IDEAL AND GENERALIZED SOFT-AND-HARD BOUNDARIES

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ABSTRACT

Ideal boundary is defined as a surface on which the complex Poynting vector does not have a normal component for any time-harmonic fields. The general ideal boundary conditions for the electromagnetic fields include those of the PEC and PMC boundaries as well as the soft-and-hard surface (SHS) defining a direction of vanishing electric and magnetic fields on the surface. It is possible to generalize the SHS conditions by requiring that the field components each vanish along a tangential complex vector which need not be the same. Implications of these conditions to electromagnetic fields are discussed in the paper.

BOUNDARY CONDITIONS

The concept of 'boundary' in electromagnetic theory relates to a surface which does not let electromagnetic fields pass through it. In this it differs from the interface of two media. The concept of 'ideal boundary' can be defined so that at its surface the complex[1] Poynting vector $\mathbf{S} = (1/2)\mathbf{E} \times \mathbf{H}^*$ has no component normal to the boundary for any possible fields: $\mathbf{n} \cdot \mathbf{S} = 0$. This definition is different from that of the 'lossless boundary', for which the real part of the Poynting vector has no component normal to the boundary.

In addition to the trivial perfect electric and magnetic conductor (PEC and PMC) boundaries, another boundary satisfying the condition of ideal boundary is the so-called soft-and-hard (SHS) boundary [1] defined by the conditions

$$\mathbf{v} \cdot \mathbf{E} = 0, \quad \mathbf{v} \cdot \mathbf{H} = 0, \quad \mathbf{v} \cdot \mathbf{n} = 0. \tag{1}$$

Here $\mathbf{v}$ is a real unit vector tangential to the surface S. In [3] this definition was generalized to

$$\mathbf{a} \cdot \mathbf{E} = 0, \quad \mathbf{a}^* \cdot \mathbf{H} = 0, \tag{2}$$

where $\mathbf{a}$ is a complex vector tangential to the surface S and satisfying $\mathbf{a} \cdot \mathbf{a}^* = 1$. To check the validity of the Poynting vector condition we can write

$$\mathbf{n} \cdot \mathbf{E} \times \mathbf{H}^* = [\mathbf{a} \times (\mathbf{n} \times \mathbf{a}^*)] \cdot (\mathbf{E} \times \mathbf{H}^*)$$

$$= (\mathbf{a} \cdot \mathbf{E})(\mathbf{n} \times \mathbf{a}^*) \cdot \mathbf{H}^* - (\mathbf{a}^* \cdot \mathbf{H})^*(\mathbf{n} \times \mathbf{a}^*) \cdot \mathbf{E} = 0. \tag{3}$$

The concept of the soft-and-hard surface is a mathematical idealization of a surface which is both electrically and magnetically ideally conducting in one direction defined by a real unit vector $\mathbf{v}$. Such a boundary can be realized by a tuned corrugated surface which has been of interest in electromagnetics since its original introduction in the 1940's. The importance of the corrugated surface in horn antennas radiating axially symmetric patterns was realized in the 1960's before the SHS concept was launched in the 1980's [1,2].

A generalization to the conditions (1) and (2) as

$$\mathbf{a} \cdot \mathbf{E} = 0, \quad \mathbf{b} \cdot \mathbf{H} = 0. \tag{4}$$

can now be introduced in terms of two tangential complex vectors $\mathbf{a}$ and $\mathbf{b}$ satisfying $\mathbf{a} \cdot \mathbf{b} = 1$. A boundary satisfying (4) will be called the generalized soft-and-hard (GSHS) boundary. In contrast to the previous special cases (1) and (2), a more general boundary defined by (4) is not ideal. In fact, requiring the condition $\mathbf{u}_z \cdot \mathbf{E} \times \mathbf{H}^* = 0$ for all fields satisfying (4) leads to the relation $\mathbf{b} = \mathbf{a}^*$ corresponding to the case (2).

[1] Time-harmonic fields with time dependence $e^{j\omega t}$ are assumed.

