ROBUST GENERALIZED PEEC METHODOLOGY FOR FULL-WAVE ANALYSIS OF INTEGRATED CIRCUITS

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ABSTRACT

A numerically stable extension of the Partial Element Equivalent Circuit interpretation of the electric field integral equation for the modeling of arbitrarily shaped interconnections and integrated passives in high-speed/high-frequency integrated circuits is presented in this paper. The proposed methodology utilizes triangular cells for the discretization of conductor surfaces and triangular prisms for the modeling of finite dielectrics. Its numerical stability at very low frequencies is achieved through an appropriate transformation of the resulting system matrix, and makes possible the broadband electromagnetic analysis from very low (almost dc) to multi-GHz frequencies.

INTRODUCTION

Continuing trends toward miniaturization of mixed-signal, performance driven, multi-functional electronics call for a more versatile and rigorous pre-prototype, broadband electromagnetic analysis for accurate interference prediction and computer-aided design support. The interconnect network and circuit density in such systems is such that all electromagnetic interactions need be taken into account for accurate design. To address this complexity a new breed of electromagnetic CAD tools is needed that seamlessly integrated electromagnetic modeling into the industry’s design flow in ways that accommodate the primarily circuit-oriented experience of the majority of the designers, without sacrificing the electromagnetic accuracy needed for accurate analysis. From the various numerical methodologies in use today for the method of moments approximation of the integral equation statement of the electromagnetic boundary value problem, the Partial Element Equivalent Circuit (PEEC) method [1] is the one that facilitates the most physically intuitive way of bridging the gap between circuit and electromagnetic field simulation. Thus, it is not surprising the fact that the majority of the modeling efforts of the circuit analysis-oriented signal integrity community toward an electromagnetically more rigorous modeling and simulation environment for high-speed circuit design has been founded on the PEEC formulation.

Another interesting attribute of this new class of mixed-signal ICs is that the mixed-signal nature of their application makes them electrically small over a significant portion of their operating frequency bandwidth. It is well known that standard electromagnetic integral equation solvers become numerically unstable and thus unreliable when applied to the solution of electrically small structures (e.g. [2]-[3]). This difficulty is closely related to the increasing decoupling of the electric and magnetic fields as the frequency tends to zero. In order to highlight one of the aspects of this decoupling, consider the equation \( \vec{E} = -j\omega \vec{A} - \nabla \Phi \) that leads to the electric field integral equation (EFIE) formulation of the electromagnetic boundary value problem. Recognizing the fact that the vector potential term is proportional to \( j\omega \vec{J} \), while the scalar potential is proportional to \( (j\omega)^{-1} \nabla \cdot \vec{J} \), it is clear that the contribution from the vector potential term will be lost in the finite-precision numerical solution of the problem as the frequency tends to zero. As explained in detail in [3], this difficulty can be overcome through a loop-star or loop-tree decomposition of the unknown current distribution, so that the solenoidal (divergence-free) component of the current, responsible for the quasi-static magnetic field, and its complementary (curl-free) component, responsible for the quasi-static electric field, are modeled independently. The two components of the current density exhibit disparate frequency dependency that is instrumental in improving the stability of the numerical solution of the EFIE at very low frequencies while maintaining solution accuracy.

The objective of this paper is two-fold. First, a systematic methodology is presented for the generalization of the PEEC model to the conforming modeling of interconnect structures of arbitrary shapes and material composition. Second, it is shown that, in the context of the PEEC interpretation of the EFIE, the benefits from the loop-tree
decomposition of the unknown current density can be realized in an alternative and more straightforward fashion, through the re-formulation of the PEEC interpretation of the EFIE in the spirit of circuit mesh analysis.

