[1]UNIVESAL MODIFICATIONS OF THE 2D AND 3D METHOD OF DISCRETE SOURCES

A. P. Anioutine, [(1)] **A. G. Kyurkchan** [(2)] **S. A. Minaev** [(3)]

[(1)] Russian New University,49 Leningradsky Prospekt, , Moscow, 125468, Russia,(095)182-2624, e-mail: anioutine@mtu-net.ru

[(2,3)] Moscow Technical University of Communication and Informatics, 8A Aviamotornaya Str., Moscow, 111024, Russia,(095)192-8475, 236-2267, e-mail: kurkchan@mtu-net.ru, kyurkchan@mtuci2.ru

ASTRACT

A new universal 2D and 3D modifications of the method of discrete sources (MMDS) were developed and applied for the scattering of an E and H polarized waves by dielectric and perfect conducting cylinders (-or convex bodies of revolution) with complicated cross section contour (-or surface). It was shown that utilizing an analytical transformation of the cylinder's contour (-or surface of the body of revolution) allows constructing of the auxiliary contours (-or surfaces) in the procedure of the method discrete sources and it leads to stable results with high accuracy in high frequency domain.

1. INTRODUCTION

In recent years one can see that such methods as the method of discrete sources (MDS) [1,2,3] and the method of auxiliary currents (MAC) [3] are widely practiced due to their simplicity. One could find three stages in developing of this method. The first one deal with work [1] in which V. D. Kupradze had proved the completeness for a set of fundamental solutions (auxiliary sources) to the 2D(or 3D) Helmholtz equation (the vector case was considered too). These fundamental solutions have singularities on a certain closed auxiliary surface Σ (or auxiliary contour Σ) inside scatter and the theorems proved by V. D.Kupradze places almost no constraints on the geometry of the auxiliary surface Σ (auxiliary contour Σ). The second stage was connected with numerous publications in which MDS were applied for numerical solution of many scattering and diffraction problems. Only one essential scientific result was achieved in this period – it was detected the fact of unstable algorithms and decreasing accuracy with increasing the number of auxiliary sources. In [3,6], the origin of this effect was found and constraints on the geometry of Σ were formulated in the form of the following theorem in which $U_0(\vec{r})$ is a incident wave and $I(\vec{r}_\Sigma)$ is a auxiliary current (see also [6]): Let a simple closed Lyapunov curve Σ be such that k is not an eigenvalue of the interior homogeneous Dirichlet problem for the region inside Σ .Then the Fredholm integral equation of the first kind

$$\int_\Sigma H_0^{(2)}(k\,|\vec{r}_S-\vec{r}_\Sigma|)I(\vec{r}_\Sigma)d\sigma=-U_0(\vec{r}_S),\quad \vec{r}_S\in S$$

has a solution if and only if Σ encloses all of the singularities of the diffracted field inside of the scatter's contour S. The same theorem was proved for 3D case [4]. But that theorem did not show the way of constructing auxiliary surface Σ (or auxiliary contour Σ) .So, the third stage was opened when in [5] was developed the way of the constructing Σ for MDS 2D Dirichlet problem.

In this paper we has developed 2D and 3D modification of the method of discrete sources (MMDS) for a case of dielectric (impedance, metal) scatterers and E (-or H) polarized plane (-or cylindrical) incident waves basing on analytical transforming of scatter's boundary S for constructing auxiliary contour Σ . The results of it's utilizing are illustrated by solving a 2D scattering problem by dielectric or perfect conducting cylinders with elliptical or complicated cross section contour (multifoil contour) and 3D problem for impedance bodies of revolution in cases of the plane (-or cylindrical) incident waves. The problems accuracy and stable are investigated when the extent of haul an auxiliary contour taut closer at singularities and interpretation of the results is made.

