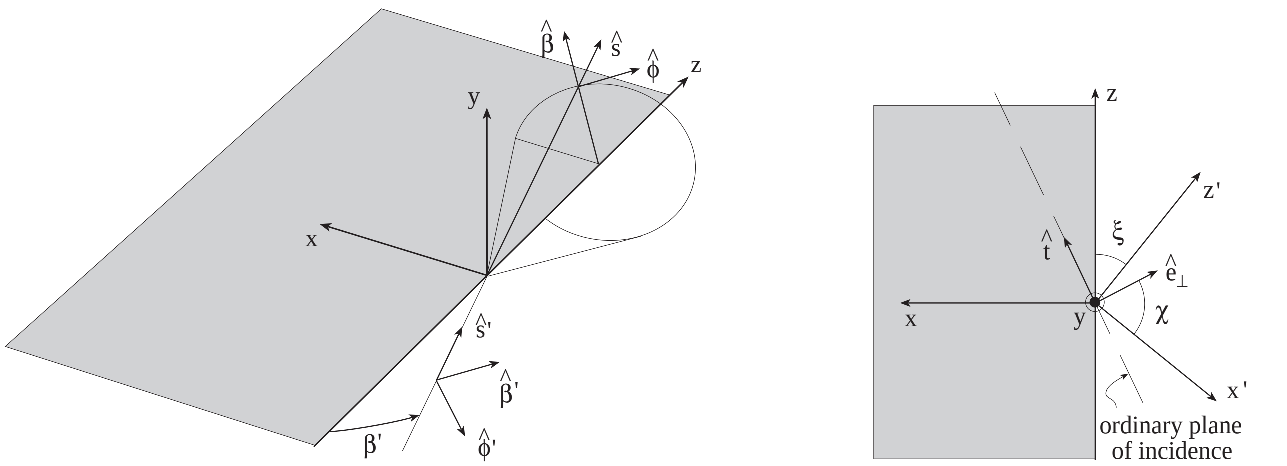

Fig. 1. Geometry of the problem.

The expressions (3) and (4) take into account that the reflected field components are related to the incident ones by the reflection matrix $\underline{\underline{R}}$ [3]:

$$R_{11} = \left[-C \cos^2\delta^i + \left(1 + AC + B^2\right)\cos\delta^i - A \right] / \Delta \quad , \tag{5}$$

$$R_{12} = -R_{21} = 2B \cos\delta^i / \Delta \quad , \tag{6}$$

$$R_{22} = \left[-C \cos^2\delta^i - \left(1 + AC + B^2\right)\cos\delta^i - A \right] / \Delta \quad , \tag{7}$$

$$A = \left(Z_{x'} \sin^2\chi + Z_{z'} \cos^2\chi\right) / \zeta \quad , \qquad B = \left(Z_{x'} - Z_{z'}\right) \sin\chi \cos\chi / \zeta \quad , \tag{8}$$

$$C = -\left(Z_{x'} \cos^2\chi + Z_{z'} \sin^2\chi\right) / \zeta \quad , \qquad \Delta = -C \cos^2\delta^i + \left(1 - AC - B^2\right)\cos\delta^i + A \quad , \tag{9}$$

χ being the angle between $\hat{x}'$ and $\hat{e}_{\perp}$. For a perfectly conducting surface the reflection matrix elements reduce to $R_{11} = 1$, $R_{12} = R_{21} = 0$, $R_{22} = -1$ and, obviously, $\underline{J}_{ms}$ is zero.

Since the dependence on the frequency of the UAPO diffraction coefficients is UTD-like, the UAPO version of the time domain diffraction matrix for an anisotropic impedance half-plane can be derived by following [1] and can be so expressed:

$$\underline{\underline{\mathcal{D}}}(t) = \underline{\underline{M}} \, \frac{1}{2\sqrt{2\pi}} \, \frac{G\left(x^{\pm}, t\right)}{\sin^2\beta' \left(\cos\phi + \cos\phi'\right)} \ , \tag{10}$$

with

$$G\left(x^{\pm}, t\right) = x^{\pm} \Big/ \left[\sqrt{\pi c t}\left(t + x^{\pm}/c\right) \right] , \qquad x^{\pm} = 2 r \sin^2\beta' \cos^2\left(\left(\phi \pm \phi'\right)/2\right) \ . \tag{11}$$

UNIT STEP RESPONSE

Let us consider a plane wave with the electric field

$$\underline{\mathcal{E}}^i(Q, t) = \begin{pmatrix} \mathcal{E}^i_{\beta'} \\ \mathcal{E}^i_{\phi'} \end{pmatrix} U\left(t - t_0\right) \tag{12}$$

be incident at the point Q on the edge of an anisotropic half-plane. In relation (12), $\mathcal{E}^i_{\beta'}$ and $\mathcal{E}^i_{\phi'}$ are the field components at the diffraction point in the incident ray-fixed coordinate system (see Fig. 1), and $U(\cdot)$ is the unit step function. For simplicity, let us assume that $t_0 = 0$. Accordingly, from (1), (10), (11), and (12), in the diffracted ray-fixed coordinate system it results:

$$\underline{\mathcal{E}}^d(r, t) = \begin{pmatrix} \mathcal{E}^d_{\beta} \\ \mathcal{E}^d_{\phi} \end{pmatrix} = \frac{1}{2\sqrt{2\pi r} \sin^2\beta' \left(\cos\phi + \cos\phi'\right)} \underline{\underline{M}} \begin{pmatrix} \mathcal{E}^i_{\beta'} \\ \mathcal{E}^i_{\phi'} \end{pmatrix} \int_0^{t-(r/c)} \frac{x^{\pm}}{\sqrt{\pi c\left(t - r/c - \tau\right)} \left[\left(t - r/c - \tau\right) + x^{\pm}/c \right]} \, d\tau, \quad t - \frac{r}{c} > 0 . \tag{13}$$