REFLECTION OF PLANE WAVE

To see the effect of the GSHS boundary on electromagnetic fields, let us consider reflection of a plane wave from the boundary plane $z = 0$. Assuming incident and reflected plane wave fields in the half space $z > 0$

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \quad \mathbf{H}^i(\mathbf{r}) = \mathbf{H}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \tag{5}$$

$$\mathbf{E}^r(\mathbf{r}) = \mathbf{E}^r e^{-j\mathbf{k}^r\cdot\mathbf{r}}, \quad \mathbf{H}^r(\mathbf{r}) = \mathbf{H}^r e^{-\mathbf{k}^r\cdot\mathbf{r}}, \tag{6}$$

satisfying the relations

$$\mathbf{H}^i = \frac{1}{k\eta}\mathbf{k}^i \times \mathbf{E}^i, \quad \mathbf{H}^r = \frac{1}{k\eta}\mathbf{k}^r \times \mathbf{E}^r, \tag{7}$$

with $k = \omega\sqrt{\mu\epsilon}$, $\eta = \sqrt{\mu/\epsilon}$, the reflection dyadic can be defined as

$$\mathbf{E}^r = \overline{\overline{R}} \cdot \mathbf{E}^i. \tag{8}$$

Applying the boundary conditions of the generalized SHS boundary at $z = 0$ we can derive the following expression for the reflection dyadic [6]:

$$\overline{\overline{R}} = -\frac{1}{J}[\mathbf{k}^r \times (\mathbf{b} \times \mathbf{k}^r)\mathbf{a} + (\mathbf{a} \times \mathbf{k}^r)(\mathbf{b} \times \mathbf{k}^i)] \tag{9}$$

or, in a more symmetrical form,

$$\overline{\overline{R}} = -\frac{1}{J}\left(\frac{1}{k^2}[\mathbf{k}^r \times (\mathbf{b} \times \mathbf{k}^r)][\mathbf{k}^i \times (\mathbf{a} \times \mathbf{k}^i)] + (\mathbf{a} \times \mathbf{k}^r)(\mathbf{b} \times \mathbf{k}^i)\right). \tag{10}$$

For real $\mathbf{b} = \mathbf{a}^*$ this expression coincides with that derived in [3].

Defining the mirroring dyadic $\overline{\overline{C}} = \overline{\overline{I}} - 2\mathbf{u}_z\mathbf{u}_z$ and

$$J = \mathbf{a} \cdot \mathbf{k}^i \times (\mathbf{b} \times \mathbf{k}^i) = \mathbf{a} \cdot \mathbf{k}^r \times (\mathbf{b} \times \mathbf{k}^r), \tag{11}$$

which satisfies $J \neq 0$ in general, let us find the eigenvalues R_i and eigenvectors $\mathbf{x}_i$ of the dyadic

$$\overline{\overline{C}} \cdot \overline{\overline{R}} = \frac{1}{J}[(\mathbf{a} \times \mathbf{k}^i)(\mathbf{b} \times \mathbf{k}^i) - \frac{1}{k^2}\mathbf{k}^i \times (\mathbf{b} \times \mathbf{k}^i)\mathbf{k}^i \times (\mathbf{a} \times \mathbf{k}^i)]. \tag{12}$$

This dyadic gives the mirror image of the reflected wave as propagating in the direction of incidence and it allows us to compare polarizations of the reflected and incident waves. If the vectors $\mathbf{E}^i$ and $\overline{\overline{C}} \cdot \overline{\overline{R}} \cdot \mathbf{E}^i$ are scalar multiples of the same vector, we can say that the incident and reflected waves have the same polarization.

The three solutions $R_i, \mathbf{x}_i$ for the eigenproblem

$$\overline{\overline{C}} \cdot \overline{\overline{R}} \cdot \mathbf{x}_i = R_i\mathbf{x}_i, \quad i = 0, 1, 2 \tag{13}$$

are

$$\mathbf{x}_0 = \mathbf{k}^i, \;\; R_0 = 0, \qquad \mathbf{x}_1 = \mathbf{a} \times \mathbf{k}^i, \;\; R_1 = +1, \qquad \mathbf{x}_2 = \mathbf{k}^i \times (\mathbf{b} \times \mathbf{k}^i), \;\; R_2 = -1, \tag{14}$$

as can be easily checked. The eigenvector $\mathbf{x}_0$ has no physical meaning since the incident electric field has no component parallel to $\mathbf{k}^i$. For the wave 1 the incident eigenfield is of the form

$$\mathbf{E}_1^i = E_1\mathbf{a} \times \mathbf{k}^i, \quad \mathbf{H}_1^i = \frac{E_1}{k\eta}\mathbf{k}^i \times (\mathbf{a} \times \mathbf{k}^i), \tag{15}$$

and the reflected field is of the form

$$\mathbf{E}_1^r = R_1E_1\overline{\overline{C}} \cdot \mathbf{a} \times \mathbf{k}^i = -E_1\mathbf{a} \times \mathbf{k}^r, \tag{16}$$

The total field at the boundary is, then,

$$\mathbf{E}_1 = E_1\mathbf{a} \times (\mathbf{k}^i - \mathbf{k}^r) = 2E_1(\mathbf{u}_z \cdot \mathbf{k}^i)\mathbf{a} \times \mathbf{u}_z, \quad \mathbf{H}_1 = -\frac{2E_1}{k\eta}(\mathbf{u}_z \cdot \mathbf{k}^i)(\mathbf{a} \cdot \mathbf{k}^i)\mathbf{u}_z. \tag{17}$$

From the last expressions we see that the GSHS boundary acts as a PMC plane for the incident eigenfield 1 which can also be labeled as the TE$_a$ field because it satisfies $\mathbf{a}\cdot\mathbf{E}_1 = 0$ everywhere.

