A COMPARATIVE RCS STUDY OF TWO CANONICAL, PARALLEL-PLATE WAVEGUIDE CAVITIES

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ABSTRACT

Two diffraction problems involving cavities formed by a semi-infinite parallel-plate waveguide and by a finite parallel-plate waveguide are rigorously analyzed for the plane wave incidence using the Wiener–Hopf technique. It is shown that the solution for the semi-infinite case is valid for arbitrary cavity dimensions, whereas the solution for the finite case is valid for the cavity depth large compared with the wavelength. Illustrative numerical examples on the radar cross section (RCS) are presented, and the far field backscattering characteristics are compared between these two cavities.

INTRODUCTION

The analysis of electromagnetic scattering from open-ended metallic waveguide cavities has received much attention recently in connection with the prediction and reduction of the radar cross section (RCS) of a target [1–3]. This problem serves as a simple model of duct structures such as jet engine intakes of aircrafts and cracks occurring on surfaces of general complicated bodies. There have been a number of investigations on the scattering by two-dimensional (2-D) and three-dimensional (3-D) cavities of various shapes based on high-frequency techniques and numerical methods. It appears, however, that the solutions obtained by these methods are not uniformly valid for arbitrary cavity dimensions.

The Wiener–Hopf technique is known as a powerful, rigorous approach for solving diffraction problems associated with canonical geometries. In the previous papers [4–7], we have considered two different parallel-plate waveguide cavities with material loading, and carried out a rigorous RCS analysis using the Wiener–Hopf technique. As a result, it has been shown that our final solutions provide accurate, reliable results over a broad frequency range. We have also verified that, for large cavities, the absorbing layer loading inside the cavities results in significant RCS reduction. The results of our recent studies on the cavity RCS are summarized in detail in [8].

This paper is in continuation with our recent research on the cavity RCS. We shall consider two canonical, 2-D parallel-plate waveguide cavities with material loading, and carry out a comparative RCS study of these cavities. The first geometry (referred to as Cavity I) is a cavity formed by a semi-infinite parallel-plate waveguide with an interior planar termination, where three different material layers are coated on the surface of the terminated plate. The second geometry (referred to as Cavity II) is a cavity formed by a finite parallel-plate waveguide with a planar termination at the aperture of the waveguide, where the same material layers as in Cavity I are coated on the surface of the terminated plate. It is noted that geometries of interior regions of the two cavities are exactly the same as each other. The exterior features are, however, totally different since Cavity I has two parallel semi-infinite plates, being joined to its outer right-angled corners. The analysis of the diffraction problems involving Cavity I and Cavity II has been carried out using the Wiener–Hopf technique in [6, 7] and [4, 5], respectively. The purpose of this paper is to compare the solutions for Cavity I and Cavity II and investigate the effect of geometrical differences of the two cavities on the far field scattering characteristics. Both E and H polarizations are treated.

The time factor is assumed to be $\exp(-iwt)$ and suppressed throughout this paper.

SUMMARY OF THE WIENER–HOPF ANALYSIS

Cavity I: Cavity Formed by a Semi-Infinite Parallel-Plate Waveguide [6, 7]

We first consider the plane wave diffraction by Cavity I. As mentioned above, Cavity I is formed by a semi-infinite parallel-plate waveguide with an interior planar termination. The problem geometry is shown in Fig. 1(a), where

