SCATTERING OF A HERMITE–GAUSSIAN BEAM BY TWO CHIRAL CYLINDERS

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ABSTRACT

The scattering of a Hermite–Gaussian beam by two chiral cylinders is examined theoretically. By using the relation between the conventional Hermite–Gaussian beam and the multipoles with complex source point, the scattered fields are expressed as a superposition of the scattered multipole fields. Electromagnetic fields are expanded in terms of the cylindrical vector wave functions. The unknown expansion coefficients for the scattered field and the internal field are obtained by using the boundary condition. As numerical examples, the effects of the chirality and the distance between two cylinders on the scattered fields are examined.

INTRODUCTION

Chiral media have attracted much attention due to many associated interesting phenomena in optical and electromagnetic activities and their potential applications in various fields [1]. The phenomena of wave interaction with chiral structures have been examined theoretically by many researchers. The scattering of a plane wave incidence by an optically active sphere [2] or cylinder [3] has been analyzed by using the vector wave functions.

On the other hand, beamlike field is important from practical points of view because it is well known that the electromagnetic fields radiated from waveguide horns and laser cavities are approximately beamlike fields. To express the Gaussian beam rigorously, Deschamps [4] introduced the complex-source-point spherical wave and showed that this wave becomes a Gaussian beam under the paraxial approximation. In this paper, the scattering of a Hermite–Gaussian beam by two chiral cylinders is analyzed by use of the relation between the complex multipole fields and the conventional Hermite–Gaussian beam. The time factor $\exp(j\omega t)$ is suppressed.

HERMITE-GAUSSIAN BEAM EXPRESSED BY COMPLEX SOURCE POINTS

The electromagnetic fields radiated from the source point located at a complex point $(0, -jb)$ are represented by the Hankel function $H_0^{(2)}(kR)$ of the second kind of order 0. R is the complex distance from the source point $(0, -jb)$ to the point $(\widetilde{x}, \widetilde{z})$. The branch of R is chosen as shown in Fig. 1 such that $R \to$ as $\widetilde{z} \to \infty$ to satisfy the radiation conditions.

The higher complex beam proposed by Siegman [5] can be generated by a multipole field that is located at a complex source point as follows:

$$G_n = \left(\frac{\partial}{\partial \widetilde{x}}\right)^n G_0 = \left(\frac{\partial}{\partial \widetilde{x}}\right)^n H_0^{(2)}(kR) \tag{1}$$

The multipole field is related to the complex beam and the complex beam is related to the conventional one. By using this relation [6], the conventional beam ψ_n is expressed in terms of a superposition of a finite number of multipole fields under the paraxial approximation as

$$\psi_n \sim (-1)^n k(\pi/2)^{1/4} \left(\frac{n! w_0}{2}\right)^{1/2} e^{-kb} \sum_{p=0}^{[n/2]} \frac{2^{-p} w_0^{n-2p}}{p!(n-2p)!} G_{n-2p} \tag{2}$$

where $[n/2]$ is $n/2$ or $(n-1)/2$ depending on whether n is even or odd, respectively.

In this paper, we study the scattering of a Hermite–Gaussian beam by two chiral cylinders as shown in Fig. 1. The incident beam whose electric field is parallel to the cylindrical axis (an E-polarized wave) has the smallest spot size $w_0 (= \sqrt{2b/k})$ at the beam waist $(-x_0, -z_0)$. Two chiral cylinders, which are consisted of the cylinder #1 with radius a_1 located at $(d_1, 0)$

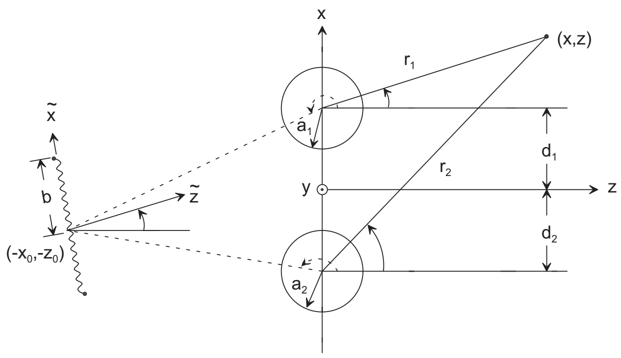

Fig. 1. Geometry of the scattering of a Hermite–Gaussian beam by two chiral cylinders

and the cylinder #2 with radius a_2 located at $(-d_2, 0)$, have relative permittivity ε_{ri}, relative permeability μ_{ri}, and chirality parameter κ_i $(i = 1, 2)$, respectively. The angle θ_0 is the angle between the propagation axis of the incident beam and z axis.