By performing the integration, the UAPO version of the unit step response can be written in terms of the normalized time ct/r as follows:

$$\begin{pmatrix} \mathcal{E}^d_{\beta} \\ \mathcal{E}^d_{\phi} \end{pmatrix} = \frac{\left|\cos\left(\left(\phi \pm \phi'\right)/2\right)\right|}{\pi \sin\beta' \left(\cos\phi + \cos\phi'\right)} \tan^{-1} \sqrt{\frac{ct/r - 1}{2\sin^2\beta' \cos^2\left(\left(\phi \pm \phi'\right)/2\right)}} \; U\left(\frac{ct}{r} - 1\right) \underline{\underline{M}} \begin{pmatrix} \mathcal{E}^i_{\beta'} \\ \mathcal{E}^i_{\phi'} \end{pmatrix} . \tag{14}$$

NUMERICAL RESULTS

A variety of numerical simulations has been performed to test the effectiveness of the derived unit step response and the correctness of its computer implementation. To save spacing, only some results are here presented for an anisotropic half-plane characterized by $Z_{x'}/\zeta = j0.5$, $Z_{z'}/\zeta = 1$ and illuminated by an incident electric field defined by $\mathcal{E}^i_{\beta'}=1$, $\mathcal{E}^i_{\phi'}=0$. All reported figures show the amplitude of the time domain electric diffracted field versus the normalized time ct/r. Obviously, according to (14), such a field is equal to zero for $ct/r<1$.

Figure 2 reports the behaviour of the β-component of the diffracted field in the case of an anisotropy angle ξ equal to 30°. The incidence and observation directions are fixed by azimuth angles equal to 30° and 45°, respectively, whereas two different values of the skewness angle β are considered.

The incidence and observation directions are fixed for the plots reported in Figs. 3 and 4. Such plots refer to the β and ϕ-components of the field diffracted by anisotropic half-planes characterized by two different values of ξ.

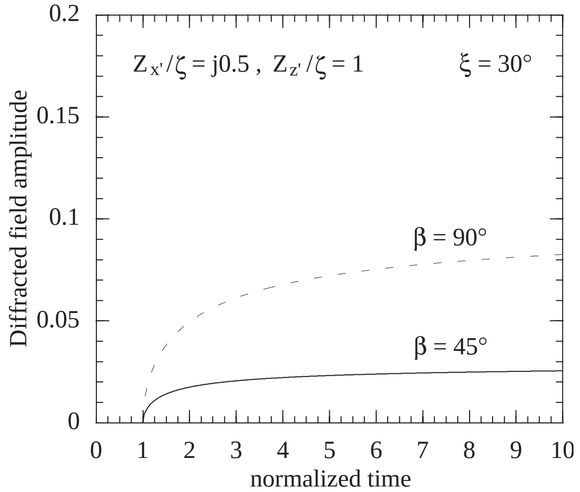

Fig. 2. β-component of the diffracted field. $\phi' = 30°$, $\phi = 45°$, $\mathcal{E}^i_{\beta'}=1$, $\mathcal{E}^i_{\phi'}=0$.

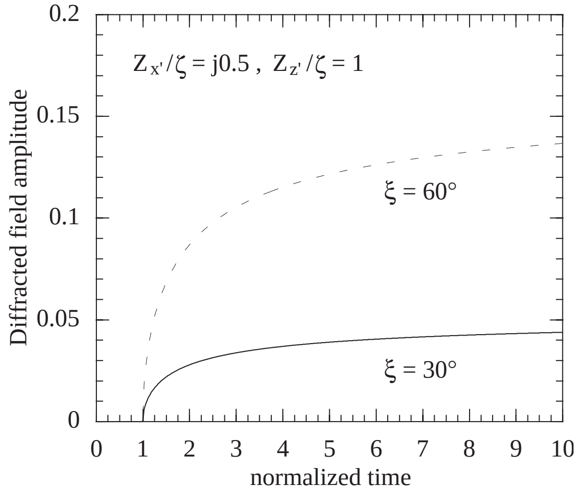

Fig. 3. β-component of the diffracted field. $\beta' = 60°$, $\phi' = 30°$, $\phi = 45°$, $\mathcal{E}^i_{\beta'}=1$, $\mathcal{E}^i_{\phi'}=0$.

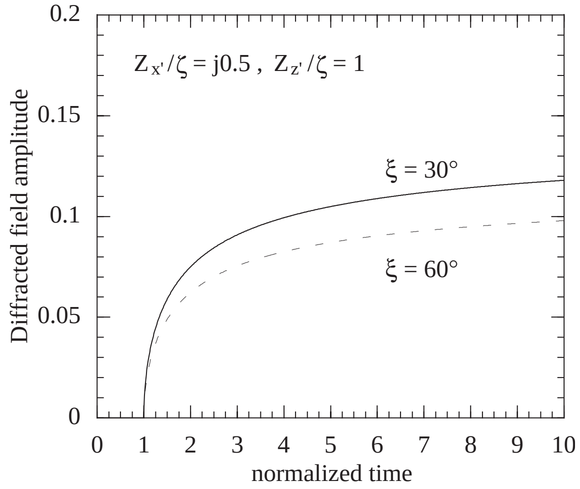

Fig. 4. ϕ-component of the diffracted field. $\beta' = 60°$, $\phi' = 30°$, $\phi = 45°$, $\mathcal{E}^i_{\beta'}=1$, $\mathcal{E}^i_{\phi'}=0$.

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