---

[1] This work was supported by the Russian Foundation for Basic Research, project no. 00-02-17639

2. 2D SCATTERING PROBLEMS

Let us consider the two-dimensional problem of diffraction of a plane (-or cylindrical) wave $U_0(r,\varphi)$:

$$U_0(r,\varphi) = \exp\{-ikr\cos(\varphi-\varphi_0)\} \quad \text{or} \quad U_0(r,\varphi) = H_0^{(2)}(k\,|\vec{r}-\vec{r}_0\,|) \tag{1}$$

by a dielectric (-or perfect conducting) cylinder in the cylindrical coordinate system (r,φ,z). We suppose that the *z*-axis is the axis of the cylinder. In (1) φ_0, k are the angles of incidence and the wave number of the plane wave respectively, $\vec{r}_0$ – coordinates of the source of cylindrical wave. Let the cylinder's cross-section have the equation in the polar coordinates

$$r\,|_S = \rho\ (\varphi), \tag{2}$$

where $\rho(\varphi)$ is an analytical function of φ subject to the condition of the theorem mentioned above .

According to the MDS, the diffracted field $U(r,\varphi)$ outside of S and the field $U^i(r,\varphi)$ inside of S could be presented in the form

$$U(r,\varphi) = \sum_{m=1}^{M} A_m H_0^{(2)}(k\,|\vec{r}-\vec{r}_m\,|)\ , U^i(r,\varphi) = \sum_{m=1}^{M} B_m H_0^{(2)}(k\,|\vec{r}-\vec{r}'_m\,|)\,. \tag{3}$$

Here, $H_0^{(2)}(k\,|\vec{r}-\vec{r}_m\,|)$ is the fundamental solution to the Helmholz equation (source); A_m, B_m are the coefficients to be determined; $|\,\vec{r}-\vec{r}_m\,| = [r^2 + r_m^2 - 2rr_m\cos(\varphi-\varphi_m)\,|^{1/2}$ is the distance between points given by the radius vectors $\vec{r}$ and $\vec{r}_m$ in polar coordinates; $\vec{r}_m, \vec{r}'_m$ are the radius vectors positions of the sources on Σ within the boundary S and $\Sigma 1$ outside of S; k_i - the wave number of the medium inside of S; M is the total number of sources on Σ or $\Sigma 1$. The system of algebraic equations for the coefficients A_m, B_m is obtained by applying the boundary condition to (3)

$$U^i(r,\varphi)\,|_S - U(r,\varphi)\,|_S = U_0(r,\varphi)\,|_S; \frac{\partial}{\partial n}\{\chi U^i(r,\varphi)\,|_S - U(r,\varphi)\,|_S\} = \frac{\partial}{\partial n} U_0(r,\varphi)\,|_S; \tag{4}$$

where $\chi = 1$ for the case of an E-polarized incident wave and $\chi = 1/\varepsilon$ for H-polarized incident waves; ε_r is the relative dielectric penetrability of the cylinder material. The parameter $\chi = 0$, $U^i(r,\varphi) = 0$, and contour $\Sigma 1$ is absent when the scattering problem by a perfect conducting cylinder is under consideration. The boundary condition (4) could then be solved using the Dirichlet or Neumann formulations.

The main arising in MDS problem is known how to detect the construction and location of two auxiliary contours $\Sigma, \Sigma 1$ (inside and outside of the original contour). In almost all published works this contours had choused similar to the original one or as a equidistant contour. In [5] was shown that such way has a limitation for accuracy and leads to unstable result. To avoid this effects we have to find all singularities of the diffracted field, to make an analytical transformation of original contour to enclose its [6] and takes this transformed contours as auxiliary contours $\Sigma, \Sigma 1$. For plane (-or cylindrical) incident wave and original contour S as ellipse or multifoil the singularities can be found in analytical form [6]. For example, in case of ellipse we have for $\Sigma, \Sigma 1$

$$r_\Sigma = |\,\zeta\,|, \varphi_\Sigma = \arg(\zeta), \zeta = \rho(\varphi + i\varphi')\exp(\varphi + i\varphi'); \varphi' = -\ln[\varepsilon/\{2-\varepsilon^2\}^{1/2}] + \delta; \tag{5}$$

$$r_{\Sigma 1} = |\,\zeta\,|, \varphi_{\Sigma 1} = \arg(\zeta), \zeta = \rho(\varphi - i\varphi')\exp(\varphi - i\varphi'); \tag{6}$$

