DIFFRACTED GAUSSIAN BEAM ANALYSIS: A MODULAR APPROACH TO THE ANALYSIS OF QUASI-OPTICAL MULTI-REFLECTOR ANTENNA SYSTEMS

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ABSTRACT

Herein we outline a modular procedure to quicken accurate analysis of Quasi-optical, multi-reflector, antenna systems. An illuminating field is analysed into a set of elementary Gaussian beams. These are propagated to a reflector. Geometric Optics initially treats reflection. The canonical problem of a 3D Gaussian beam incident on an opaque Kirchoff half-screen treats edge diffraction. Its asymptotic solution yields a boundary diffraction wave that complements the geometrical wave (that corresponds to the reflected beam). The modular analysis is verified against a GRASP Physical Optics / Physical Theory of Diffraction analysis of a tri-reflector antenna system.

INTRODUCTION

Antenna systems employed, for example, in radiometry and radio astronomy, are generally complex. They typically consist of multiple quasi-optical (QO) reflectors that are cascaded to form a feed system to an electrically large main reflector. With the operation of such QO antenna systems tending to increasingly higher (THz) frequencies, faster and more accurate design and verification computing is required. The power of the Diffracted Gaussian Beam Analysis (DGBA) approach lies in its modularity. Modularity requires that the fields at the interface between two modules (for example reflectors) need to be fully specified. Simply specifying the transverse fields on some intermediate surface (preferably normal to the main collimated field) does not give the direction of any scattered ray that forms part of this field. The only way to fully specify the fields on a surface, and hence replace the "sources" inside that surface, is to invoke the field equivalence principle and specify currents on the surface. Integration of these currents is then required to find the field (or current) on the next reflector. This is equivalent conceptually to Physical Optics. It is for this reason that the Unified Theory of Diffraction (UTD) is unsuitable for this application. Large multi-reflector Quasi-optical systems usually contain within the system of reflectors signal processing components such as dichroic reflectors and wire grids. The modularity of our approach allows for such elements to be analysed by a suitable external method, and the processed field to be re-injected into the DGBA analysis chain. For example we are currently using the periodic boundary conditioned FDTD to analyse wire grids and dichroic reflectors. The steps in a modular analysis are sketched in Fig. 1.

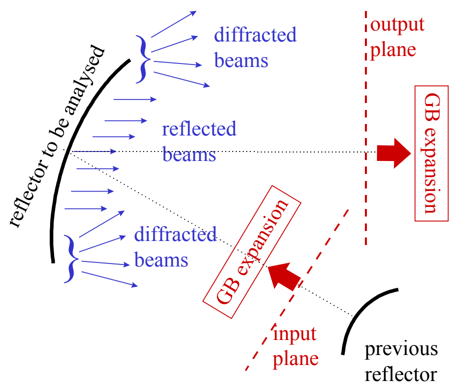

Fig.1 Sketch of stages in a module of Diffracted Gaussian Beam Analysis (DGBA)

The procedure of DGBA is firstly to expand the source field (from either the feed or the previous reflector) at some input aperture plane, into an array of fundamental 3D Gaussian beam modes. Each beam has a beam width of several wavelengths. The elementary beams are secondly propagated to the reflector where the transformation of their complex beam parameters is determined according to the curvature properties of the reflector. The reflected beam is thirdly propagated to an output plane. At this point the analysis may stop or, continue, by decomposing the field at this output plane into a new set of fundamental Gaussian beam modes that are propagated to the next reflector to repeat the above sequence of steps. Herein lies the modularity of the procedure.

THEORY

Gaussian Beam Expansion

A module of DGBA begins by decomposing the illuminating field, defined at an appropriately determined aperture input plane, into Gaussian elementary beam modes [1,2]. A transverse scalar field component, say $E_x = E_x(x, y, z)$, may be synthesized from the following beam modes expansion

$$E_x(x, y, z) = \sum_{m,n,\mu,\nu} \exp[i(mn + \mu\nu)K_0 L_0] A^x_{m,n,\mu,\nu} B_{m,n,\mu,\nu}(x, y, z). \tag{1}$$

Firstly, in (1), $A^x_{m,n,\mu,\nu} = \iint dx\, dy\, E_x(x, y)\tilde{w}^*_{m,n}(x)\tilde{w}^*_{\mu,\nu}(y)$ are the expansion coefficients in the series representation $E_x(x, y, z=0) = \sum_{m,n,\mu,\nu} A^x_{m,n,\mu,\nu}\, w(x - mL_o)\exp[inK_0 x]w(x - \mu L_0)\exp[i\nu K_0 y]$ of the aperture plane field, in terms of a spatially shifted and spectrally rotated 2D windowed Fourier transform [3]. In these expressions the integers m and μ increment the spatial shift L_0, and n and ν the spectral shift K_0 of the windowing function, here, a Gaussian, $w(x, y) = w(x)w(y) = \sqrt{2}/L \exp[-\pi(x^2 + y^2)/L^2]$, in the x and y directions respectively. L is the supporting width of the windowing function that is related to the Gaussian beam waist. L_0 and K_0 are chosen so that $L_0 K_0 \le 2\pi$ in order to ensure stable convergence of (1). $\tilde{w}$ is the dual function to w [4].
Secondly,