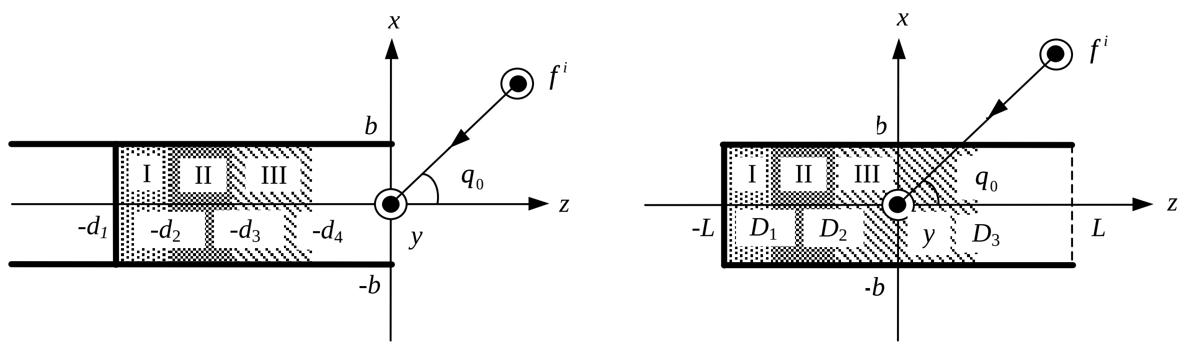
$-\infty < -d_1 < -d_2 < -d_3 < -d_4 < 0$, and the upper and lower plates at $x = \pm b$ and the endplate at $z = -d_1$ are infinitely thin, perfectly conducting, and uniform in the *y*-direction. The material layers I $(-d_1 < z < -d_2)$, II $(-d_2 < z < -d_3)$, and III $(-d_3 < z < -d_4)$ inside the waveguide are characterized by the relative permittivity/permeability (e_{rm}, m_{rm}) for m = 1, 2, and 3, respectively. Let the total field f^t be defined by $f^t(x,z) = f^i(x,z) + f(x,z)$, where $f^i(x,z) = \exp[-ik(x\sin q_0 + z\cos q_0)]$ is the incident field of E or H polarization for $0 < q_0 < p/2$ with $k[\equiv w(m_0 e_0)^{1/2}]$ being the free-space wavenumber. The term $f(x,z)$ is the unknown scattered field and satisfies the 2-D Helmholtz equation. As usual in the Wiener–Hopf analysis, we assume that the vacuum is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real *k* is obtained by letting $k_2 \to +0$ at the end of analysis. Referring to the geometrical classification proposed in [9], Cavity I belongs to a class of the modified Wiener–Hopf geometry of the second kind.

Introducing the Fourier transform for the scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the modified Wiener–Hopf equations of the second kind [9], which are solved exactly in a formal sense via the factorization and decomposition procedure. It should be noted, however, that the formal solution involves infinite series with unknown coefficients. Applying a rigorous asymptotics with the aid of the edge condition, approximate expressions of the infinite series are derived leading to an efficient approximate solution of the Wiener–Hopf equations. The approximate solution involves numerical inversion of matrix equations. The results are uniformly valid for arbitrary cavity dimensions. The scattered filed in the real space is evaluated asymptotically by taking the inverse Fourier transform and applying the saddle point method.

Cavity II: Cavity Formed by a Finite Parallel-Plate Waveguide [4, 5]

We now consider the diffraction problem for Cavity II, which is formed by a finite parallel-plate waveguide with a planar termination at the aperture. The geometry of the cavity is shown in Fig. 1(b), where f^i is the incident field of *E* or *H* polarization, where $-L < D_1 < D_2 < D_3 < L$, and the cavity plates are perfectly conducting and of zero thickness. The material layers I $(-L < z < D_1)$, II $(D_1 < z < D_2)$, and III $(D_2 < z < D_3)$ inside the cavity are characterized by the relative permittivity/permeability (e_{rm}, m_{rm}) for m = 1, 2, and 3, respectively. Comparing Cavity I and Cavity II, it is seen that the interior cavity geometries are exactly the same as each other.

Taking the Fourier transform of the Helmloltz equation and solving the resultant transformed wave equations, we may derive a scattered field expression in the Fourier transform domain. We apply boundary conditions to the scattered field representation in the transform domain. It should be noted that the cavity is now formed by a finite parallel-plate waveguide and hence, the problem is formulated in terms of the modified Wiener–Hopf equations of the third kind [9]. This is the essential difference in the structure of the Wiener–Hopf equations. The Wiener–Hopf equations are solved exactly in a formal sense via the factorization and decomposition procedure as in the case of Cavity I leading to a formal solution. Due to the geometrical differences between Cavity I and Cavity II, the structure of the formal solution for Cavity II is totally different from that for Cavity I. The solution now involves branch-cut integrals with unknown integrands as well as infinite series with unknown coefficients. Assuming that the cavity depth 2*L* is large compared with the wavelength, we can derive high-frequency asymptotic expansions of the branch-cut integrals. As for the infinite series contained in the formal solution, we apply the edge condition to obtain their approximate expressions. This procedure leads to an approximate solution of the Wiener–Hopf equations, which is valid for the cavity depth large compared with wavelength. As in Cavity I, the solution involves numerical inversion of matrix equations. The scattered field in the real space is derived by taking the Fourier inverse and applying the saddle point method.