For arbitrary analytical contours $\Sigma, \Sigma 1$ a numerical procedure could be developed. The accuracy of the solving problems we estimate as the residual Δ of the boundary conditions on S. It was detected that applying of these contours leads to stable results and the residual Δ becomes much better when auxiliary contours approach to singularities and number of the auxiliary sources is fixed or number of the auxiliary sources is increasing and location of the auxiliary contour is fixed. This effect is illustrated in Tab.1.

	$\delta=10^{-3}$	$\delta=10^{-4}$	$\delta=10^{-5}$	$\delta=10^{-6}$	$\delta=10^{-7}$
N=128	0.01256	0.0005825	0.0005807	0.00058057	0.00058055
N=256	$6.9*10^{-8}$	$5.03*10^{-9}$	$3.54*10^{-10}$	$8.71*10^{-11}$	$6.44*10^{-11}$
N=512	$1.53471*10^{-10}$	$1.53252*10^{-10}$	$1.89562*10^{-10}$	$4.03475*10^{-10}$	$9.07961*10^{-11}$

Tab.1

So we have a possibility to achieve a high accuracy and stable results by changing of these two parameters. We applied this method for calculation of the scattering pattern $g(\theta)$ in case of perfect conducting or dielectric corrugate cylinders; cylinders with ellipse or multifoil cross-sections and E (-or H) polarized incident plane (-or cylindrical) waves. For perfect conducting elliptical cylinder with kb=120; ka=40; N=800; $\delta = 10^{-6}$, E polarized incident plane wave with $\varphi_0 = \pi/2$ we had max(Δ)=$9.27 * 10^{-6}$. In case of perfect conducting cylinder with multifoil cross-section and q=4,ka=65; kb=6;N=800; H polarized incident plane wave with $\varphi_0 = 0$ we had max(Δ)=0.004. The scattering pattern $g(\varphi)$ for corrugate dielectric and perfect conducting cylinder (multifoil with q=24; ka=8,8; kb=1,2;N=1400; δ=0,0000001); H polarized incident plane wave with $\varphi_0 = 0$ are presented at Fig.1, 2. The scattering pattern $g(\varphi)$ for perfect conducting cylinder with multifoil cross-section and q=4; ka=30; kb=15;N=800; δ=0,0001; H polarized plane incident wave with $\varphi_0 = 0$ and max(Δ)=$1{,}05 * 10^{-7}$ are presented at Fig3.

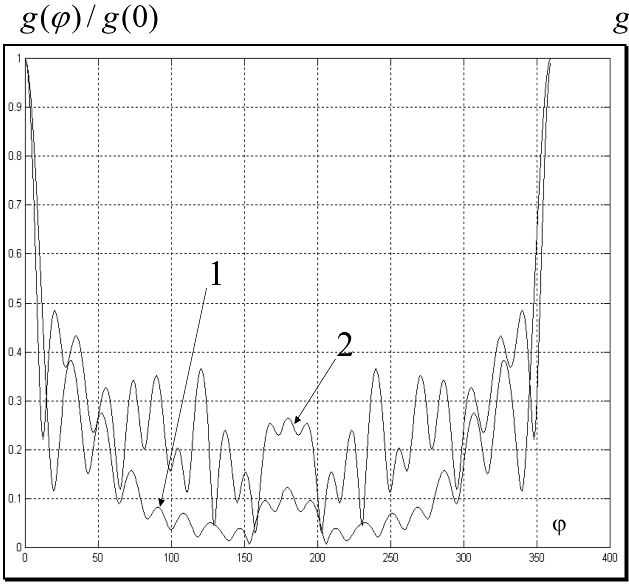

Fig.1 (curves 1,2- for corrugate and circular cylinders)

