FINITE-DIFFERENCE ANALYSIS OF A WAVEGUIDE SYSTEM

Akira Komiyama

Osaka Electro-Communication University
Hatsu-cho Neyagawa-shi 572-8530, Japan
E-mail:komiyama@isc.osakac.ac.jp

ABSTRACT

Discretizing the wave equation for a TE wave in a one-dimensional ordered waveguide system composed of identical slab cores by grids a set of second-order differential equations for a lattice with equal grid spacing is derived. The equations are transformed into a matrix eigenvalue problem and the distribution of eigenvalues is numerically obtained. By comparing the results obtained with the results by the coupled mode theory and the results by the exact theory it is shown that the method allows accurate calculations of the modal density of states of a large-scale waveguide system.

INTRODUCTION

The localization of mode waves in a disordered waveguide system composed of randomly different cores in size has been treated by the coupled mode theory[1,2]. A simple description of localized modes has been obtained. However, since the coupled mode theory gives the apporoximate description of eigenmodes in a waveguide system, it is expected that the results by the coupled mode theory deviate from the results obtained by solving exactly the wave equation. A number of methods for calculating band structures of photonic crystals, including the finite-difference method[3], the plane wave method[4] and the multipole method[5], are applicable to the mode analysis of a waveguide system.

In this paper, the finite-difference method is applied to calculations of the modal density of states of a one-dimensional ordered waveguide system composed of identical slab cores of equal spacing. It is shown that the method allows accurate calculations of the modal density of states of a large-scale waveguide system.

WAVEGUIDE SYSTEM

A one-dimensinal ordered waveguide system composed of N identical slab cores of equal spacing is assumed. A core width and a spacing between cores are denoted by a and d, respectively. For a TE wave the wave equation is given by

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k_0^2 n^2(x) E_y = 0 \tag{1}$$

where z is the distance along the waveguide axis, E_y is the y-component of the electric field, k_0 is the wave number in a free space and $n(x)$ is the refractive index along the x axis.

We are interested in effects of the disorder of the system on the crosstalk in an image fiber[6] and assume in our waveguide model that a core width is $5\mu m$ and the numerical aperture(NA) of the waveguide is 0.24. Then each core can support four TE modes at wavelengths in the red. Since the coupling between the highest order modes gives rise to the power transfer between neighbouring cores in an image fiber we treat only the coupling between the TE_3 modes.

COUPLED MODE THEORY

The coupled mode equations for the one-dimensional ordered waveguide system are given by

$$\frac{dc_n}{dz} = -j\beta c_n - j\kappa(c_{n-1} + c_{n+1}) \tag{2}$$

where c_n is the amplitude of the mode in the n-th core and β is the propagation constant of the mode in the isolated core. κ is the mode coupling coefficient. For an infinite system we have the following dispersion relation,

$$\gamma = \beta + 2\kappa \cos(kd) \tag{3}$$

where γ is the propagation constant of an eigenmode and k is the wave number in the first Brillouin zone. The modal density of states can be analytically calculated from the dispersion relation(3), which is given by

$$D(\gamma) = \frac{1}{2\pi\kappa d}\frac{1}{\sqrt{1-u^2}}, \quad |u| < 1 \tag{4}$$

where

$$u = \frac{\beta - \gamma}{2\kappa}. \tag{5}$$

The region of propagation constant can be separated into a region where eigenmodes exist and a region where no eigenmode exists. Modes are densly distributed in the band region with its center at β and with the width of 4κ. Many modes appear near the edges of the propagation constant band and the density of states diverges at the band edges as shown in Fig.1. The density of states has symmetry with respect to $\beta(u = 0)$.

EXACT DISPERSION RELATION

The exact dispersion relation can be derived by imposing a quasi periodic condition on the wave equation(1).

$$2pq\cos(kd) + (p^2 - q^2)\sin(pa)\sinh(q(d-a)) - 2pq\cos(pa)\cosh(q(d-a)) = 0 \tag{6}$$

where

$$p = \sqrt{k_0^2 n_1^2 - \gamma^2}, \tag{7}$$

$$q = \sqrt{\gamma^2 - k_0^2 n_2^2}. \tag{8}$$

n_1 and n_2 are refractive indices of a core and a cladding, respectively. The center and the boundary of the Brillouin zone($k = 0$ and $k = \pi/d$) give the edges of the propagation constant band. The dependence of edge positions on the spacing between cores is shown in Fig.2. The wavelength is assumed to be $0.633\mu m$. Solid lines indicate the results by the dispersion relation(6) and dotted lines the results by the coupled mode theory. Regions between solid lines or dotted lines are propagation constant bands where eigenmodes exist. The propagation constant band by the coupled mode theory deviates from the exact band with narrowing the spacing between cores. The dispersion relation(6) shows that the density of states diverges at the band edges.

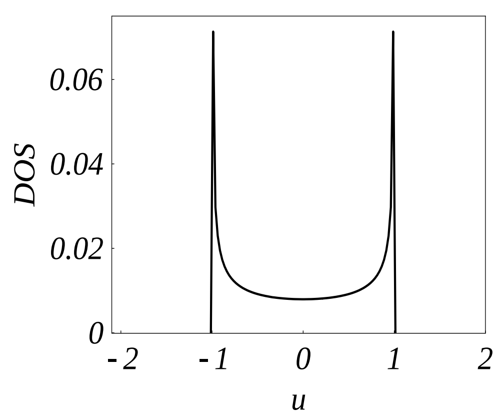

Fig.1 Density of states of an ordered waveguide system, which is the theoretical result by the coupled mode theory.

