MINIMUM ENTROPY-BASED FUZZY EDGE DETECTION

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ABSTRACT

In this paper, a new fuzzy logic-based edge detection technique is proposed. The drawbacks of the conventional gradient-based techniques are efficiently overcome in the proposed technique. Using the relation between the probability partition and the fuzzy 2-partition of the image gradient, the best gradient-threshold is automatically and efficiently selected based on the condition of minimum entropy. The objective is to find the best compact edge representation. The excellent performance of the proposed technique is illustrated by the simulation results. It is shown that the extracted and purified edges provide an efficient presentation of images.

1. INTRODUCTION

Edge detection is a very important process in image processing and analysis [1-2]. Its importance arises from the fact that edges carry the most important information in image. In images, edges are marked with discontinuities or significant variations in intensity or gray levels, providing the locations of objects' contours. Methods of identifying the intensity discontinuities that mark object edges in images are usually based on the calculation of the intensity gradient through the whole image. The occurrence of a high local gradient is evidence for the existence of edge discontinuities.

In gradient-based edge detection methods, such as Sobel and Prewitt operator-based techniques, a gradient threshold level must be imposed, above which a pixel is classified as an edge point. The choice of the gradient threshold level is left to experience or trial-and-error iterations until the best result is obtained. Hence, some meaningful discontinuities may not be detected because of the suboptimal gradient threshold selection.

In the literature, several fuzzy logic-based edge detection and enhancement techniques have been proposed, e.g. [3-6]. Recently, Zhao, Fu and Yan [7] proposed a new technique for three-level thresholding for image segmentation. Using both the relations between a probability partition and a fuzzy c-partition and the principle of maximum entropy, a technique for selecting the parameters of the best fuzzy 3-partition is derived. The necessary condition for the entropy function to reach its maximum is derived. Based on this condition, an effective algorithm for the three-level thresholding is deduced.

In this paper, we extend the results of [7] to fuzzy edge detection. A new fuzzy logic-based edge detection technique, in which the drawbacks of the conventional gradient-based techniques are eliminated, is thus introduced. Our edge detection technique has two major features. First, the edge-detection problem is actually reduced to a two-level thresholding problem, which result in the partition of the image into two regions, according to its gradient values. One region constitutes pixels of low local gradient values (smooth region) and another that constitutes pixels of high local gradient values (edge region). Second, the criterion upon which the gradient threshold level is selected is the minimum entropy rather than the maximum entropy as in [7]. This is because in edge detection, we aim to find the best compact representation of the image edges and contours. The superior performance of the proposed scheme is justified through simulations and comparison with Sobel operator-based edge detection results.

2. PROBLEM FORMULATION

Let $G(x,y)$ denotes the gradient of an image, $f(x,y)$, at the pixel location (x,y). The gradient of the image can be determined using one of the derivative operators such as Sobel operator . The normalized gradient is then defined as:

$$NG(x,y) = round\left(\left(G(x,y)/max(G)\right)\times 100\right) \tag{1}$$

where $x=\{0,1,..,M-1\}$ and $y=\{0,1,...,N-1\}$ for an M-by-N image. Through normalization, the NG values are restricted to integers between zero and 100. This permissible approximation serves to simplify the mathematical operations performed later. The NG is defined by its domain D and range R, namely, $NG(x,y)\in R \quad \forall (x,y)\in D$, where $R = \{0,1,..K, K=100\}$ and $D = \{D(x,y);\ x = 0,1,..,M-1,\ y = 0,1,..,N-1\}$. The histogram of the NG is defined as, $HG=\{hg_0,hg_1,.......,hg_K\}$, where $hg_k = n_k/MN$, $k=0,1,....,K$ and n_k denotes the number of points that have an NG value equal to k. It is clear that $0 \le hg_k \le 1$ and $\sum_k hg_k = 1$.

A probability partition PP of the image normalized gradient domain D, is defined as $\Pi_g = \{D_0, D_1, ..., D_K\}$, which is characterized by a probabilistic distribution,

$$p_k \equiv P(D_k) = hg_k, \quad k=0,1,....,K \tag{2}$$

where $D_k = \{(x,y) : NG(x,y)=k,\ (x,y)\in D\}$.

