FORMATION OF A STATIONARY STATE OF STRIATIONS IN A VERTICALLY INHOMOGENEOUS IONOSPHERE

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Abstract

A new theory of a stationary state of striations is presented. Striations are treated as slowly varying with the height wave-guides that trap upper-hybrid resonance (UHR) oscillations. Modes with different discrete eigennumbers are retained in striations. Each mode interacts with a pump wave within a restricted range of heights. Thus the vertical ionospheric inhomogeneity determines the amplification of UHR oscillations. Anomalous absorption of the pump wave associated with the excitation of UHR oscillations in striations is calculated. Typical transversal scales and increase of the electron temperature in striations are estimated.

INTRODUCTION

One of the most interesting physical phenomena associated with the HF heating of the ionosphere is the formation of striations - elongated plasma depletions [1]. According to the theory striations are created as a result of conversion of the EM pump wave into UHR oscillations on small scale plasma irregularities [2]. UHR oscillations are trapped in striations and increase their amplitude due to the resonance (thermal parametric) instability. The mechanism of amplification of UHR oscillations and the formation of the stationary state of striations is a complicated theoretical problem. The existing modern theories treat this problem taking into account only the transversal plasma inhomogeneity [3, 4]. At the same time it is well-known that the longitudinal inhomogeneity of plasma changes the process of the waves interaction. The instability instead of absolute becomes of a convective type thus restricting the amplitude of the growing waves.
We present a new approach to the investigation of the stationary state of striations in which not only transversal but also longitudinal inhomogeneity of plasma is included into consideration. It is shown that such inhomogeneity plays an important role in the formation of a stationary state of striations. In our theory each striation is treated as a waveguide for UHR oscillations. The height variation of such a waveguide is slow enough to compare with the scale determined by the typical longitudinal wave number of UHR oscillations. It causes significant simplification of the problem and allows to use well-known methods in the theory of irregular waveguides.

BASIC EQUATIONS

Equation for UHR oscillations in an inhomogeneous ionosphere is obtained with the help of a continuity equation for electrons and the Poisson equation [5]. In stationary conditions we arrive at the fourth order partial differential equation for electric potential describing interaction of UHR oscillations with a pump electromagnetic wave

$$\frac{\partial^4\Psi_1}{\partial x^4} + \frac{\partial}{\partial x}\frac{\omega_0(\omega_0 - i\nu_e) - \omega_{Pe}^2 - \omega_{He}^2}{\gamma_\perp v_{Te}^2}\frac{\partial\Psi_1}{\partial x} + \frac{\Omega^2}{\gamma_\perp v_{Te}^2}\frac{\partial^2\Psi_1}{\partial z^2} =$$

$$-\frac{\omega_{Pe}^2}{\gamma_\perp v_{Te}^2}\frac{\partial ln\, N}{\partial x}E_{0,x}\exp(-ik_0\sqrt{\epsilon_p}z) \qquad (1)$$

Here $\omega_{Pe} = (\frac{e^2N}{\epsilon_0 m})^{1/2}$ is the electron plasma frequency, m is the mass of electron, ω_{He} is the electron Larmour frequency, $\omega_{He} = eH_0/m$, H_0 is the Earth's magnetic field, the speed of light $c = 1$, $\gamma_\perp$ is the adiabatic coefficient, v_{Te} is the thermal speed of electrons, $N(x, z) = N_0 + n_{st}$ is the total plasma concentration, $N_0(z)$ is a background concentration, n_{st} is a variation of plasma concentration in a striation, ν_e is the collision frequency of electrons, $E_{0,x}(z)\exp i(\omega_0 t - k_0\sqrt{\epsilon_p}z)$ is the x-component of the EM pump wave, $k_0 = \omega_0$ is the vacuum wave number, ϵ_p is the dielectric permeability for the ordinary pump wave, $\Omega^2 = (\omega_0(\omega_0 - i\nu_e) - \omega_{Pe}^2)(1 - \omega_{He}^2/\omega_0^2)$. In deriving (1) we neglected thermal corrections in the longitudinal motion of electrons and also the term $\sim \frac{\partial lnN_0}{\partial z}\frac{\partial\Psi_1}{\partial z}$. The reason for this is two-fold: first, we discuss quasitransversal perturbations and, second, plasma is assumed to vary smoothly in the z-direction. Also it was assumed that perturbations are large-scale to compare with the electron Larmour radius $\rho_{He} = v_{Te}/\omega_{He}$. The frequency ω_0 is supposed to be large enough $\omega_0 \gg \omega_{He}$ and not too close to multiple electron cyclotron frequencies. The solution of this equation is presented as a combination of different transversal eigenmodes trapped in a striation