Similarly, for the wave 2 the incident eigenfield can be expressed as

$$\mathbf{E}_2^i = -\frac{\eta H_2}{k}\mathbf{k}^i \times (\mathbf{b}\times\mathbf{k}^i), \quad \mathbf{H}_1^i = H_2\mathbf{b}\times\mathbf{k}^i, \tag{18}$$

and the reflected field

$$\mathbf{E}_2^r = -R_2\frac{\eta H_2}{k}\overline{\overline{C}}\cdot[\mathbf{k}^i\times(\mathbf{b}\times\mathbf{k}^i)] = \frac{\eta H_2}{k}\mathbf{k}^r\times(\mathbf{b}\times\mathbf{k}^r). \tag{19}$$

The total field at the boundary is

$$\mathbf{E}_2 = \frac{2\eta H_2}{k}(\mathbf{u}_z\cdot\mathbf{k}^i)(\mathbf{b}\cdot\mathbf{k}^i)\mathbf{u}_z, \quad \mathbf{H}_2 = 2H_2(\mathbf{u}_z\cdot\mathbf{k}^i)\mathbf{b}\times\mathbf{u}_z, \tag{20}$$

or the GSHS boundary acts as a PEC plane for the incident eigenfield 2 which can be labeled as the TM$_b$ field because it satisfies $\mathbf{b}\cdot\mathbf{H}_2 = 0$ everywhere.

The reflected field of any given incident plane wave can be found simply by decomposing the wave into its TE$_a$ and TM$_b$ parts and applying the respective PMC and PEC reflection coefficients:

$$\mathbf{E}^i = \overline{\overline{I}}\cdot\mathbf{E}^i = \frac{1}{J}\left([\mathbf{k}^i\times(\mathbf{b}\times\mathbf{k}^i)](\mathbf{a}\cdot\mathbf{E}^i) + (\mathbf{a}\times\mathbf{k}^i)(\mathbf{b}\times\mathbf{k}^i\cdot\mathbf{E}^i)\right). \tag{21}$$

Here the last term gives the TE$_a$ component and, the one before that, the TM$_b$ component, of the electric field. In doing this one should note that the TE and TM parts are not power orthogonal because, assuming real $\mathbf{k} = \mathbf{u}k$ with real unit vector $\mathbf{u}$ and the electric field in the form

$$\mathbf{E} = E_{TE}\mathbf{a}\times\mathbf{u} + E_{TM}\mathbf{u}\times(\mathbf{b}\times\mathbf{u}), \tag{22}$$

the Poynting vector for a plane wave can be expressed as

$$\frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{\mathbf{u}}{2\eta}|\mathbf{E}_{TE}+\mathbf{E}_{TM}|^2 = \frac{\mathbf{u}}{2\eta}(|\mathbf{E}_{TE}|^2 + |\mathbf{E}_{TM}|^2 + \mathbf{u}\cdot 2\Re\{E_{TE}E_{TM}^*\mathbf{a}\times\mathbf{b}^*\}). \tag{23}$$

Thus, the decomposed parts are power orthogonal only in some special cases like when $\mathbf{b}$ is a multiple of $\mathbf{a}^*$. The non-orthogonality of the decomposition does not affect its use in computing fields but one must be careful in making conclusions about the power carried by the two waves.

Since the conditions $\mathbf{a}\cdot\mathbf{E} = 0$ and $\mathbf{b}\cdot\mathbf{H} = 0$ do not depend on the wave-vector $\mathbf{k}^i$ of the plane wave, the decomposition method is equally valid for a sum or an integral of plane waves and, thus, for any electromagnetic field outside of its sources where the field can be expressed as a plane-wave expansion. This means that, if we can split the source of the field in two parts which create TE$_a$ and TM$_b$ fields in the homogeneous space, introducing a GSHS plane will not couple the TE$_a$ and TM$_b$ fields. Thus, the problem is split in two simpler problems where the GSHS plane can be replaced by a PMC plane for the TE$_a$ problem and by a PEC plane for the TM$_b$ problem. Because the classical image principles apply for each of the problems, the original GSHS plane can be removed and replaced by the PMC image of the TE part of the source and the PEC image of the TM part of the source. This idea was elaborated in more detail for the special case $\mathbf{b} = \mathbf{a}^*$ in the reference [3] and a method for finding the source decomposition was sketched for that special case.