THE GENERALIZED PEEC FORMULATION

The PEEC formulation is based in the enforcement of a discrete approximation of the equation for the electric field,

\[ \vec{E}(\vec{r}) = -j\omega \vec{A}(\vec{r}) - \nabla \Phi(\vec{r}) \]

through a Galerkin’s testing at each and every element in the discrete model of the interconnect structure. In the above equation, \( \vec{A}(\vec{r}) \) and \( \Phi(\vec{r}) \) are, respectively, the magnetic vector and electric scalar potentials. In the proposed formulation, polarization currents are introduced to account for the presence of inhomogeneous dielectric volumes. Therefore, the Green’s function used in the Poisson’s integrals for the potential is the free-space Green’s function. Furthermore, a triangular mesh is used for the discretization of conductor surfaces. Dielectric volumes are discretized by means of prisms. Prisms are used also for the discretization of the volume of conductors when a volumetric rather than a surface model is used to account for skin effect. For the case of a surface model for the representation of electric current flow in the conductors, the surface electric current density \( \vec{J} \) and the surface electric charge density \( \rho \) are expanded, respectively, into the roof-top Rao-Wilton-Glisson (RWG) expansion functions [4], and pulse (piece-wise constant) functions [5]. Use of these expansion functions in conjunction with the Galerkin’s testing of the EFIE yields the familiar PEEC statement of the problem (when only conductors are involved),

\[ Z_a I_a + \sum_{n=1}^{N} j\omega L_{an} I_n - \sum_{m=1}^{M} p_{pm} Q_m + \sum_{m=1}^{M} p_{ym} Q_m = V_a^{(s)} \]  

where \( Z_a \) is the surface impedance for element \( a \), and the remaining coefficients are given by,

\[ L_{an} = \frac{\mu}{4\pi} \int_{\tau_n^+} \int_{\tau_n^-} \vec{f}_a(\vec{r}) d\tau \cdot \int_{C_n} G_0(\vec{r}, \vec{r}') \vec{f}_a(\vec{r}') d\tau' \]

\[ p_{pm} = \frac{1}{4\pi\epsilon_0} \int_{\tau_m^+} \int_{\tau_m^-} q_\beta d\tau \cdot \int_{C_m} G_0(\vec{r}, \vec{r}') q_\alpha d\tau' \]

\[ p_{ym} = \frac{1}{4\pi\epsilon_0} \int_{\tau_m^+} \int_{\tau_m^-} q_\alpha d\tau \cdot \int_{C_m} G_0(\vec{r}, \vec{r}') q_\beta d\tau' \]

In the above \( G_0 \) denotes the free-space Green’s function, and the source \( V_a^{(s)} \) is introduced to account for the excitation. The presence of finite dielectric volumes introduces extra unknowns, associated with the discretization of the bound charge and polarization current density. More specifically, proper expansion functions are used to represent the polarization current density \( \vec{J}_d = j\omega(\epsilon_\alpha - 1)\epsilon_\alpha \vec{E} \) inside the triangular prisms used for the discretization of the finite dielectrics. These expansion functions may be thought of as the generalization of the roof-top basis functions stated earlier to three dimensions, where the represented transverse current densities are assumed to be constant in the vertical direction, while the vertical component of the current density is assumed constant in the transverse direction and exhibiting a linear (roof-top) function variation in the vertical direction. The details of the PEEC interpretation of the dielectric volume modeling can be found in [5].