Fuzzy 2-Partition of Normalized Image Gradient: The problem of fuzzy edge detection is actually a two-level thresholding problem, where the goal is to partition the image domain according to the gradient value into two fuzzy partitions (regions) D_{smooth} and D_{edge}. The region, D_{smooth}, constitutes the pixels that acquire low local gradient value (smooth region) and the region, D_{edge}, constitutes the pixels that acquire high local gradient value (edge region). The problem is to find the unknown probabilistic fuzzy 2-partition of D, $\Pi_2 = \{D_{smooth}, D_{edge}\}$, which is characterized by the probability distribution of p_{smooth} and p_{edge}, i.e. $p_{smooth} \equiv P(D_{smooth})$ and $p_{edge} \equiv P(D_{edge})$. The two fuzzy partitions D_{smooth} and D_{edge} are characterized by two membership functions μ_{smooth} and μ_{edge} respectively. Many membership functions have been introduced in the literature. In the proposed edge detection scheme, trapezoidal membership functions are used. These two membership functions have two parameters, t_1 and t_2 as shown in Fig. 1, where $0 \le t_1 \le t_2 \le 100$. In fact, the membership functions μ_{smooth} and μ_{edge} represent the conditional probability that a pixel is classified into the smooth region and edge region, respectively, under the condition that it has a gradient of k, i.e. $\mu_{smooth}(k) = p_{smooth|k}$ and $\mu_{edge}(k) = p_{edge|k}$. Since $p_{smooth} + p_{edge} = 1$, it follows that $\mu_{smooth} + \mu_{edge} = 1$. If t_1 and t_2 are selected, then the fuzzy 2-partition of the image gradient domain $\Pi_2 = \{D_{smooth}, D_{edge}\}$, is found. It is characterized by the probabilistic distributions:

$$p_{smooth} \equiv \sum_k P(D_k)\cdot P(D_{smooth,k}|D_k) \equiv \sum_k hg_k \cdot \mu_{smooth}(k) \quad \text{and} \quad p_{edge} \equiv \sum_k hg_k \cdot \mu_{edge}(k) \tag{3}$$

It is shown that p_{smooth} and p_{edge}, represent the weighted area on the gradient histogram where the weights are the membership functions μ_{smooth} and μ_{edge}, respectively.

Minimum Fuzzy Entropy Criterion: Based on the information theory, the entropy function is given as:

$$H(t_1,t_2) = -p_{smooth}\log_2(p_{smooth}) - p_{edge}\log_2(p_{edge}) \tag{4}$$

Where p_{smooth} and p_{edge} are defined previously. In [7], the best-selected set of thresholds is the one that corresponds to maximum entropy. This is because in image segmentation, the aim is to retain most of the image information (content) after thresholding to a specified number of gray levels (3 gray levels are considered in [7]). In the problem of edge detection, the case is different, since we aim at finding the parameters' values that result in the best compact edge-representation of images. Hence the parameters' values that correspond to minimum entropy $H(t_1,t_2)$ are selected.

Necessary Condition for Minimum Fuzzy Entropy: The necessary conditions for the entropy function $H(t_1,t_2)$ to reach a minimum/maximum value is given by:

$$\partial H(t_1,t_2)/\partial t_1 = 0 = -\left(\partial p_{smooth}/\partial t_1 \times \log\left(p_{smooth}/1 - p_{smooth}\right)\right) \tag{5}$$

$$\partial H(t_1,t_2)/\partial t_2 = 0 = -\left(\partial p_{smooth}/\partial t_2 \times \log\left(p_{smooth}/1 - p_{smooth}\right)\right) \tag{6}$$

It is thus clear that the condition for the entropy to reach a minimum value is $p_{smooth} = p_{edge} = 1/2$ since $\partial p_{smooth}/\partial t_1$ and $\partial p_{smooth}/\partial t_2$ cannot be zero. Based on this necessary condition, the effective search scheme for the parameters' selection introduced in [7], is followed, with the attention paid to that the parameters' selection is based on minimum entropy. The reader is kindly referred to [7] for a detailed description of the search algorithm. After the search procedure is terminated, there is a set of R candidate parameters' pairs $(t_1(r), t_2(r))$ where $r = 1,2,...,R$. These candidate pairs satisfy: $|p_{smooth} - 1/2| \le \varepsilon$ and $|p_{edge} - 1/2| \le \varepsilon$, where ε is a small positive number, typically in the order of 0.05. The best set of parameters, $(\tilde{t}_1, \tilde{t}_2)$, is the one that satisfies: $(\tilde{t}_1, \tilde{t}_2)$ $where\ H(\tilde{t}_1, \tilde{t}_2) = \min_{r=1,2,....,R} (H(t_1(r), t_2(r)))$.

