POLARISATION EFFECTS IN SURFACE AND VOLUME SCATTERING

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ABSTRACT

In this paper we review recent developments in scattering theory related to applications involving polarised electromagnetic waves. We concentrate on applications in random media where second and higher order moments of the scattered field are important. We split the discussion into surface and volume scattering effects and consider each separately. However, we employ a unified approach based on the construction of an 8x8 scattering coherency matrix, encompassiing current applications in Polarimetry and Polarimetric Interferometry. We then use this matrix as a template to explore the limitations of currently used scattering theories and highlight areas for possible future study.

INTRODUCTION

In this paper we consider the subject of scattering polarimetry [1]. This is to be distinguished from the more mature topic of propagation and specular polarimetry. While the Jones calculus is widely employed for the analysis of polarisation transformation due to propagation through cascaded devices, there is a relatively poor analytical appreciation of how to deal with the structure of depolarisation effects caused by wave scattering. This is despite a long history of depolarisation measurements in astronomy, remote sensing, atmospheric optics and surface roughness studies.

A key distinguishing feature of scattering polarimetry is the presence of depolarisation, to be distinguished from cross-polarization. While the latter is a deterministic effect, the former is fundamentally a stochastic process whereby incident radiation is made noise-like due to the randomness of a scattering volume. The classical methods for describing such phenomena are based on the degree of polarisation, defined from the Stokes vector. These may be formally connected to a 2 x 2 Hermitian wave coherency matrix [J], leading to the wave dichotomy and associated Hermitian eigenvalue problem. However, there has been a recent trend towards using active sensors (particularly polarized Radar and Lidar) for remote sensing purposes. These systems can fully populate the scattering matrices of polarimetry and this has lead to a more sophisticated understanding of the generalised relationship between depolarisation and scattering. This in turn has lead to improved methods for remote sensing using polarized waves. Here we review the implications of these developments for scattering theory.

COHERENCY MATRIX FORMULATION OF POLARIMETRY AND INTERFEROMETRY

A key development has been the mapping of the real 4x4 Mueller matrix into a 4 x 4 Hermitian coherency matrix. Figure 1 shows how this coherency matrix can be derived from Maxwells equations. Here we derive the covariance [C] and in the lower diagram we show schematically how [C] may then be transformed into 2 other equivalent forms [1], the 4 x 4 Mueller matrix [M] or 4 x 4 hermitian coherency [T]. All contain information about scattering amplitudes, amplitude ratios, relative phase and coherence. Unlike the Mueller matrix, [T] and [C] are positive semi-definite Hermitian with a real non-negative eigenvalue spectrum. These eigenvalues can be used to interpret the depolarisation properties of the medium. Further, the eigenvectors provide an analytical parameterisation of the ways in which systems can depolarise by using N-1 dimensional unitary groups [2,3]. For backscatter systems, the reciprocity theorem restricts N = 3 and hence SU(2) governs the possible types of depolarisation. For general scattering N = 4 and so SU(3) is the governing group. Fully characterising the way in which a system can depolarise waves is an important step towards the development of data inversion schemes for radar and optical remote sensing.

These techniques have recently been applied to measured data from surface and volume scatterers [4,5] but quantitative interpretation of the results requires support from improved scattering theories. For example, when we examine the polarisation structure of the classical approximation methods available we find widespread differences in their predictive properties.

$$\varepsilon(\underline{r}) = \langle \varepsilon \rangle + \varepsilon_f(\underline{r})$$

Fluctuations in permittivity

$$\nabla \times \nabla \times \underline{E} - k_m^2 \underline{E} = Q(\underline{r})\underline{E} \qquad \begin{cases} k_m^2 = \omega^2 \mu \langle \varepsilon \rangle \\ Q(\underline{r}) = \omega^2 \mu \varepsilon_f(\underline{r}) \end{cases}$$

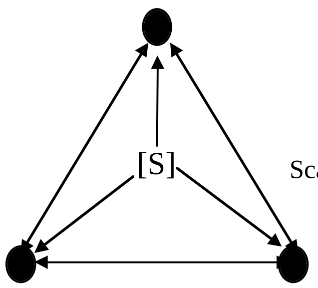
Scattering Sources

$$\underline{E}(\underline{r}) = \underline{E}_0 + \int_V \overline{\overline{G}}(\underline{r}, \underline{r}_1) Q(\underline{r}_1).\underline{E}(\underline{r}_1) d^3 \underline{r}_1$$