In (2) conservation of electric charge has not been enforced. In the spirit of PEEC the enforcement of this condition is interpreted as the generalized form of Kirchhoff’s current law. Let \( \mathbf{V}_m \) denote the vector of “voltages” across the “inductive” branches in the PEEC circuit, \( \mathbf{V}_p \) the vector of the “voltages” at the “capacitive” nodes, while the vectors \( \mathbf{I}_m \) and \( \mathbf{I}_p \) contain the corresponding currents, respectively. The vector of unknown charges is related to \( \mathbf{I}_p \) through \( j\omega \mathbf{Q}_p = \mathbf{I}_p \). The combination of the discrete form of the conservation of charge equation with (2) yields the complete form of the PEEC approximation of the electromagnetic problem, which may be cast in matrix form as follows,
This is typically referred to as the nodal analysis PEEC model [1], and its matrix properties are identical to those of the method of moments matrix obtained for the EFIE using a Galerkin’s approach with the RWG as the expansion and testing functions. Hence, it also exhibits the low-frequency ill-conditioning mentioned in the Introduction. This ill-conditioning, in the context of PEEC, has also been reported in [6]. It was noted in [6] that, contrary to the above nodal analysis formulation of the discrete problem, a mesh analysis yields a more stable discrete system at low frequencies. In view of the discussion in the Introduction of the effective use of the loop-star decomposition of the current density to improve the low-frequency numerical stability of the approximation, the improvement obtained by the application of mesh analysis makes sense. At the same time it motivates a systematic methodology for the transformation of the discrete problem of (4) into one that is free from low-frequency numerical instability. This methodology is outlined in the following.

Once the discrete PEEC model has been generated, application of mesh analysis requires the identification of all independent loops in the network graph that describes the discrete model. Associated with these loops is a loop matrix $M$. The identification of the loops requires the selection of a tree first. The proposed procedure makes the selection of the tree for the network that results after all “capacitive” branches are opened. Hence, the independent loops obtained from this tree contain only “inductive” branches. Subsequently, the “capacitive” branches are reintroduced in the network and the remaining independent loops are identified. Once all independent loops have been identified, the mesh currents associated with these loops become the new unknown quantities. Let $I_m$ be the vector of the mesh currents. The branch currents are obtained from it through the mapping,

$$
I_b = M^T I_m.
$$

The loop matrix $M$ can be used to express in a discrete form Faraday’s law for each one of the loops. More specifically, it is,

$$
M \begin{bmatrix} V_{bl} \\ V_{bp} \end{bmatrix} = V_S
$$

where the vector, $V_s$, of external voltage source terms has been identified explicitly. Clearly, (5) and the relationship between mesh currents and branch currents given above, can be utilized in (4) to recast the discrete PEEC model in the following form,

$$
( M_{bl} (Z + j\omega L) M_{bl}^T + (j\omega)^{-1} M_{bp} P M_{bp}^T ) I_m = V_S
$$

This completes the development of the alternative, mesh analysis-based formulation of the PEEC interpretation of the EFIE statement of the electromagnetic problem. Its superior numerical stability as the operating frequency tends to zero is demonstrated in the following section through its application to the broadband electromagnetic analysis of high-speed interconnect structures.

APPLICATIONS AND CONCLUDING REMARKS

The proposed generalized PEEC model has been applied to the electromagnetic analysis of the interconnect structure of Fig. 1. The strip width and separation is 0.4 mm in the near end and 0.1 mm in the far end. The lengths of the near-end uniform portion, the transition portion and the far-end uniform portion are, respectively, 2, 3 and 1 mm. The conductivity of the strips is $5.8 \times 10^7$ S/m. The near-end ports are odd numbered (1,3,5,…,17) while the far-end ports are even numbered (2,4,6,…,18). First, the indefinite $Y$ parameters for the multiport are extracted. Indefinite scattering parameters are then generated using a reference impedance of 100 Ohm at all ports. Figure 2 depicts the magnitude of the calculated scattering parameters over the bandwidth $0.001 – 100$ GHz. It is noted that the erratic behavior observed in the values of the coupling coefficients at lower frequencies are attributed to numerical noise that corrupts the negligibly small values of the electromagnetic coupling at such frequencies. Coupling starts becoming noticeable in the GHz regime, while a richer behavior in the electromagnetic transmission and coupling manifests itself in the frequency range beyond 50 GHz where the interconnect length becomes comparable to the wavelength. This application demonstrates the ability of the proposed generalized PEEC methodology to calculate with electromagnetic accuracy the response of interconnect structures from dc to multi-GHz frequencies. On-going enhancements of this methodology include its acceleration through the application of pre-corrected FFT and adaptive integral method fast solution processes.
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REFERENCES