We investigate the scattered field for the multipole field G_n, since the conventional beam is expressed in terms of the multipole fields as shown in Eq. (1).

Let the cylindrical vector wave functions $\mathbf{N}_m^{(p)}(k, \mathbf{r}_i)$ and $\mathbf{M}_m^{(p)}(k, \mathbf{r}_i)$ be defined as

$$\mathbf{M}_n^{(p)}(k, \mathbf{r}_i) = \nabla \times \left[\widehat{y} f_n^{(p)}(k, \mathbf{r}_i)\right] \tag{3}$$

$$\mathbf{N}_n^{(p)}(k, \mathbf{r}_i) = \frac{j}{k} \nabla \times \mathbf{M}_n^{(p)}(k, \mathbf{r}_i) \quad p = 1, 2 \tag{4}$$

where $\widehat{y}$ is the unit vector in the y direction and the functions $f_n^{(p)}(k, \mathbf{r}_i)$ are defined by

$$f_n^{(p)}(k, \mathbf{r}_i) = Z_n^{(p)}(kr_i) \exp(jn\theta_i) \tag{5}$$

where

$$Z_n^{(1)}(kr_i) = J_n(kr_i) \tag{6}$$

$$Z_n^{(2)}(kr_i) = H_n^{(2)}(kr_i) \tag{7}$$

and $J_n(kr_i)$ is the Bessel function of the first kind and $H_n^{(2)}(kr_i)$ is the Hankel function

The incident electromagnetic fields are given by using the addition theorem for the Bessel functions in the coordinate system (r_i, θ_i) as

$$\mathbf{E}^i(k, \mathbf{r}_i) = \widehat{y} G_n = \frac{-j}{k} \begin{cases} \displaystyle\sum_{m=-\infty}^{\infty} \alpha_m(i, n) \mathbf{N}_m^{(2)}(k, \mathbf{r}_i)\,, & |\rho_{i0}| < r_i \\ \displaystyle\sum_{m=-\infty}^{\infty} \beta_m(i, n) \mathbf{N}_m^{(1)}(k, \mathbf{r}_i)\,, & |\rho_{i0}| > r_i \end{cases} \tag{8}$$

$$\mathbf{H}^i(k, \mathbf{r}_i) = \frac{j}{\omega\mu_0} \begin{cases} \displaystyle\sum_{m=-\infty}^{\infty} \alpha_m(i, n) \mathbf{M}_m^{(2)}(k, \mathbf{r}_i)\,, & |\rho_{i0}| < r_i \\ \displaystyle\sum_{m=-\infty}^{\infty} \beta_m(i, n) \mathbf{M}_m^{(1)}(k, \mathbf{r}_i)\,, & |\rho_{i0}| > r_i \end{cases} \tag{9}$$

where the coefficients $\alpha_m(i, n)$ and $\beta_m(i, n)$ are given explicitly by

$$\alpha_m(i, n) = \left(\frac{jk}{2}\right)^n \sum_{p=0}^{n} \binom{n}{p} J_{m+n-2p}(k\rho_{i0}) \exp[-j(m + n - 2p)\phi_{i0} + j(n - 2p)\theta_0] \tag{10}$$

$$\beta_m(i,n) = \left(\frac{jk}{2}\right)^n \sum_{p=0}^{n} \binom{n}{p} H^{(2)}_{m+n-2p}(k\rho_{i0}) \exp[-j(m+n-2p)\phi_{i0} + j(n-2p)\theta_0] \quad (11)$$

and

$$\rho_{i0} = [(\pm d_i + x_0 + jb\sin\theta_0)^2 + (z_0 + jb\cos\theta_0)^2]^{1/2} \quad (12)$$