The problem how to realize a boundary approximating the GSHS conditions (4) in practice can be approached in a way similar to that discussed in [3], as based on the reflection of eigenpolarized waves. If we can devise a polarization filter which reflects the TM$_b$ part at the boundary as from a PEC plane and lets the TE$_a$ part pass through, we can put another PEC plane at the depth of quarter wavelength to make the necessary 180° phase shift to the TE$_a$ part. However, the GSHS boundary is nonreciprocal in general. This means that the realization would most probably require some nonreciprocal medium behind the polarization filter. Attempts to realize the GSHS boundary in terms of anisotropic media were made in [6] for the special case $\mathbf{b} = \mathbf{a}^*$.

APPLICATIONS

As possible applications of the GSHS boundary we can consider polarization transformers. In [3] a polarization transformer principle corresponding to the special case $\mathbf{b} = \mathbf{a}^*$ was suggested. In fact, because the boundary gives 180° phase

difference to the two eigenpolarizations reflected from the surface, the polarization of the incident wave is changed in reflection, in general. By rotating the surface in its plane the transformation can be changed. It appears obvious that by choosing the second vector $\mathbf{b}$ properly, rotation becomes unnecessary. To see this, let us consider the reflection dyadic for normal incidence, $\mathbf{k}^i = -\mathbf{u}_z k$:

$$\overline{\overline{R}} = (\mathbf{a} \times \mathbf{u}_z)(\mathbf{b} \times \mathbf{u}_z) - \mathbf{ba}. \tag{24}$$

Now let us assume that the polarizations of the incident and reflected fields are defined by the respective complex vectors $\mathbf{c}$ and $\mathbf{d}$, both orthogonal to $\mathbf{u}_z$. Recalling that $\mathbf{a} \cdot \mathbf{b} = 1$, from (24) we obtain two orthogonality conditions

$$\mathbf{a} \cdot \mathbf{d} = \mathbf{a} \cdot \overline{\overline{R}} \cdot \mathbf{c} = -\mathbf{a} \cdot \mathbf{c} \quad \Rightarrow \quad \mathbf{a} \cdot (\mathbf{d} + \mathbf{c}) = 0, \tag{25}$$

$$\mathbf{b} \times \mathbf{u}_z \cdot \mathbf{d} = \mathbf{b} \times \mathbf{u}_z \cdot \overline{\overline{R}} \cdot \mathbf{c} = \mathbf{b} \times \mathbf{u}_z \cdot \mathbf{c} \quad \Rightarrow \quad (\mathbf{b} \times \mathbf{u}_z) \cdot (\mathbf{d} - \mathbf{c}) = 0. \tag{26}$$

Now if we wish the boundary to change a given polarization $\mathbf{c}$ to another given polarization $\mathbf{d}$, it is obvious that we must choose the vectors $\mathbf{a}$ and $\mathbf{b}$ defining the GSHS boundary as

$$\mathbf{a} = \alpha \mathbf{u}_z \times (\mathbf{d} + \mathbf{c}), \quad \mathbf{b} = \beta(\mathbf{d} - \mathbf{c}), \tag{27}$$

where the scalars α and β must be chosen so that $\mathbf{a} \cdot \mathbf{b} = 1$. It is obvious that a further generalization of the SHS boundary from one defined by the vectors $\mathbf{a}$ and $\mathbf{a}^*$ to one defined by $\mathbf{a}$ and $\mathbf{b}$ makes it possible to transform any polarization to any other polarization. A device performing this can be made quite convenient if the complex vectors $\mathbf{a}$ and $\mathbf{b}$ of the boundary could be changed electronically. How to do this in practice is a topic of another study.

CONCLUSION

In the present analysis the classical soft-and-hard surface (SHS) boundary concept was generalized so that, instead of a single real or complex vector, the boundary condition was defined by two complex vectors $\mathbf{a}$ and $\mathbf{b}$ tangential to the surface. The basic property of the SHS boundary was seen to remain intact in the generalization: a plane wave field is reflected from the generalized SHS plane so that its TE_a component is reflected as from a perfect magnetic conductor (PMC) plane while its TM_b component is reflected as from a perfect electric conductor (PEC) plane. Because this property is valid for the field of any source, by splitting the source in its TE and TM parts one can use the classical image principle to arrive at the solution for the boundary-value problem [5]. If any vectors $\mathbf{a}$ and $\mathbf{b}$ can be realized for a GSHS boundary, a device transforming any given polarization to any other polarization in reflection appears feasible.

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