APPLICATIONS OF HOMOGENIZATION IN ELECTROMAGNETICS

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ABSTRACT

We review in this paper some of our recent work in electromagnetic homogenization together with our current research in this topic. There are two highlights in this paper. The first is surprising results that arise from effective media treatment of lattices containing cube-shaped particles. Such a lattice produces an homogenized medium that displays nearly the minimal possible polarizability per unit volume. The second highlight is a description of our recent work in modeling fibrous electromagnetic space sailcraft. The computation of the force and torque electromagnetically induced on these sails is described along with our use of homogenization.

INTRODUCTION

Homogenization is an active area of electromagnetics research, and has been so for quite some time. Examples of recent work in this exciting area include high impedance surfaces [1, 2], homogenization of woven fabrics [3], "negative" media (also called "left-handed metamaterials") [4, 5], metal-clad spheres containing nonlinear dielectrics [6], "wire media" [7] and microwave aquametry [8]. While not inclusive, this list does give an indication of the wide range of current and future applications for electromagnetic homogenization.

Homogenization in electromagnetics has been around for roughly a century and a half, which is a testament in itself to the successfulness and usefulness of this research topic. Some of the first applications were Mossotti's and later Clausius's work concerning the effective permittivity for weakly interacting dielectric particles [9-11].

In-between these two researchers was James Clerk Maxwell whom himself dabbled in homogenization. In fact, the wildly successful "Maxwell Garnett" formula [12] was actually derived and studied by J. C. Maxwell some 30 years earlier in his famous treatise [13]. Specifically, in Article 314, J. C. Maxwell computed the static "specific resistance" for a collection of finitely conducting and weakly interacting spheres. Later in Article 430, he computed the "coefficient of magnetic induction" (effective permeability) for a similar collection, but containing magnetic spheres. In both cases, the modern researcher would recognize his equations as the "Maxwell Garnett formula".[†] This wonderful equation has proven to be very accurate for weakly interacting particles – so much so that Lord Rayleigh referred to this as a "remarkable" formula [14].

Broadly speaking, all of the above-mentioned topics can be grouped into the two categories of (1) effective media description of composite materials, and (2) sheet description of materials. One aim of this paper is to describe our recent research activities in both of these categories.

EFFECTIVE MEDIA DESCRIPTION OF COMPOSITE MATERIALS

In the first area, we have investigated the quasistatic effective permittivity for (potentially) strongly interacting systems of complex-shaped dielectric and/or conducting particles. In other words, systems where the volume fraction of particles may be very large (or maximum) and the particles have complicated shapes, with corners or edges for example. These investigations have included (i) *T*-matrix solutions for multiphase lattices of spheres [15] and cylinders [16]; and moment method solutions for (ii) lattices of complex-shaped dielectric and conducting particles (cubes, "rounded-square cylinders, etc.) [17, 18], (iii) anisotropic lattices of conducting particles [19], and (iv) "pseudorandom" isotropic lattices of 2-D particles [20].

In the case of (ii), a particularly interesting result was observed for lattices of cube particles. As the volume fraction of particles is increased, we observed that due to mutual coupling the effective permittivity of the lattice actually decreased [18]. In fact, for highly conducting cubes the effective permittivity was reduced almost to the minimum possible polarization per unit volume as predicted from the Hashin-Shtrikman lower bound [21]. Measurements for quasistatic effective conductivity confirmed this prediction [18]. Shown in Fig. 1 is the apparatus that was used to measure the effective quasistatic conductivity for a simple cubic (SC) lattice of conducting cubes [18, 22].

Fig. 1 Experimental apparatus used to measure the quasistatic effective conductivity for a simple cubic lattice of conducting cubes.

Shown in Fig. 2 is the computed quasistatic effective permittivity for this SC lattice together with the measured data from the apparatus in Fig. 1 (differently sized Lexan boxes where used to discretely vary the volume fraction) [18, 22]. What is most apparent in this data is that the effective permittivity (conductivity) has been reduced almost to the minimum possible value, as mentioned earlier. This is opposite to the effect observed for spherical particles. Mutual interaction between particles is often associated with an *increase* in the effective permittivity (or other related quantities, such as quasistatic effective conductivity or permeability). We see here that mutual interaction between particles can significantly reduce the material's polarizability per unit volume. A description as to why this is occurring may be found elsewhere [18].