Scattered Field

$$\sigma_{ijkl} = \lim_{\substack{r \to \infty \\ A \to \infty}} \frac{4\pi r^2}{A} \frac{\langle E_i(\underline{r}) E_k^*(\underline{r}) \rangle}{E_j E_l^*} \longrightarrow [C] = \begin{bmatrix} \sigma_{XXXX} & \sigma_{XXXY} & \sigma_{XXYXV} & \sigma_{XXYY} \\ \sigma_{XXXY}^* & \sigma_{XYXY} & \sigma_{XYYX} & \sigma_{XYYY} \\ \sigma_{XXYX}^* & \sigma_{XYYX}^* & \sigma_{YXYX} & \sigma_{YXYY} \\ \sigma_{XXYY}^* & \sigma_{VHVV}^* & \sigma_{YXYY}^* & \sigma_{YYYY} \end{bmatrix}$$

[C] - 4 x 4 Covariance



Scattering amplitude matrix

[T] – 4 x 4 Coherency   [M] - 4 x 4 Mueller Matrix

Figure 1 : Coherency and Scattering Matrices in Polarimetry

Figure 2 shows how this coherency formalism may be extended to include polarimetric interferometry. Here we retain the polarimetric information from two ends of a baseline as sub-matrices [T_{11}] and [T_{22}] but also employ phase and coherence information from the off-diagonal 3x3 complex matrix [$\mathbf{\Omega}_{12}$]. Given that current measurement systems can fully populate these matrices, the challenge to surface and volume scattering theories is to provide analytical and numerical estimates of this full 8 x 8 matrix. Here we provide a review of the status of such efforts.

We show in figure 2 how the interferometric coherence and phase can be estimated for arbitrary polarisation vectors w. Also shown is coherence optimisation as an eigenvalue problem. The eigenvectors then locate low noise phase centres in the random medium. These can be used for inversion and imaging applications. Interpretation is then centred on the eigenvalue spectra of these matrices.

$$[P] = \left\langle \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix} \begin{bmatrix} \underline{k}_1^{*T} & \underline{k}_2^{*T} \end{bmatrix} \right\rangle = \begin{bmatrix} [T_{11}] & [\Omega_{12}] \\ [\Omega_{12}]^{*T} & [T_{22}] \end{bmatrix}$$

Interferogram Formation ↓

$$\left. \begin{aligned} s_1 &= \underline{w}_1^{*T} \underline{k}_1 \\ s_2 &= \underline{w}_2^{*T} \underline{k}_2 \end{aligned} \right\} \Rightarrow s_1 s_2^* = \begin{cases} \gamma = \dfrac{\left|\langle s_1 s_2^* \rangle\right|}{\sqrt{\langle s_1 s_1^* \rangle . \langle s_2 s_2^* \rangle}} & \text{coherence } 0 \le \gamma \le 1 \\ \phi = \arg\left(\underline{w}_1^{*T} \Omega_{12} \underline{w}_2\right) & \text{interferometric phase } \phi \end{cases}$$

Coherence Optimiser ↓

$$\left. \begin{aligned} [T_{22}^{-1}][\Omega_{12}]^{*T}[T_{11}^{-1}][\Omega_{12}]\,\underline{w}_2 &= \lambda \underline{w}_2 \\ [T_{11}^{-1}][\Omega_{12}][T_{22}^{-1}][\Omega_{12}]^{*T}\,\underline{w}_1 &= \lambda \underline{w}_1 \end{aligned} \right\} 0 \le \lambda = \gamma_{opt}^2 \le 1$$

Figure 2 : Extension of the Coherency Formalism to Polarimetric Interferometry

SURFACE AND VOLUME SCATTERING

In surface scattering for example, standard scattering theories provide for very restricted types of depolarisation. The vector small perturbation method is an example of a non-depolarising stochastic scattering system, setting three of the four eigenvalues to zero. This prediction is not matched by experimental observations, even for small roughness scales ks < 0.3, where classically the Bragg model is applied. A recent development has been the extended Bragg model [5], which expands the range of validity of the small perturbation model for backscatter problems by adding a depolariser with the restriction that it be reflection symmetric in the plane of polarisation. This model has been compared with microwave chamber data for surfaces and shows good agreement with the depolarisation properties up to ks = 1 [5]. Another popular surface model, the integral equation method, while providing improved copolar scattering coefficients, makes no predictions of surface depolarisation and hence is not yet suitable for use in depolarisation studies.