(a) Cavity I. (b) Cavity II.

Fig.1. Geometry of the cavities.

NUMERICAL RESULTS AND DISCUSSION

We shall now show representative numerical examples of the RCS for both *E* and *H* polarizations to discuss the far field backscattering characteristics of the cavities. In particular, the effect of geometrical differences of the two cavities on the scattered far field is investigated. The RCS per unit length is defined by $s = \lim_{r \to \infty} (2pr \,|\, f / f^{i} \,|^{2})$, where the cylindrical coordinate (r, q) has been introduced as in $x = r \sin q$, $z = r \cos q$ for $-p < q < p$.

Figures 2(a-c) and 2(d-f) show numerical results of the normalized monostatic RCS s / l [dB] as a function of incidence angle q_0 for *E* and *H* polarizations, respectively, where l is the free-space wavelength. In numerical computation, cavity dimensions have been taken as *kb* = 3.14, 15.7, 31.4 with $d_1 / 2b = L / b = 1.0$. As an example of existing three-layer materials, we have chosen Emerson & Cuming AN-73 [1], where the material constants are $e_{r1} = 3.4 + i10.0$, $e_{r2} = 1.6 + i0.9$, $e_{r3} = 1.4 + i0.35$, $m_{r1} = m_{r2} = m_{r3} = 1.0$, and the thickness of each layer is such that $d_1 - d_2 = d_2 - d_3 = d_3 - d_4 = D_1 + L = D_2 - D_1 = D_3 - D_2 (= t / 3)$ (see Fig. 1). The total thickness of the three-layer material is taken as *kt* = 2.08.

It is seen from the figure that, the RCS curves for Cavity I and Cavity II with *kb* = 3.14 and *E* polarization (Fig. 2(a)) show close features for $0° < q_0 < 55°$, whereas in all the other examples, there are some differences on the backscattering characteristics between the two cavities. These differences are clearly observed in the *H*-polarized case, since the RCS curves for Cavity II oscillate rapidly in comparison to those for Cavity I. This oscillation for Cavity II is due to the fact that the multiple diffraction occurs between the leading edges at the aperture and the outer edges of the right-angled back corners.

CONCLUSIONS

In this paper, we have carried out a rigorous Wiener–Hopf analysis of the plane wave diffraction by two different cavities with three-layer material loading, formed by a semi-infinite parallel-plate waveguide (Cavity I) and by a finite parallel-plate waveguide (Cavity II). We have presented numerical examples of the monostatic RCS to discuss the far field backscattering characteristics of the cavities. Comparing the RCS results between Cavity I and Cavity II, some differences on the backscattering characteristics have been observed. It is therefore confirmed that the backscattering from these cavities is affected not only by the interior irradiation but also by the diffraction by the outer edges.