$$\Psi_1(x,z) = \sum_n b_n(z)\tilde{\Psi}_{1,n}(x,z) \tag{2}$$

Propagating inside of a striation along the magnetic field line these modes grow in magnitude due to interaction with a pump wave. The matching conditions between the longitudinal wave numbers of the pump wave and excited UHR oscillations are fulfilled at different altitudes (depending on the mode number) and within restricted height intervals. It is shown that these conditions determine maximal amplitudes of UHR eigenmodes. This result presents one of the main distinctions of our approach to compare with the vertically homogeneous model in which amplification of UHR waves is connected with collisions. According to our theory the matching conditions for the longitudinal wave numbers of the interacting waves in a vertically inhomogeneous ionosphere cause amplification of UHR oscillations in striations at a lower level than collisions. The corresponding equation that determines the growth of UHR oscillations follows from (1), (2)

$$\frac{d^2 b_n}{dz^2} + \left[k_0^2\epsilon_p + \frac{\omega_{Pe}^2}{\omega_{He}^2}\left(\frac{z-z_n}{L_z} + i\frac{\nu_e}{\omega_0}\right)D_n\right]b_n = -E_0\frac{\omega_{Pe}^2}{\omega_{He}^2}R_n\exp(-ik_0\sqrt{\epsilon_p}z), \tag{3}$$

where L_z is the vertical scale of the ionosphere, z_n is the height at which the exact matching condition between the longitudinal wave numbers takes place,

$$D_n = \frac{\int(\frac{\partial^2\tilde{\Psi}_{1,n}}{\partial x^2})^2\,dx}{\int(\frac{\partial\tilde{\Psi}_{1,n}}{\partial x})^2\,dx},\quad R_n = \frac{\int\frac{\partial\tilde{\Psi}_{1,n}}{\partial x}\frac{\partial^2 n_{st}}{N_0\partial x^2}\,dx}{\int(\frac{\partial\tilde{\Psi}_{1,n}}{\partial x})^2\,dx}.$$

Note that the efficiency of the waves transformation depends on the mode number. The smaller is the mode number of UHR oscillations trapped in a striation, the higher is the amplitude of the corresponding HF electrostatic field. This statement is confirmed by calculations. For the lowest modes the distribution of plasma in a cross-section of a striation can be modelled by a parabola. It allows us to find approximate analytical solutions for the lowest eigenmodes, their maximal amplitudes and attenuation scales.

HEATING OF ELECTRONS IN STRIATIONS

While propagating in a striation each eigenmode of UHR oscillations is attenuated due to collisions, the leakage of the energy from the waveguide and the transformation into EM wave. We estimate the role of these different factors. The collisions of electrons with heavy particles cause an increase of the electron temperature. This process in stationary conditions is determined by equation, see [1]

$$\frac{\partial}{\partial z}\left(\kappa_\parallel\frac{\partial T_e}{\partial z}\right) + \frac{\partial}{\partial x}\left(\kappa_\perp\frac{\partial T_e}{\partial x}\right) - \delta\nu_e(T_e - T_e^{(0)}) = -Q_T(z,x) \tag{4}$$

Here $\kappa_\parallel, \kappa_\perp$ are the longitudinal and the transversal components of the thermal conductivity $\kappa_\parallel = 5.9\frac{T_e}{m\nu_e}, \kappa_\perp = 1.8\frac{T_e\nu_e}{m\omega_{He}^2}$, δ is the averaged fraction of energy lost by an electron in collisions with ions and neutrals, $T_e^{(0)}$ is the background temperature of electrons, T_e is the temperature of electrons inside a striation heated by UH waves, the energy source $Q_T(z,x)$ takes the form $Q_T(z,x) = \frac{\sigma_\perp}{N^{(0)}}|E_\perp(z,x)|^2$, where $\sigma_\perp$ is the transversal conductivity, $E_\perp$ is the transversal component of the electric field. It is argued that the most efficient heating is produced by the lowest modes of UHR oscillations. Note that during day-time the typical scale of attenuation associated with collisions in the waveguide is only of the order of a few hundred meters which is much less than the distance between the UHR level and the reflection height of the pump wave. It means that the heating source is more localized along the magnetic field line that was assumed before [3].