$$\phi_{i0} = \tan^{-1}\frac{\pm d_i + x_0 + jb\sin\theta_0}{z_0 + jb\cos\theta_0} \quad (13)$$

$$r_i = [(x \mp d_i)^2 + z^2]^{1/2} \quad (14)$$

$$\theta_i = \tan^{-1}(x \mp d_i)/z \quad (15)$$

$\binom{n}{p}$ is the binomial coefficient. $\pm(\mp)$ signs correspond to $i = 1$ and 2, respectively. The scattered fields by the cylinder #i are expressed as

$$\mathbf{E}^s(k,\mathbf{r}_i) = \sum_{m=-\infty}^{\infty} \left\{\alpha^s_m(i,n)\mathbf{P}^{(2)}_m(k,\mathbf{r}_i) + \beta^s_m(i,n)\mathbf{Q}^{(2)}_m(k,\mathbf{r}_i)\right\} \quad (16)$$

$$\mathbf{H}^s(k,\mathbf{r}_i) = \frac{k}{j\omega\mu_0} \sum_{m=-\infty}^{\infty} \left\{\alpha^s_m(i,n)\mathbf{P}^{(2)}_m(k,\mathbf{r}_i) - \beta^s_m(i,n)\mathbf{Q}^{(2)}_m(k,\mathbf{r}_i)\right\} \quad (17)$$

where $\mathbf{P}^{(p)}_m(k,\mathbf{r}_i)$ and $\mathbf{Q}^{(p)}_m(k,\mathbf{r}_i)$ are defined as follows:

$$\mathbf{P}^{(p)}_m(k,\mathbf{r}_i) = \mathbf{M}^{(p)}_m(k,\mathbf{r}_i) + j\mathbf{N}^{(p)}_m(k,\mathbf{r}_i) \quad (18)$$

$$\mathbf{Q}^{(p)}_m(k,\mathbf{r}_i) = \mathbf{M}^{(p)}_m(k,\mathbf{r}_i) - j\mathbf{N}^{(p)}_m(k,\mathbf{r}_i) \quad (19)$$

The scattered fields given by (16) and (17) can be expressed with respect to the coordinate system (r_l, θ_l) of the lth cylinder by using the addition theorem in order to impose the boundary condition at $r_l = a_l$.

The constitutive relations for a chiral medium are given by [1]

$$\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} - j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{H} \quad (20)$$

$$\mathbf{B} = j\kappa\sqrt{\varepsilon_0\mu_0}\,\mathbf{E} + \mu_o\mu_r\mathbf{H} \quad (21)$$

In chiral medium, there exists two eigenvalues k^+_i and $-k^-_i$ given by

$$k^\pm_i = \omega\sqrt{\varepsilon_0\mu_0}\,(\sqrt{\varepsilon_{ri}\mu_{ri}} \pm \kappa_i) \quad (22)$$

Then, the electromagnetic field inside each chiral cylinder can be expressed in terms of the cylindrical vector wave functions with the argument $k^\pm_i$ as

$$\mathbf{E}^t(k,\mathbf{r}_i) = \sum_{m=-\infty}^{\infty} \left\{a_m(i,n)\mathbf{P}^{(1)}_m(k^+_i,\mathbf{r}_i) + b_m(i,n)\mathbf{Q}^{(1)}_m(k^-_i,\mathbf{r}_i)\right\} \quad (23)$$

$$\mathbf{H}^t(k,\mathbf{r}_i) = \frac{1}{j\omega\mu_i} \sum_{m=-\infty}^{\infty} \left\{(k^+_i + \omega\kappa\sqrt{\varepsilon_0\mu_0})a_m(i,n)\mathbf{P}^{(1)}_m(k^+_i,\mathbf{r}_i) - (k^-_i - \omega\kappa\sqrt{\varepsilon_0\mu_0})b_m(i,n)\mathbf{Q}^{(1)}_m(k^-_i,\mathbf{r}_i)\right\} \quad (24)$$

The magnetic fields can be expressed in the same way as the electric fields. The unknown expansion coefficients for the internal fields and the scattered fields are determined by the boundary conditions on the surface of each chiral cylinder.