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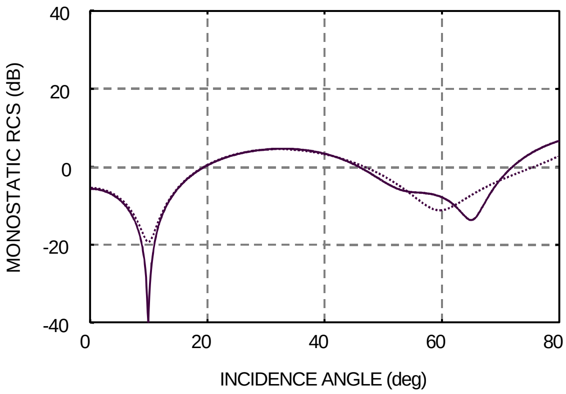
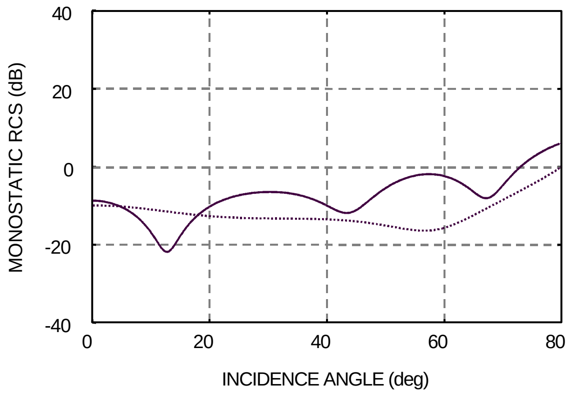
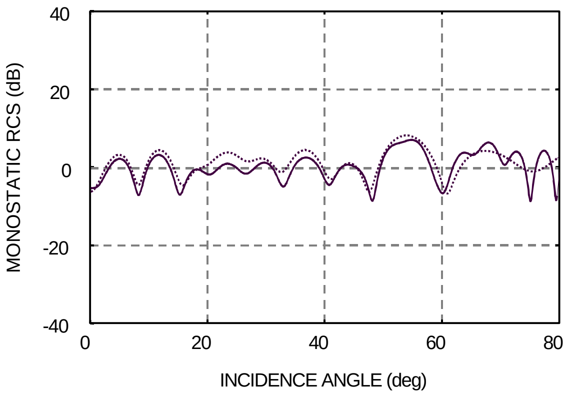


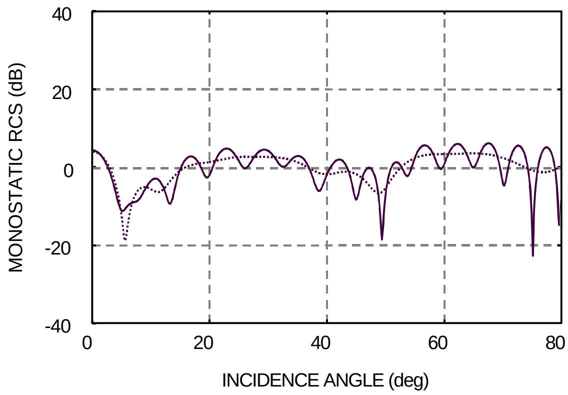
(a) E polarization, $kb = 3.14$.



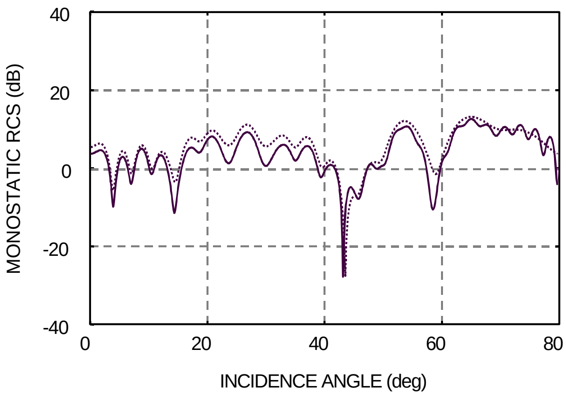
(d) H polarization, $kb = 3.14$.



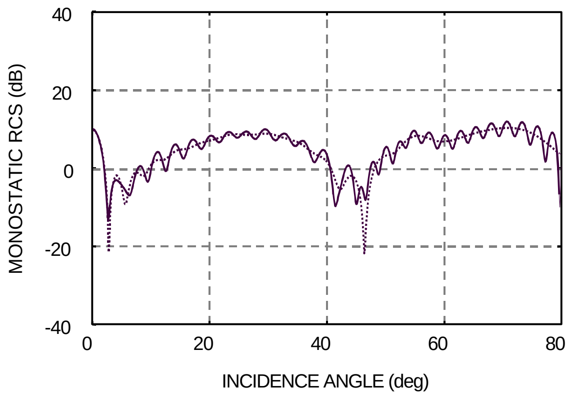
(b) E polarization, $kb = 15.7$.



(e) H polarization, $kb = 15.7$.



(c) E polarization, $kb = 31.4$.



(f) H polarization, $kb = 31.4$.

Fig. 2. Monostatic RCS versus incidence angle for $d_1 / 2b = L / b = 1.0$. ·········· : Cavity I, ——— : Cavity II. The three-layer material inside the two cavities is Emerson & Cuming AN-73 with $kt = 2.08$.