$$B_{m,n,\mu,\nu}(x, y, z) =$$

$$1/4\pi^2 \iint d\xi\, d\eta\, \hat{w}(\xi - nK_0)\hat{w}(\eta - \nu K_0)\exp\left[i\left(\xi(x - mL_0) + \eta(y - \mu L_0) + \sqrt{k_0^2 - \xi^2 - \eta^2}\, z\right)\right]$$

describes the elementary beam modes where $\hat{w}(\xi, \eta)$ is the Fourier transform of $w(x, y)$. $k_0 = 2\pi/\lambda$ is the vacuum wavenumber. For practical purposes, an asymptotic approximation to (1) yields

$$B_{m,n,\mu,\nu}(\rho_t, z_t) \cong \sqrt{2}/L\, q_{n\nu}(0)/q_{n\nu}(z_t)\exp\left[ik_0\left(z_t + \rho_t^2/(2q_{n\nu}(z_t))\right)\right].$$

The transformed coordinates ρ_t and z_t, are linked with the Cartesian coordinates x, y, z and the spectral components of the wave vector via, the transformations

$$z_t = \left[nK_0(x - mL_0) + \nu K_0(y - \mu L_0) + \sqrt{k_0^2 - (nK_0)^2 - (\nu K_0)^2}\, z\right]/k_0$$

$$\rho_t^2 = (x - mL_0)^2 + (y - \mu L_0)^2 + z^2 - z_t^2.$$

z_t can be viewed as the projection of the vector pointing from the source point $(mL_0, \mu L_0, 0)^t$ to the observation point $(x, y, z)^t$, onto the direction $\hat{z}_t = (1/k_0)\left(nK_0, \nu K_0, \sqrt{k_0^2 - (nK_0)^2 - (\nu K_0)^2}\right)^t$ of the wave vector. The complex beam parameter $q_{n\nu}(z_t)$ absorbs all the information about the local spot size $\omega(z_t)$ and the local radius of curvature $R(z_t)$ of the beam at each axial position z_t. To summarise,

$$q_{n\nu}(z_t) = \left(1/R(z_t) + j\lambda/(\pi\omega^2(z_t))\right)^{-1} = z_t - j(L^2/\lambda)\left(1 - (nK_0/k_0)^2 - (\nu K_0/k_0)^2\right).$$

Gaussian Beam Reflection

Geometrical Optics (GO) is applied [5] to determine how the complex beam parameter q changes when a Gaussian beam impinges upon a curved reflector surface. At the reflection point, the spot size of the beam stays the same while the curvature, which can be generally described by a set of two principal radii of curvature R_1, R_2 changes according to the local curvature of the reflecting surface. The details of application of GO to DGBA may be found in [6].

Gaussian Beam Diffraction

In [7], diffraction of a Gaussian beam with a circular spot size normally incident upon an opaque Kirchhoff half-screen was investigated based on the boundary-diffraction wave (BDW) theory. There, the evaluation of the boundary-diffraction wave by the steepest descent method yielded the uniform asymptotic representation of the total diffracted field consisting of the Geometrical Optics component (geometrical wave) and diffraction components in terms of complementary error functions (boundary-diffraction wave). One is again referred to [6] for detail as well as for consideration of field representation in the backward-scattering region and generalisation of analysis to oblique incidence.

RESULTS

The results to follow are presented for a spherical tri-reflector Compact Antenna Range (CATR) test case carrying a 90 GHz signal. The QO arrangement of the system is depicted in Fig. 2. Figure 3a following shows the co-ordinate geometry for the simpler case of normal beam incidence. Figure 3b next, plots a field comparison between GRASP PO/PTD and DGBA output at $z = 300\lambda$ for a beam with beam waist $\omega_0 = 8\lambda$.