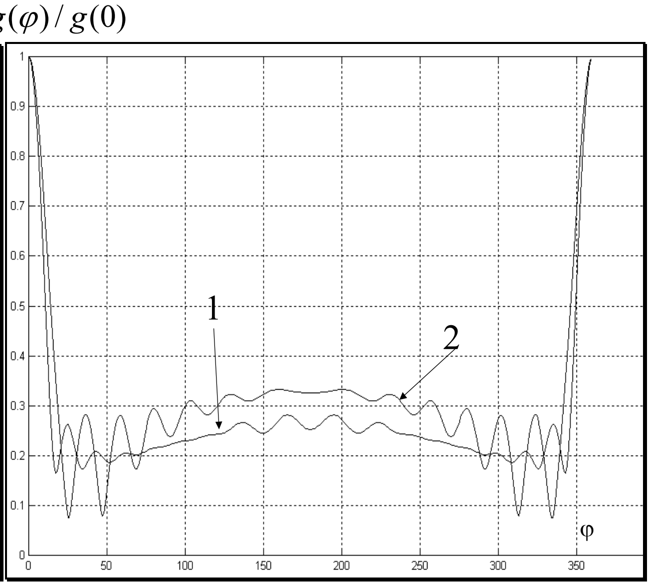

Fig.2 (curves 1,2- for corrugate and circular cylinders)

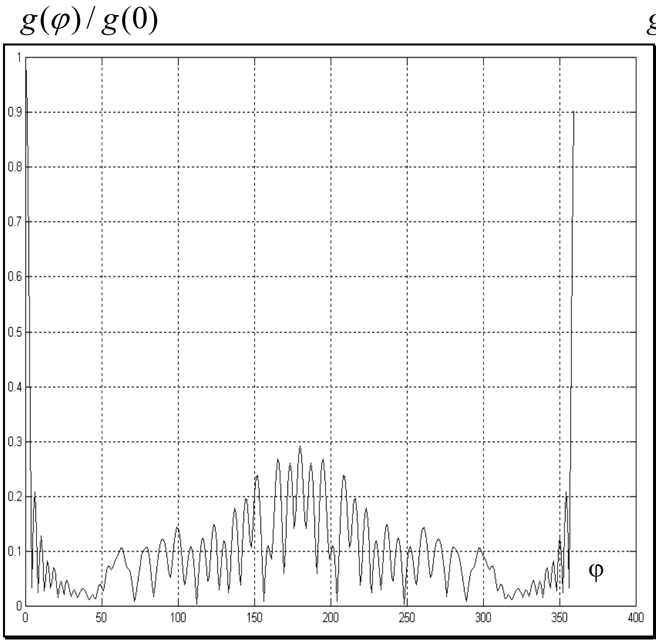

Fig.3

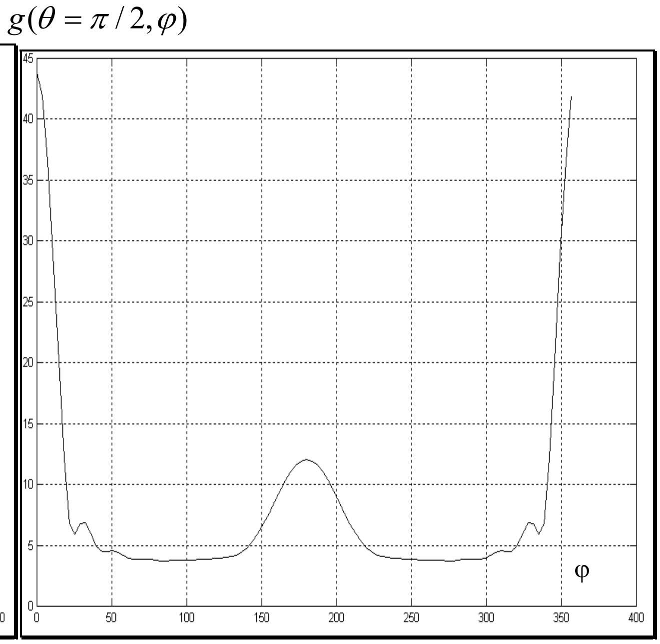

Fig.4

3. 3D SCATTERING PROBLEMS

Let us consider the 3D problem of the scattering of a plane wave by impedance body of revolution. According to MMDS, the diffracted field $u^1(\vec{r})$ outside of the scatterer could be presented in form of set with auxiliary sources

$$u^1(\vec{r}) = \sum_{n=0}^{N} a_n \exp\{-ik\,|\vec{r} - \vec{r}_{\Sigma_n}|\} / |\vec{r} - \vec{r}_{\Sigma_n}| \quad . \tag{7}$$

