WAVELETS IN 2D AND 3D SCATTERING AND WAVE PROPAGATION PROBLEMS[1]

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ABSTRACT

Wavelets technique is applied for solving of the 2D (-or 3D) Dirichlet, Neumann or mix boundary problems when scattering bodies are perfect conducting (-or dielectric) cylinders with complicated cross sections, screens and bodies of revolution. The developed method utilizes the Haar's wavelets functions and linear Battle-Lemarie wavelets functions for solving the Fredholm integral equation of fist kind with smooth or singular kernel. The problems of accuracy, choosing auxiliary contour Σ and stable results are discussed.

1. INTRODUCTION

It is known that wavelet technique is widely practiced in such domains as time- or frequency analyses of the signals and image process [1,2]. One of the most attractive ideas was appeared in last years had been connected with utilizing of wavelets as basis functions in method of the moments. It is clear that wavelets technique allows to create a faster algorithms then ordinary one due to applying a specific attributes of its as a basis functions.

This paper is concerned the extending wavelets technique (Haar and linear Battle-Lemarie wavelets functions) for solving 1D (2D) auxiliary currents integral equations (ACIE) of the first kind with smooth kernel [3,4], 1D Abel integral equations and 1D (2D) currents integral equations (CIE) of the first kind with singular kernel [5,6] (or system of integral equations). It is well known that all these integral equations are widely used for solving many scattering (diffraction) problems or wave propagation problems [5,6].

2. 2D SCATTERING PROBLEMS

Let us at first consider the scattering of an E polarized (-or H-polarized) time-harmonic wave $u_0(\vec{r})$ by perfect conducting cylinder with cross-section contour S described by the equation $r(\varphi)$ in a cylindrical system of coordinates (z,r,φ) with z along the cylinder axis. The diffracted field $u^1(\vec{r})$ outside of S is given by a solution to the Helmholts equation

$$\Delta u^1 + k^2 u^1 = 0 \quad , \tag{1}$$

that satisfies the Dirichlet (-or Neumann) boundary condition on S

$$u(\vec{r})\,|_S \equiv [u_0(\vec{r}) + u^1(\vec{r})]\,|_S = 0 \quad \text{or} \quad \frac{\partial}{\partial n}\,[u_0(\vec{r}) + u^1(\vec{r})]\,|_S = 0 \quad , \tag{2}$$

and Sommerfeld's radiation condition

$$\frac{\partial u^1(\vec{r})}{\partial r} + iku^1(\vec{r}) = o(r^{-1/2}), |\,r\,| \to \infty \quad , \tag{3}$$

where k - is a wave number in free space.

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In accordance with the method of an auxiliary current [3], the boundary problems (1)-(3) could be reduced to the Fredholm integral equations of the first kind with smooth kernel

$$u_0(\vec{r}_S) = \int_{\Sigma} \mu\ (\vec{r}_\Sigma) H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|) d\sigma. \quad \text{or} \quad \frac{\partial}{\partial n_S} u_0(\vec{r}_S) = \int_{\Sigma} \mu(\vec{r}_\Sigma) \frac{\partial}{\partial n_S} H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|) d\sigma. \quad (4)$$

and in case of mixed boundary conditions (when scatterer is a dielectric cylinder) we have a system of integral equations of the first kind with smooth kernel

$$u^0(\vec{r}_S) + \int_{\Sigma} I(\vec{r}_\Sigma) H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|) d\sigma \;=\; \int_{\Sigma 1} I_1(\vec{r}_{\Sigma 1}) H_0^{(2)}(k_i\,|\,\vec{r}_S - \vec{r}_{\Sigma 1}\,|) d\sigma_1, \quad (5)$$

$$\frac{\partial}{\partial n_S}[u^0(\vec{r}_S) + \int_{\Sigma} I(\vec{r}_\Sigma) H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|) d\sigma] = \chi \frac{\partial}{\partial n_S}[\int_{\Sigma 1} I(\vec{r}_\Sigma) H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|) d\sigma_1\ ].$$

Here, $\mu(\vec{r}_\Sigma), I(\vec{r}_\Sigma), I(\vec{r}_{\Sigma 1})$ - located on Σ and $\Sigma 1$ an auxiliary currents; Σ is a closed contour within S; $\Sigma 1$ is a closed contour outside S; $\vec{r}_\Sigma$ is the radius-vector of the integration points on Σ, $\vec{r}_{\Sigma 1}$ is the radius-vector of the integration points on $\Sigma 1$, $\vec{r}_S$ is the radius-vector of the points on S; $d\sigma$ is the length of an element's shaft-bow on Σ; $|\,\vec{r}_S - \vec{r}_\Sigma\,| = [r^2(\varphi) + \rho^2(\theta) + 2r(\varphi)\rho(\theta)\cos(\varphi - \theta)]^{1/2}$; $r(\varphi)$ - is the equation of the contour S and $\rho(\theta)$ - is the equation of the contour Σ inside S in a cylindrical coordinate system, $\chi = 1$ for E polarized incident wave and $\chi = 1/\varepsilon_r$ for H one, ε_r - is a relative dielectric penetrability of the cylinder's dielectric.