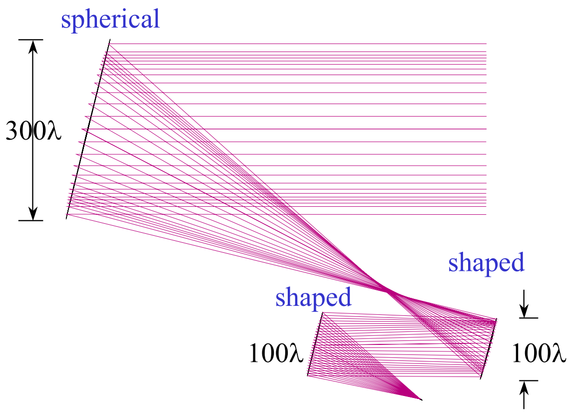

Fig. 2. DGBA is demonstrated upon this test case of a CATR tri-reflector at 90 GHz

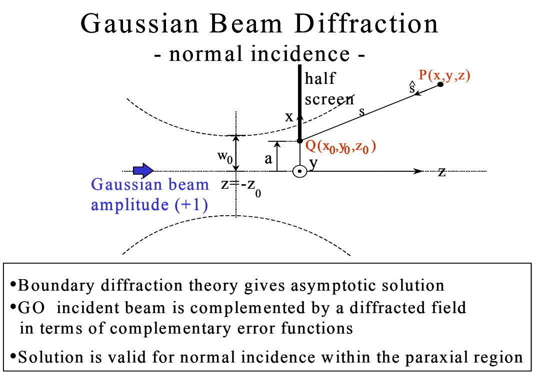

Fig. 3a. Co-ordinate geometry

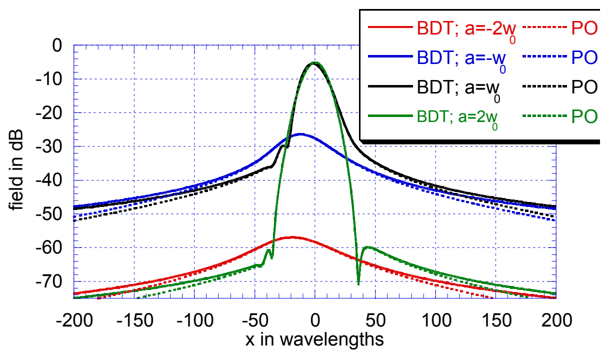

Fig. 3b. GRASP vs. DGBA: normal incidence

The final results to be shown are of comparative field plots between GRASP PO/PTD and the DGBA. The first of these, Fig 4a., is of the incident field upon the second sub-reflector. The second in Fig. 4b., is of the field on the focal plane between the second and main reflectors. The third in Fig. 4c., is of the field on the main reflector. Lastly in Fig. 4d., a comparative field plot is shown in the quiet zone at 1200λ from the main reflector.

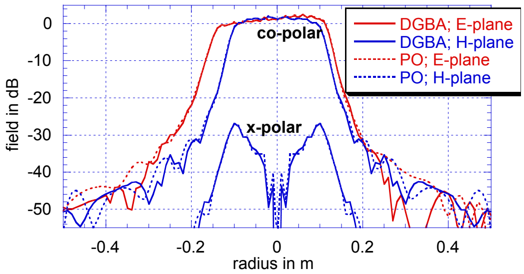

Fig. 4a. Incident field at second sub-reflector

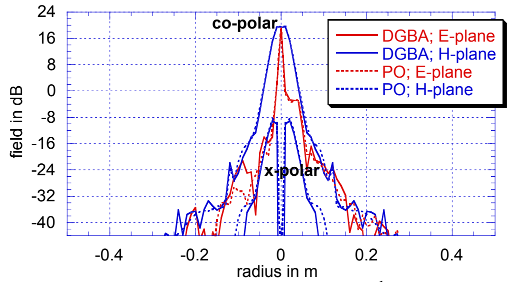

Fig. 4b. Field on focal plane between 2nd sub-reflector and main reflector

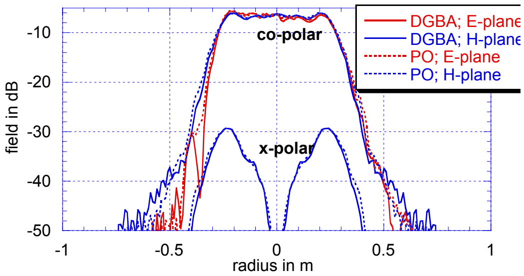

Fig. 4c. Incident field on main reflector

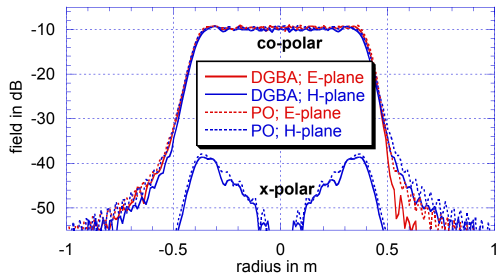

Fig. 4d. Field in quiet zone of main reflector at 1200λ

CONCLUSIONS

A novel diffracted Gaussian beam approach for analysing a quasi-optical multi-reflector system has been proposed. The method is especially well suited for the analysis of electrically large multi-reflector systems where Physical Optics is computationally expensive. As opposed to the Geometrical Theory of Diffraction (and UTD), the new method is highly modular and does not suffer from caustic problems. It has been tested for a tri-reflector antenna configuration and good agreement with PO has been obtained.

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