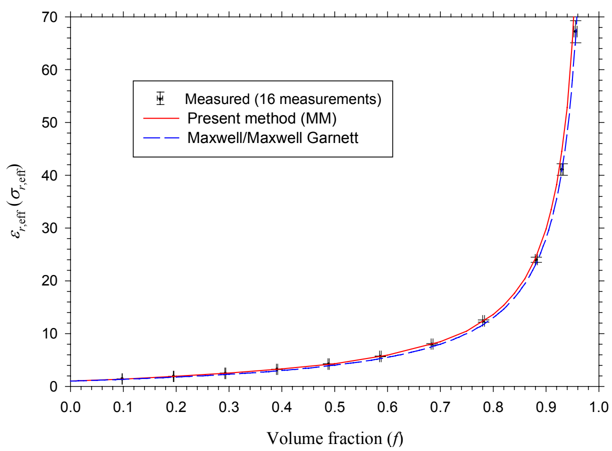


Fig. 2 Predicted quasistatic effective permittivity and measured effective conductivity for a simple cubic lattice of conducting cubes.

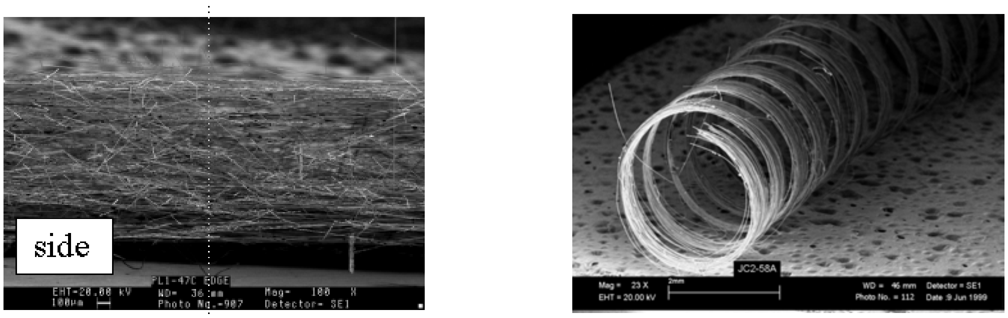
Also in the results of Fig. 2, we see that even though the microstructure of this material is very complicated, the effective permittivity can be easily and quite accurately predicted using the simple Maxwell/Maxwell Garnett formula. That is very convenient from a designer's standpoint. For example, materials with this microstructure could be designed and incorporated efficiently into optimization codes to design materials with tailorable electromagnetic characteristics. Potential applications include lightweight substrates for microwave-frequency circuits, perforated substrates that allow convective heat transfer in high power situations, and finely graded materials for non-reflective coatings or lenses, among others.

SHEET DESCRIPTION OF MATERIALS

In the second category of research mentioned in the Introduction, our activity has recently centered on the electromagnetic force and torque exerted on carbon fiber solar sails [23, 24]. Solar sails are ultra-lightweight spacecraft that use electromagnetic momentum to derive some or all of their propulsion. Near-term missions for such craft include Cosmos I, which is due for orbital launch in the spring 2002 (which would be the first solar sail flight) [25] and GEOSTORMS which is a mission sponsored by NASA, NOAA and the USAF to monitor solar wind [26]. Future missions are envisioned to be much more ambitious including those to planets in our solar system and those to outside, perhaps extending to the heliopause and beyond.

In this talk, we will describe our recent efforts in studying the effects of fiber microstructure on the behavior of the sailcraft. In particular, sails with chiral and/or anisotropic features that may provide additional degrees of control than what is normally possible with conventional sail materials such as Kapton or Mylar.

The sails we are studying are constructed from carbon fiber with a novel “microtruss” structure in which individual fibers are node-bonded by a carbon chemical vapor deposition process. The resulting material can be fabricated in extremely lightweight and thin sheets as illustrated in Fig. 3(a). The microstructure of the material can be tailored for specific applications that may require anisotropic or perhaps chiral features, as illustrated in Fig. 3(b). Our work in modeling these materials and equivalent sheet modeling techniques will be described.



(a) (b)

Fig. 3 Carbon fiber samples. (a) Microtruss sheet material approximately 1 mm thick with an areal mass density of 9 g/m^2, and (b) carbon fiber helix with an approximate 2-mm diameter.

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† To honor the great work of J. C. Maxwell, we suggest that the “Maxwell Garnett formula” be renamed the “Maxwell-Garnett formula” where the hyphenation indicates the contributions of both J. C. Maxwell and J. C. M. Garnett. This is somewhat ironic, however, in that the hyphenated form is often used to refer (incorrectly) only to the Garnett paper.