According to experimental data in day-time conditions the enhancement of the electron temperature is not too strong [6]. In this case it is convenient to introduce as an independent variable a reduced electron temperature integrated along the magnetic field line. The equation for such variable is a second order ordinary differential equation with the source in the right-hand side. This source determines the heating produced by different UHR modes. The maximal amplitude of each UHR mode is proportional to the electric field of the pump wave at the heights of interaction. It is known that the electric field of the pump wave is strongly attenuated due to the excitation of UHR oscillations. This effect is included into consideration in such a manner that allows us to take into account subsequent interaction of a pump wave with different eigenmodes of UHR oscillations trapped in striations.

ANOMALOUS ABSORPTION OF THE PUMP WAVE

To analyse the magnitude of anomalous absorption we start with two coupled equations in the geometric optics approximation for the interaction of the pump wave with UHR oscillations. This system is reduced to one equation of the second order that determines the attenuation of the pump wave due to the excitation of different eigenmodes of UHR oscillations trapped in striations. Equation describing interaction with a single UHR mode in striations can be presented in the form

$$\frac{d^2\tilde{E}_0}{dz^2} - \left[T_n - \frac{1}{4}\left(\frac{d\Delta\varphi}{dz}\right)^2 + \frac{i}{2}\frac{d^2\Delta\varphi}{dz^2}\right]\tilde{E}_0 = 0, \quad (5)$$

where $\tilde{E}_0 = E_0 \exp(i\frac{\Delta\varphi}{2})$, $\Delta\varphi$ is the phase variation along the z-axis that corresponds to the difference of the longitudinal wave numbers of the pump wave and UHR oscillations. For a parabolic model of plasma distribution in a striation

$$T_n = \frac{k_0}{4k_n\sqrt{<\epsilon_p>}} < \left(1 - \frac{x^2}{l_\perp^2}\right)\frac{\partial\tilde{\Psi}_{1,n}}{\partial x} > \frac{\omega_{Pe}^2}{\omega_{He}^2}\left(\frac{n_m}{N_0}\right)^2 \frac{\rho}{l_\perp^2} B_n, \quad (6)$$

where $l_\perp$ is a scale of a parabolic depletion, n_m is the maximal variation of plasma concentration in a striation, the brackets $<>$ mean space averaging across the magnetic field line, k_n is a longitudinal wave number of UHR oscillations, B_n is a numerical factor of the order of unity. The obtained above equation is similar to equation describing the convective type instability. Only in our case the decreasing solution should be selected. The magnitude of the effect depends on the scale that is determined by the matching conditions between longitudinal wave numbers of the interacting waves. There are several significant distinctions of our results from the previous calculations of anomalous absorption, see e.g. [7]. First, we take into account that the real longitudinal scale that determines coupling of waves is proportional but not equal to the vertical scale of the ionosphere. Second, the distribution of the electric field of UHR oscillations in a cross-section of a striation is included in our calculations. Third, it follows from our results that the anomalous absorbtion of the pump wave depends in the explicit form on the transversal scale of striations.

PLASMA CONCENTRATION IN STRIATIONS

To find parameters of striations in a stationary state a two dimensional model is used. We start with the continuity equations for electrons and ions and equation for the temperature of electrons. For simplicity it is assumed that the electron temperature in striations is not strongly enhanced. In addition to the height integrated reduced electron temperature as a second independent variable a height integrated reduced plasma concentration is used. As a result two coupled ordinary differential equations for plasma distribution and electron temperature in a cross section of a striation are obtained. Equation describing excitation of different UHR eigen modes should be added to this system. The analysis of the obtained equations allows to find parameters of striations. It is shown that striations with different reduced plasma concentration and different temperature enhancements are present in stationary conditions.

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