NUMERICAL RESULTS

We consider the scattered near field of the lowest–order beam ($n = 0$) incidence whose beam waist is located at $(0, -\lambda)$. The wavelength λ is assumed to be 1.55μm, the smallest spot size of the lowest–order beam is $w_0 = 2\lambda$, and the refractive index is $\varepsilon_{r1} = \varepsilon_{r2} = \varepsilon_r = 4.0$. The relative permeability $\mu_{r1} = \mu_{r2} = \mu_r = 1.0$. The radius of each cylinder is $a_1 = a_2 = a = 0.5\lambda$. The truncation size in this example is chosen to be $m = \pm 30$. In what follows, the scattered fields are calculated at $z = \lambda$.

Figs. 2 (a) and (b) show the normalized scattered near fields by two chiral cylinders for $d_1 = d_2 = d = 1.6\lambda$ and $d = 2.5\lambda$, respectively. The chiral parameter κ is 0.1. When the distance between the cylinders becomes large, the effect of the direct scattering is dominant and the interaction between the cylinders is small. Figs. 3 (a) and (b) show the scattered near fields for the chiral parameter $\kappa = 0.3$. The other parameters are the same as those of Fig. 2. When the chiral parameter becomes large, the effect of the interaction between the cylinders is small.

CONCLUSIONS

The scattering of a Hermite–Gaussian beam by two chiral cylinders has been analyzed by use of the relation between the multipole fields and the conventional Hermite–Gaussian beam. The scattered field has been derived by expansion of the electromagnetic fields with the cylindrical vector wave functions. As numerical results, the effects of the chirality and the distance between two cylinders on the scattered fields have been examined.

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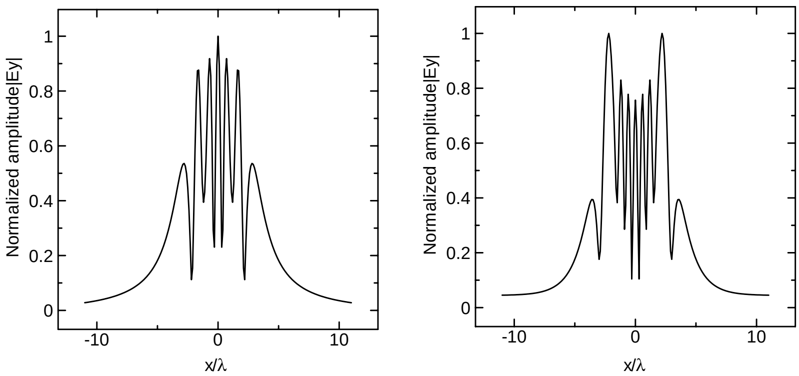
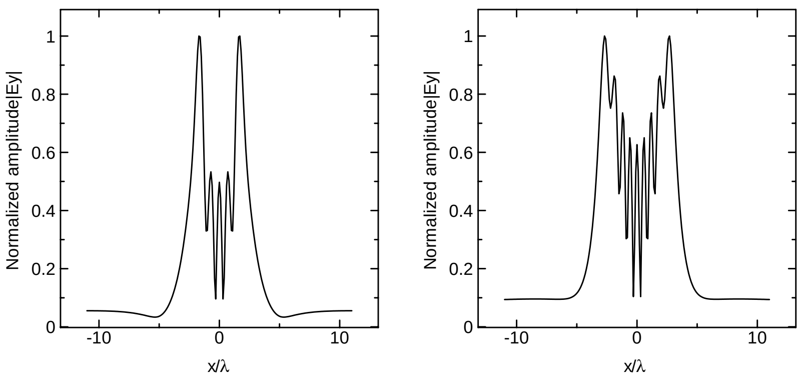


(a) $d = 1.6\lambda$ (b) $d = 2.5\lambda$

Fig. 2. Normalized scattered near fields for the chiral parameter $\kappa = 0.1$.



(a) $d = 1.6\lambda$ (b) $d = 2.5\lambda$

Fig. 3. Normalized scattered near fields for the chiral parameter $\kappa = 0.3$.