Fractal surface scattering models are also currently being developed as more realistic descriptors of natural surfaces than classical single or dual scale statistical models. However, scattering solutions are often restricted analytically to 1-D surface roughness profiles and this limits the nature of the depolarisation by always setting one of the eigenvalues to zero. This in turn limits the interpretation of the scattering behaviour of natural surfaces, even if ÒexactÓ numerical methods and multi-scale statistics are used for the solution.

Turning to volume scattering, polarisation plays an important role in the interpretation of scattering from clouds of non-spherical particles. The excellent review edited by Mischenko et al [6] provides an overview of the most recent numerical and experimental methods in this important subject. Significant progress has recently been made in predicting the polarisation structure of scattering from random particle clouds. These techniques provide fast averaging methods over particle size, shape and orientation distributions for full depolarisation calculations and are being applied to a range of problems in optics and microwave remote sensing [6].

While it is often sufficient to assume that all orientations of non-spheroidal particles are equi-probable, sometimes external magnetic, electrostatic or aerodynamic forces provide an axis of symmetry for nontrivial orientation distributions. In the former case depolarisation is a function of particle shape and size distribution and the scattering matrix is sparse, while in the latter case all 16 elements of the scattering matrix must be considered. Again practical interest centres largely on understanding the depolarising properties of volumes [3,6]. In particular, it is often important to ascertain the isotropy of the depolarisation across polarisation states as this impacts for example on the potential for polarisation diversity to provide vegetation structure information in radar remote sensing and enhanced imaging through turbid media in optics.

A second important development in recent years has been the remote sensing of volume scattering by employing interferometric techniques [7,8]. Here the correlation between particle shape distribution and spatial density fluctuations can be mapped by combining polarimetry with interferometry. For example, such a technique has recently been used to measure the thickness of a random layer of non-spherical particles above a rough surface [7]. It has also been shown that a similar method can provide information on the thickness of a slab of uniaxial scattering media, where particle orientation effects become important [8].

However, further advances in these techniques require improved scattering models, which encompass not only non-sphericity in particle shape but also better spatial density models. For example, in vegetation scattering for forestry applications, detailed fractal growth models are now coupled with particle shape diversity and Monte Carlo techniques to provide better predictions of coherent volume scattering. These and other trends offer the potential for further improved applications of scattering theory in remote sensing.

In this paper we provide examples of the polarisation structure of surface and volume scattering problems and highlight possible future trends.

REFERENCES

[1] S R Cloude, ÒPolarimetry in Wave Scattering ApplicationsÓ Chapter 1.6.2 in SCATTERING, Volume 1,Eds R Pike, P Sabatier, Academic Press, 2001, ISBN 0-12-613760-9
[2] S R Cloude, ÒA New Method for Characterising Depolarisation Effects in Radar and Optical Remote SensingÓ Proceedings of IEEE International Geoscience and Remote Sensing Symposium (IGARSS 2001), Sydney, Australia, July 2001
[3] S.R. Cloude, "Lie Groups in EM Wave Propagation and Scattering", Chapter 2 in Electromagnetic Symmetry, Eds. C Baum, H N Kritikos, Taylor and Francis, Washington, USA, ISBN 1-56032-321-3, pp 91-142, 1995
[4] Y Q Jin, S R Cloude, "Numerical Eigenanalysis of the Coherency matrix for a Layer of Random Non-Spherical Scatterers", IEEE Transactions on Geoscience and Remote Sensing , Vol. 32., pp 1179-1185 November 1994
[5] S R Cloude ÒEigenvalue Parameters for Surface Roughness StudiesÓ Proceedings of SPIE Conference on Polarization: Measurement, Analysis and Remote Sensing II, Volume 3754, Denver, Colorado, USA, July 1999
[6] M I Mishchenko, J.W. Hovenier, L.D. Travis (eds.), 2000, ÒLight Scattering by Nonspherical ParticlesÓ Academic Press, NY
[7] K.P. Papathanassiou, S R Cloude, ÒSingle Baseline Polarimetric SAR InterferometryÓ IEEE Transactions Geoscience and Remote Sensing, Vol 39/11, pp 2352-2363, November 2001
[8] S R Cloude, K P Papathanassiou, W M Boerner, ÒThe Remote Sensing of Oriented Volume Scattering Using Polarimetric Radar InterferometryÓ Proceedings of International Symposium on Antennas and Propagation, ISAP 2000, Fukuoka, Japan, August 2000, pp 549-552