THE CONCEPT OF PATH LENGTH FACTOR IN THE PREDICTION OF RAIN ATTENUATION IN TERRESTRIAL LINKS

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ABSTRACT

The concept of path length factor is quite useful for overcoming the problem associated to the non-uniform distribution of precipitation rate along the propagation path. However, considering that is not feasible to derive a rigorous mathematical expression for this factor, an alternative is the use of experimental data for fitting an empirical solution based on a given rain cell model. In this context, a truncated exponential rain cell was used here. The corresponding path length factor obtained with this procedure was slightly adjusted to minimize the mean relative error and the standard deviation.

INTRODUCTION

An important problem to be considered when deriving a prediction model for the evaluation of rain attenuation in terrestrial links is the non-uniformity of precipitation rate along the propagation path. The concept of path length factor (or distance factor) is used by several rain attenuation prediction methods for solving this problem [1-3]. In a given link, this factor corresponds to the ratio between an equivalent path length over which the precipitation rate (R) is assumed to be constant and the actual path length. The path length factor is evaluated by dividing the rain attenuation (measured or predicted) exceeded for a given percentage of time by the specific attenuation ($\gamma = kR^{\alpha}$) for the same percentage of time, k and α being primarily functions of frequency and polarization. The mathematical derivation of this factor requires the definition of the geometrical shape of rain cells. Based on experimental observations, rain cells have been modeled by cylindrical [4], exponential [5,6] and Gauss [7] shapes. In this paper it is assumed a truncated exponential rain cell, which combines the cylindrical and the exponential shapes and avoids the restriction associated to a measured path length factor greater than one, common to other predicting methods.

2. TRUNCATED EXPONENTIAL MODEL

According to this model the rain cell has a rotational symmetry, being defined by,

$$R(x) = R = R_m / e \qquad \text{for} \quad |x| \le \rho$$

and

$$R(x) = R_m \exp[-x/\rho] \quad \text{for} \quad \rho \le |x| \le d/2$$

The parameter ρ for which R(x) decays by a factor 1/e is to be achieved empirically from the experimental data available. The flat circular area with a diameter 2ρ corresponds to a precipitation rate $R = R_m/e$ exceeded in the same percentage of time as the rain attenuation considered in the numerical analysis.

Introducing the concept of equivalent precipitation [4], i.e.,

$$R_e = \left[\frac{1}{d} \int_{-d/2}^{+d/2} R(x)^{\alpha} dx \right]^{1/\alpha},$$

the path length factor (r) for the truncated exponential cell is given by,

$$r = \rho / d + \exp(\alpha)[\exp(-\alpha/2) - \exp(-Y)] / 2Y \qquad (1)$$

where $Y = \alpha d / 2\rho$.

3. NUMERICAL RESULTS

Equation (1) was slightly adjusted to be used for fitting an empirical expression for ρ. The reason for this procedure was to minimize the mean relative error when comparing experimental and predicted values of the path length factor. The new equation for r is expressed by

$$r = \rho / d + [\exp(-\alpha / 2) - \exp(-Y)] / Y \tag{2}$$

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Aiming to have accurate measurements in the numerical analysis, experimental data were taken from the ITU-R data bank [9] corresponding to radio links located in temperate regions in which precipitation rate and rain attenuation are exceeded for 0.01% of time (annual basis). The best empirical expression for ρ was proved to be,

$$\rho(km) = 25.6[R(mm / h)]^{-0.5} \tag{3}$$

Equations (2) and (3) were used in the prediction of the path length factor for all data available (ITU-R data bank supplemented with additional information from Brazil and India). Table 1 shows the mean relative error and the standard deviation when comparing experimental and predicted values of r for different data sets. Details about this comparison are shown in Table 2.