The system of algebraic equations for coefficients a_n of this set is obtained by placing set (7) under boundary condition

$$u^0(r,\theta,\varphi) + u^1(r,\theta,\varphi)|_S = \frac{W}{k}\frac{\partial}{\partial n}[u^0(r,\theta,\varphi) + u^1(r,\theta,\varphi)]|_S \tag{8}$$

and satisfying the boundary conditions in matching points. In (5),(6) $u^0(r,\theta,\varphi)$ - incident wave, r,θ,φ – spherical coordinates of $\vec{r}$, $\vec{r}_{\Sigma_n}$ - radius vector of the location's points for auxiliary sources. In this case the problem of detecting the location of the auxiliary suffices arises too. We have to execute a procedure of finding all singularities of the diffracted field and then repeat a procedure of analytical transformation for original surface to enclose its as we had made in 2D case. We had made this procedure and had calculated the scattering pattern for such bodies as ellipsoid or multifoil of revolution when the incident wave was a plane wave. The result of calculations of the $g(\varphi)$ for perfect conducting multifoil of revolution with q=3; ka=5; kb=1; N=1400; $\delta = 0.000001$; H polarized incident plane wave $u^0 = \exp(-ikz)$ is presented at Fig.4.

4. PULSE PROBLEM

The developed 2D results for harmonic fields allow us to calculate a pulse scattering by cylinder with ellipse cross section. We applied the Fast Fourier Transform to harmonic solution of the scattering problem and got the solution of the pulse problem. As an incident pulse we took a radio impulse with rectangular envelope and constant carrier frequency.

5. CONCLUSION

The numerical solutions of a number of model diffraction problems provided above allow us to conclude that the approach proposed in this paper has advantages over the traditional techniques . This modification of the MDS largely extends the class of admissible scatter's shapes and frequency domain , to a great extensive improves the accuracy, and, simultaneously, retains the simplicity of realization and versatility typical of the MDS. The method can easily be extended to plane-layered media and vectors problems.

References

[1] V.D.Kupradze, "On the approximate solutions of problems in mathematical physics", *Success Mathematical Sci*., vol. 22,pp. 58-108,1967.

[2] A.G.Kyurkchan, "On the method of auxiliary currents and sources in problems of waves diffraction", *Radio Eng. Electron. Phys*., Vol. 28,pp. 2129-2139, August 1984.

[3] A.G.Kyurkchan, "Representation of diffraction fields by wave potentials and the method of auxiliary currents in problems of the diffraction of electromagnetic waves", *Soviet Journ. of Commun. Techn. and Electron*., Vol.31 ,pp. 20-27, May 1986.

[4] A.G.Kyurkchan, A.I.Sukov and A.I.Kleev "Singularities of wave fields and numerical methods of solving the boundary-value problems for Helmholtz equation", in: Generalized Multipole Techniques for Electromagnetic and Light Scattering ,Ed. T.Wriedt, Elsevier, Amsterdam,pp,81-109, 1999.

[5] A.G.Kyurkchan, S.A.Minaev and A.L.Soloveichik "A modification of the method of discrete sources based on a priori information about the singularities of the diffracted field", *Journ. of Commun. Techn. and Electron*. , vol. 46, pp. 615-621, June 2001.

[6] A.G.Kyurkchan, "Analytical continuation of wave fields", *Soviet Journ. of Commun. Techn. and Electron*., vol. 31, pp., 59-69, November 1986.