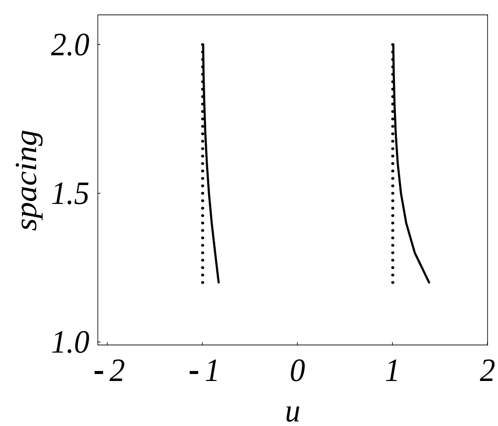

Fig.2 Dependence of edge positions of the propagation constant band on the spacing between cores. The ordinate indicates the ratio d/a.

FINITE-DIFFERENCE ANALYSIS

Discretizing the wave equation(1) by grids along the x axis, a set of second-order differential equations for a lattice with equal grid spacing is derived, which is given by

$$\frac{d^2u_m}{dz^2} = -\frac{1}{\Delta x^2}u_{m-1} - \left(k_0^2 n_m^2 - \frac{2}{\Delta x^2}\right) u_m - \frac{1}{\Delta x^2}u_{m+1} \tag{9}$$

where

$$u_m = E_y(m\Delta x, z) \tag{10}$$

$$n_m = n(m\Delta x) \tag{11}$$

Δx is a spacing between grids along the x axis. The differential equations(9) are transformed into a matrix eigenvalue problem,

$$\gamma^2 U = KU \tag{12}$$

where U is an eigenvector associated with an eigenvalue γ^2 and the coefficient matrix K has a tridiagonal form. The integrated distribution of eigenvalues which gives the number of eigenvalues less than a value is determined by the method introduced by Dean and Martin[7] to study the vibrational properties of a disordered atomic system. The density of states can be obtained by differentiating the integrated distribution. Since it is difficult to incorporate the guided mode condtion in the equations(9), the electric wall condition is applied in the x direction. A spacing between the core and the electric wall is so large that effects of the electric wall on eigenvalues can be ignored.

The density of states of a waveguide system with a spacing of $d = 10\mu m$ is shown in Fig.3. The number of cores is assumed to be $N = 5000$. The system size is sufficiently large for calculations of the density of states. The grid spacing is taken to be $\Delta x = 0.005\mu m$. The results are in good agreement with the results by the coupled mode theory. A difference between heights of peaks at the band edges is caused by the grouping of eigenvalues. The density of states for $d = 6\mu m$ is shown in Fig.4. It is slightly unsymmetrical with respect to the center. The exact edges of the propagation constant band obtanied by the dispersion relation(6) are $u = -0.829$ and $u = 1.385$, respectively. In Fig.4 the band edges are positioned at intervals -0.875 to -0.85 and 1.375 to 1.4, respectively and their intervals include the exact band edges. This shows that we can calculate accurately the density states by the finite-difference method.

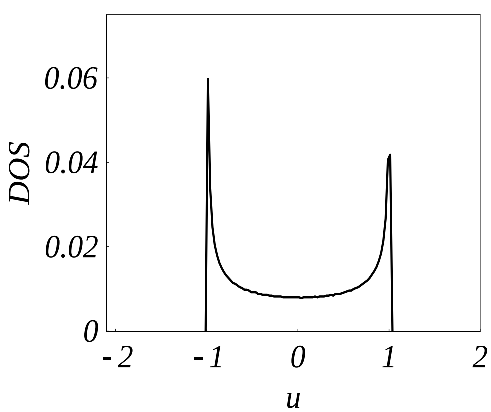

Fig.3 Density of states of the system with $d = 10\mu m$.

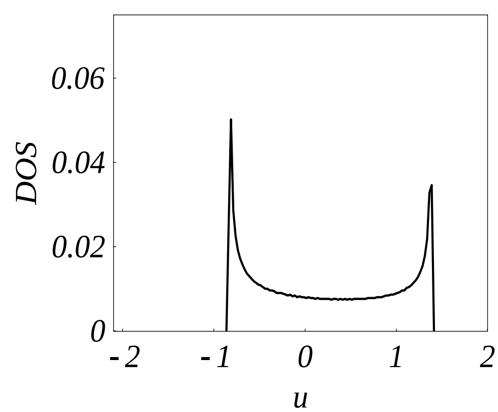

Fig.4 Density of states of the system with $d = 6\mu m$.

CONCLUSIONS

The finite-difference method has been applied to the calculations of the modal density of states of a one-dimensional ordered waveguide system and by comparing the results obtained with the results by the coupled mode theory and the results by the exact theory it has been shown that the method allows the accurate calculations of the density of states. The propagation constant band where eigenmodes exist deviates from the results by the coupled mode theory with narrowing the spacing between cores and the symmetry of the density of states with respect to its center is slightly broken. By the method we can treat accurately a disordered waveguide system.

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