3. EDGE DETECTION AND PURIFICATION

The output from the proposed fuzzy edge detection scheme $Edge\ Image(x, y)$ is calculated as follows:

$$Edge\ Image\,(x,y) = \begin{cases} 1 & if\ NG(x,y) \ge t \\ 0 & if\ NG(x,y) < t \end{cases} \tag{7}$$

where $t = (\tilde{t}_1 + \tilde{t}_2)/2$ is the optimal, automatically-determined, value of the gradient threshold. Spurious or weak edges (intensity discontinuities) may result in the image edge representation due to many factors; among them are noise and breaks in the boundary between two regions due to nonuniform illumination, introducing a noise-like appearance in the edge-detected image. Here, we introduce a simple yet effective procedure for removing spurious or weak edges. This procedure is called edge purification, since it purifies the edge detected image from these edges that do not contribute effectively to the edge-representation of the image, rather they introduce a noise-like appearance and increase the amount of information, while our aim is to obtain the best compact edge representation of images. The purification procedure is shown below. Practically, a value of S = 10 is appropriate for a 5×5 window.

♦ Run a 5×5 window on the edge detected image, where the center of the window imposed on each location *(x,y)* of that image.
♦ Within the window, estimate the number of points that have been classified as edge-points:

If the number of these points $\ge$ ***S***
 Leave these edge points
Else
 Remove these edge points!!
 They represent weak or spurious edges.
End

4. SIMULATION RESULTS

The performance of the proposed fuzzy edge detection technique has been evaluated through the simulation results (using MATLAB) for a set of five test images, namely, *Peppers*, *Trees*, *Baboon*, *Lena* and *Barbara* images. For calculating the image gradient, any derivative operator can be used. In this paper, the image gradient is calculated using Sobel operator. Table 1 demonstrates the set of optimally selected t_1 and t_2 parameters' values for the five test images. As example, Fig. 2 demonstrate the results of the proposed edge detection technique for the Trees image. (a) Corresponds to the edge-detected image before purification of edges. (b) Corresponds to the membership function for the *smooth* and *edge* regions combined with the normalized gradient histogram. (c) Corresponds to the results after the edge purification procedure. (d) Corresponds to the results using Sobel method after many iterations until the proper gradient threshold is selected. The excellent performance of the proposed technique is exercisable through the simulation results. It is shown how the extracted, enhanced, and purified edges provide an efficient edge representation for images, and how the drawbacks of conventional gradient-based techniques are overcome efficiently.

5. CONCLUSION

Throughout this paper, we introduced a new fuzzy logic-based edge detection technique, Minimum Entropy-Based Fuzzy Edge Detection, through which the drawbacks of the conventional gradient-based techniques are efficiently overcome. Through the simulation results, it is shown how the extracted and purified edges provide an efficient representation of the images edges and contours.

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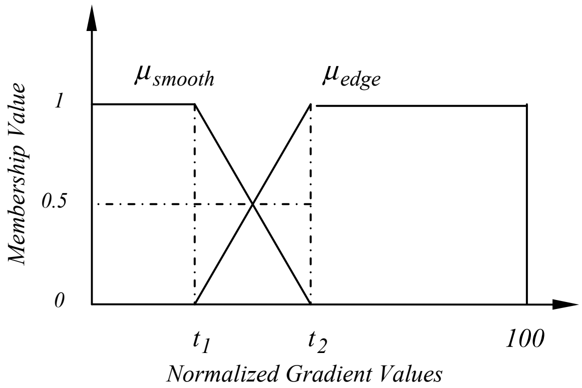


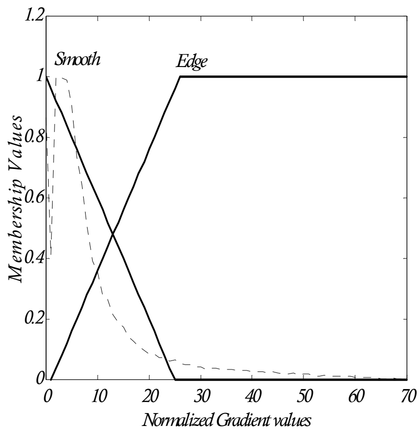
Fig.1. Membership Functions for Smooth and Edge Regions.

Table 1. Set of Optimally Selected Membership Functions' Parameters

Image	*t_1-parameter*	*t_2-parameter*
Peppers	3	13
Trees	1	25
Baboon	1	25
Lena	3	13
Barbara	8	25

(a)

(b)



(c)

(d)

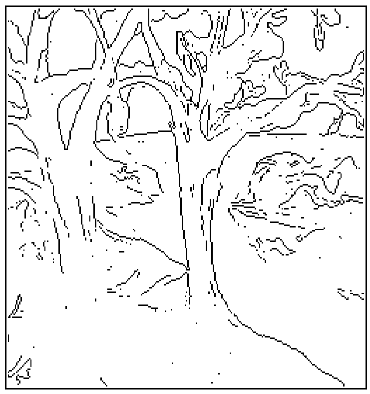


Fig.2. Simulation Results for the Trees Image.