TABLE 1

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL VALUES OF THE PATH LENGTH FACTOR – SUMMARY

Data Set	Mean relative error (%)	Standard deviation (%)
Temperate climate	0.54	31.29
Tropical Climate	9.06	31.29
All Data	2.67	31.30

4. CONCLUDING REMARKS

An accurate expression for the path length factor is fundamental in the prediction of rain attenuation. In this context, the path length factor model based on a truncated exponential rain cell described in this paper seems to be a good starting point for attaining this objective. Additionally, this factor has the advantage of predicting values greater than one. This happens for short links, where ρ is larger than path length (d) and constitutes a limitation of other available models. On the hand, although the results presented here were derived from the precipitation rate exceeded for 0.01% of time of an average year, there is some evidence that this path length factor permits the evaluation of rain attenuation in a point-to-point basis. This question is being investigated, as well as a possible improvement in the prediction of rain attenuation in tropical areas.

5. REFERENCES

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TABLE 2

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL VALUES OF THE PATH LENGTH FACTOR – DETAILED RESULTS

SITE	COUNTRY	Path Length Factor		SITE	COUNTRY	Path Length Factor	
		Measured	Predicted			Measured	Predicted
Mendlesham	GB	0,76	0,92	Chilbolton	GB	2,43	2,58
Mendlesham	GB	0,80	0,91	Chilbolton	GB	2,43	2,98
Mendlesham	GB	0,67	0,61	Chilbolton	GB	2,74	2,98
Mendlesham	GB	1,08	1,35	Chilbolton	GB	2,14	2,41
Mendlesham	GB	1,22	1,18	Chilbolton	GB	2,33	2,79
Mendlesham	GB	0,94	0,94	Chilbolton	GB	2,56	2,79
Mendlesham	GB	0,91	1,34	Chilbolton	GB	2,63	2,69
Mendlesham	GB	0,78	0,92	Uvaly	CZ	0,80	0,56
Mendlesham	GB	0,73	0,86	Mostova	CZ	0,41	0,24
Mendlesham	GB	0,82	1,20	Pisek	CZ	0,60	0,25
Mendlesham	GB	0,50	0,49	Strahov	CZ	0,68	0,27
Kjeller	NO	0,49	0,49	Strahov	CZ	0,58	0,27
Stockholm	SE	0,81	0,56	Piaseczno	PL	0,82	0,51
Stockholm	SE	0,64	0,59	Piaseczno	PL	0,74	0,53
Stockholm	SE	0,53	0,62	Dubna 3	SU	0,73	0,60
Darmstadt	DE	0,52	0,47	Dubna 3	SU	0,89	0,58
Darmstadt	DE	0,54	0,48	Chilbolton	GB	2,30	3,20
Darmstadt	DE	0,46	0,51	Chilbolton	GB	2,55	2,82
Leidschendam	NL	0,68	0,71	Chilbolton	GB	2,32	2,65
Paris	FR	0,33	0,14	Calicut	IN	0,37	0,33
Paris	FR	0,54	0,59	Delhi	IN	0,83	0,55
Paris	FR	0,60	0,59	Ranchi	IN	0,56	0,61
Paris	FR	0,58	0,49	Ranchi	IN	0,94	0,64
Dijon	FR	0,32	0,15	Fortaleza	BR	0,35	0,35
Dijon	FR	0,46	0,35	Porto Alegre	BR	0,87	0,60
Fucino	IT	0,78	0,66	Bradesco II	BR	0,62	0,43
Fucino	IT	0,63	0,69	Bradesco II	BR	0,61	0,42
Fucino	IT	0,67	0,68	Cenesp 15	BR	0,51	0,42
Rome	IT	0,48	0,24	Cenesp 15	BR	0,53	0,41
Turin	IT	0,54	0,30	Cenesp 18	BR	0,47	0,44
Merrimack valley	US	0,90	0,94	Scania	BR	0,58	0,31
Palmetto	US	0,60	0,77	Scania	BR	0,59	0,31
Holmdel	US	0,85	0,74	Barueri	BR	0,41	0,27
Tokyo	JP	0,90	1,49	Rio de Janeiro	BR	0,30	0,52
Tokyo	JP	0,87	1,41	Rio de Janeiro	BR	0,37	0,51
Tokyo	JP	0,95	1,34	Rio de Janeiro	BR	0,40	0,49
Xixiang-Henan	CN	0,79	1,23	Rio de Janeiro	BR	0,50	0,48
Chilbolton	GB	2,30	3,20	Rio de Janeiro	BR	0,40	0,52