One of the main problems of the method ACIE is the constructing of the auxiliary contours and detecting its location. We constructed these contours as a result of the analytical transformation of the original contour S: $\zeta = \rho(\varphi)\exp\{i\varphi\}$; $\varphi = \varphi' + i\varphi''$; $r_\Sigma = |\,\zeta\,|$; $\theta = \arg\zeta$. This sort of transformation is possible as long as the analytical transformation remain a one-to-one mapping. The points at which this one-to-one correspondence is violated and singular points of the analytical extension of the diffracted field to the region inside (outside) of the original contour constitute a set of so-called principal singularities of the diffracted field [8]. As it was shown in [3] for integral equations mention above to have a solution, it is sufficient for auxiliary contour to enclose these singularities. For such types of original contours as ellipse (-or multifoil): $\rho(\varphi) = a/\sqrt{1 - \varepsilon^2\cos(\varphi)}$; $\varepsilon = a/b$; (- or $\rho(\varphi) = a + b\cos(q\varphi)$) these singularities could be calculated in analytical form [3,8]. For arbitrary analytical original contour these singularities could be calculated by developed numerical procedure.

Another problem is the problem of the choosing the basis functions for numerical solution of the integral equations. We had solved this problem by applying the wavelet technique for presentation of the auxiliary currents in form of set with 1D Haar functions or linear Battle-Lemarie functions [1,2]. It allows creating fast and stable algorithms of solving the integral equations of the first kind with smooth kernel for all scatterers mention above and for cases of the plane or cylindrical incident waves [9].

The auxiliary currents having been found, well-knownn relations could be used for calculation the scattering pattern $g(\varphi)$ without any difficulties. The accuracy of the solving problems we estimate as the residual Δ of the boundary conditions. It was shown that Δ depends as on the number N and type of the basis functions as on the degree δ of the closeness auxiliary contour at singularities. It is illustrated in Tab. l for a case of dielectric cylinder with multifiol cross-section and q=5; ka=10; kb=2; ε=4; Haar wavelets functions.

	N=128			N=256			N=512
max(Δ)	0,013	$5,8*10^{-4}$	$0,58*10^{-4}$	$6,9*10^{-9}$	$5*10^{-9}$	$8,9*10^{-11}$	$1,45*10^{-10}$
δ	10^{-3}	10^{-4}	10^{-5}	10^{-3}	10^{-4}	10^{-6}	10^{-3}

Tab.1

It was detected that the Haar wavelets gives much better accuracy then linear Battle-Lemarie wavelets when the total number of the basis functions is the same. It was shown that the stable results could be obtained when descriptive points are choosing not only in the middle of the intervals but also in the points closed at the ends of the intervals. Developed method was applied for calculation a scattering pattern $g(\varphi)$ in high frequency region. For example, we had calculated $g(\varphi)$ for E polarized incident wave $u_0 = \exp\{-ikr\cos(\varphi - \varphi_0)\}$ and dielectric cylinder with elliptic cross-section with kb=30, ka=10, $\varepsilon = 4$, N=256, $\delta = 10^{-6}$, max(Δ)=$6,2*10^{-7}$, $\varphi_0 = \pi/2$ (k_rD=120, Dmaximum size of the scattering region); for metal cylinder with multifoil cross-section and q=4, ka=30, kb=15, N=256, $\delta = 10^{-5}$, $\varphi_0 = 0$, max(Δ)= $1{,}85 * 10^{-7}$, (kD=90); for dielectric cylinder with multifoil cross-section and q=24, ka=8,2 , kb=1,2, $\varepsilon = 4$, N=256, $\delta = 10^{-5}$, $\varphi_0 = 0$, max(Δ)= $0{,}018$ (k_r D=40).

Solution of the 2D scattering problems on the base of currents integral equations of the first kind with singular kernel we had made by developed new method "prolonged" boundary conditions and applying Haar or linear Battle-Lemarie wavelets as a basis functions. We had considered the scattering of the plane wave by system of M perfect conducting bands, corner reflector with corner's angle ψ . We had calculated $g(\varphi)$ of one band when ka =200 (a is the band's width), N=128, max(Δ)< 10^{-3} for E polarized plane incident wave with $\varphi_0 = \pi / 2$. The scattering pattern $g(\varphi)$ by 4 perfect conducting bands and corner reflector, E polarized plane incident wave is shown at Fig.1,2.

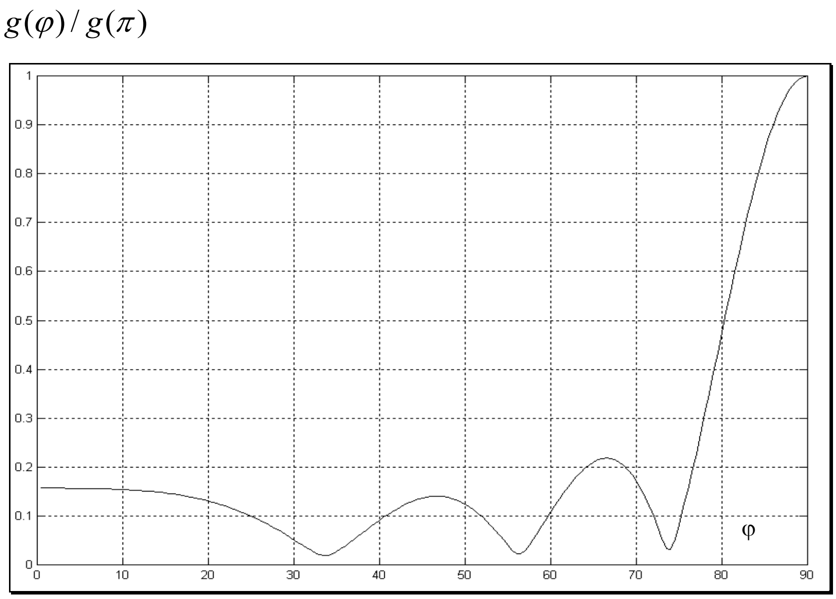

Fig. 1 Scattering pattern $g(\varphi)/g(\pi)$ for 4 bands ($ka = 0{,}637\lambda; kb = 0{,}956\lambda; \varphi_0 = \pi/2; N = 64$)

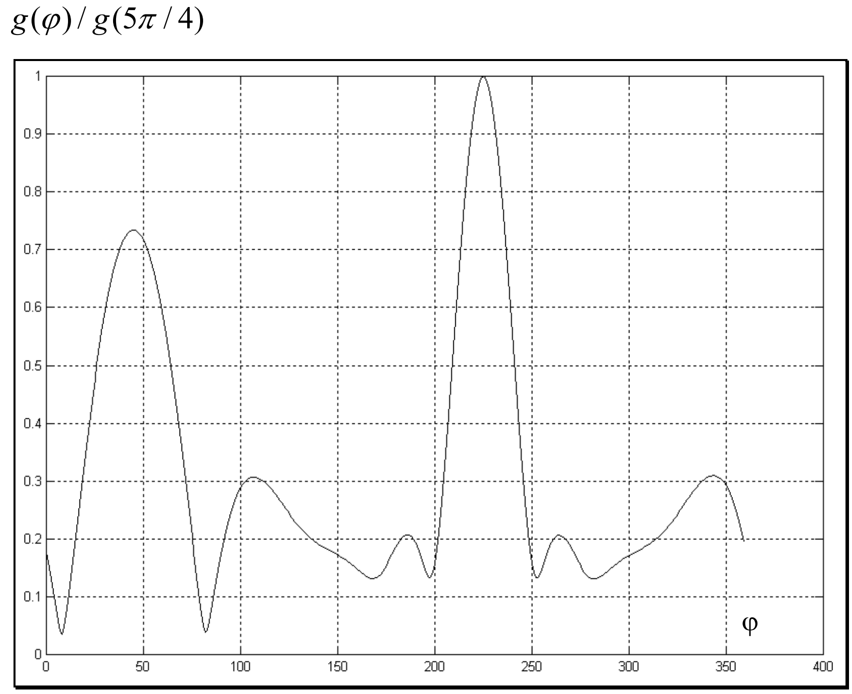

Fig. 2 Scattering Pattern $g(\varphi)/g(0)$ for corner antenna $(ka = 1{,}5\lambda; \psi = \pi/2; \varphi_0 = \pi/4; N = 128)$

At least, the ordinary way (with isolation of the kernel's singularities) and Haar wavelets as a basis functions we applied for solving the Abel integral equation which is arise when trans-ionosphere inverse problem is under consideration. It allows making a stable and fast algorithm for making solution of such type inverse problem.

3. 3D SCATTERING PROBLEMS

A case of 3D scattering problem was considered for ideal and impedance boundary conditions and bodies of revolution such as ellipsoid of revolution and multifoil of revolution. The 2D auxiliary currents integral equations of the first kind with smooth kernel [7] were used. The 2D auxiliary contours were obtained by revolution of 1D contours. The unknown 2D function - auxiliary current $\mu(\theta,\varphi)$ was presented as a set with 2D Haar wavelet functions [1]. The procedure of the point-matching method was applied when the integral equations had been solved. It was shown that arising matrix close at sparse one. So, well-known methods of getting a solution of these type linear equations were applied when the order of matrix is large (more then some thousands). We had calculated $g(\theta,\varphi=\pi/2)$ for ellipsoid of revolution with kb=8, ka=4; $\delta=10^{-5}$, E polarized plane incident wave $u_0=\exp\{-ikz)$ with max(Δ)=2,4$*10^{-4}$.

4. PULSE PROBLEM

The obtained results of solving the harmonic scattering problems mention above allow us making an effective algorithms for solving the pulse scattering problems. We used Fast Fourier Transform and algorithms described above for considering a pulse scattering by perfect conducting cylinder with elliptic cross section or band. As incident pulses we took a radio signals with rectangular or sinusoidal envelopes.

5. CONCLUSION

The developed methods allow making the essential step into high frequency domain (kD>>1) and in solving of the pulse problems. It decreases the time of calculation and has a good accuracy and stable. The method can easily be extended to plane-layered media and vector fields.

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