Monte-Carlo simulation of surface clutter in Ground Penetrating Radars

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ABSTRACT

In this paper the surface clutter caused by scattering from a rough air-ground interface is analyzed. To determine statistical properties of the clutter we have used the Monte-Carlo approach. An ensemble of surface profiles with Gaussian probability distribution and a power-law spectrum for the surface correlation function is simulated numerically. For each realization of the interface profile the scattered field has been calculated using a small-slope approximation. Averaging of the scattered filed over ensemble of the surface realizations has been done numerically. Magnitude of the surface clutter as well its statistical properties (like correlation function) have been investigated for a variety of surface and ground parameters.

INTRODUCTION

Study of electromagnetic field scattering from the air-ground interface is of prime importance for theoretical analysis of Ground Penetrating Radar (GPR) performance. Scattered field is measured by GPR as clutter, which masks the response from buried targets and limits practically the available dynamic range of GPR [1]. In numerous previous studies of wave scattering from rough surfaces far-field characteristics of the scattered field have been investigated. Much less attention has been paid to the scattered field near the rough surface. Main reason for this is the difficulty to derive any analytical approximation for the scattered field. With the development of direct numerical methods for the scattering field computation this problem can be overcome. In this paper we investigate fluctuations of the scattered field near a rough interface between two dielectric media using a fast forward solver [2] and Monte-Carlo method [3,4].

FORMULATION OF THE SCATTERING PROBLEM AND ITS SOLUTION

We consider an interface, rough in one dimension and described by a profile function \( z = \zeta(x) \) (Fig. 1). The surface roughness is characterized by RMS height \( \sigma \) and the correlation function \( W(x) \). To achieve realistic results, we used the fractional Brownian motion model with a power-law spectrum for the surface correlation function. In this case the spatial spectrum of the correlation function can be represented by a band-limited power-law spectrum [5]:

\[
W(k) = \begin{cases} 
\frac{2\pi(s-1)k_{l}^{s-1}}{\kappa_{l}^{s}} k^{-s}, & \kappa_{l} \leq k \leq \kappa_{u}, \\
1 - \left(\frac{k_{l}}{k_{u}}\right), & 0, \\
\end{cases}
\]

(1)

where the exponent \( s \) is usually between 3 and 4, and \( k_{l} \) and \( k_{u} \) are the lower and upper cut-off wavenumbers, respectively. The choice of the power-law spectrum is motivated by recent studies [5-6], which show that the spectrum (1) is much more suitable for the description of natural surfaces than the Gaussian spectrum.
The medium above the interface is free space, while the medium below is a homogeneous lossy dielectric characterized by a real part of the dielectric permittivity $\varepsilon_r$ and ohmic losses $\sigma$.

A plane incident wave, described by the y-component of electric field, is assumed to impinge from the upper half-space, with an angle of incidence $\theta_{\text{inc}}$, measured from the z-direction.

The forward scattering problem is solved applying the small-slope approximation to a system of two simultaneous integral equations derived via the well-known extended boundary condition (EBC) approach [2]. The developed forward solver is much faster than direct numerical solvers based, e.g. on the Method of Moments. Limitation of the applicability area of the solver to relatively smooth surfaces seems to be not a problem while the solver is applied to realistic air-ground interfaces because for a wide variety of natural surfaces the surface slope remains relatively small. Due to these two circumstances the developed forward solver is well suited for the Monte-Carlo simulations of a natural surface clutter.

**NUMERICAL ANALYSIS AND RESULTS**

To examine the statistical properties of the field reflected from the air-ground interface (in other words, clutter) we use the Monte-Carlo approach, which employs three major steps: generation of an ensemble of surface profiles with the desired probability distribution and power spectrum; calculation of the reflected field for each realization of the surface; and calculation of statistical moments of the reflected field by numerical averaging over the whole ensemble. To generate an ensemble of statistically rough surfaces with pre-defined statistical properties we use the algorithm described in papers [3,4]. The main difference of our approach from previous studies is utilization in the Monte-Carlo simulations of a band-limited power-law spectrum (1). The following parameters of the power-law spectrum are used in simulations:

$$s = 3.5, \quad \kappa_i = \frac{0.1}{\lambda}, \quad \kappa_u = \frac{5}{\lambda},$$

where $\lambda$ is the wavelength of the incident field. The frequency of the incident wave is chosen as 100 MHz and the incidence angle is chosen equal to zero ($\theta_{\text{inc}} = 0^\circ$).

We have analyzed dependencies on RMS height $\sigma_h$ of the mean value of the reflected field $E_{\text{r,mean}}$

$$E_{\text{r,mean}} = \frac{1}{N} \sum_{n=1}^{N} \langle E_r(x_n) \rangle,$$

and that of the standard deviation of the reflected field $\sigma_{E_r}$ (which has a meaning of surface clutter), where
Here $E_r$ stands for the reflected field magnitude and the angle brackets $\langle \ldots \rangle$ mean ensemble averaging. These dependencies are plotted in Fig. 2. While the mean value of the reflected field decreases with an increase of $\sigma_h$, the fluctuations of the reflected field $\sigma_e$ increase. The dotted line in Fig. 2 shows the dependence of $\sigma_e$ predicted by the Born approximation. It can clearly be seen that already for a very small value of the surface RMS height ($\sigma_h = 0.03 \lambda$) the “exact” value of $\sigma_e$ deviates from the Born approximation. For the Gaussian surface power spectrum such a deviation has been observed for considerably larger values of RMS height ($\sigma_h = 0.05 \lambda$) [7].

\[ \sigma_e^2 \equiv \langle (E_r - \langle E_r \rangle)(E_r - \langle E_r \rangle) \rangle. \]  

(4)

Fig. 2. Mean value of the reflected field ($E_{r,\text{mean}}$) and surface clutter ($\sigma_e$) as functions of the surface RMS height $\sigma_h$. The elevation of the observation point above the mean position of the rough surface equals 0.33$\lambda$. The relative dielectric permittivity of the ground $\varepsilon_r$ equals 6, and the ground conductivity $\sigma$ equals 0.01S/m.

We have also found that the standard deviation of the reflected field $\sigma_e$ becomes larger than the mean value $E_{r,\text{mean}}$ at RMS height $\sigma_h \geq 0.065 \lambda$. It means that near the rough surface, the non-coherent component of the reflected field already starts to dominate the coherent component even for reasonably smooth surfaces. In other words, the magnitude of the surface clutter reaches already the mean value of the reflected signal for such smooth surfaces.

Both the mean value and the standard deviation of the reflected field increase with the real part of dielectric permittivity of the ground. However, while for small values of the ground dielectric permittivity (e.g. 4) the standard deviation of the reflected field $\sigma_e$ equals 1/7 of the field mean value $E_{r,\text{mean}}$, for large values of the ground dielectric permittivity (e.g. 20) $\sigma_e$ reaches a level of 1/3 for $E_{r,\text{mean}}$. It means that with an increase of the real part of the dielectric permittivity, the surface clutter magnitude increases relatively the mean value of the ground reflection.

Finally, the correlation function of the reflected field has been investigated. We have found that for very smooth surfaces ($\sigma_h = 0.01 \lambda$) the correlation function of the reflected field repeats the correlation function of the rough surface (Fig. 3). For larger values of the surface RMS height ($\sigma_h \geq 0.05 \lambda$) the field correlation function starts to deviate from the correlation function for the surface roughness. Taking into account the previous results, we can state that the field correlation function coincides with the correlation functions of the rough surface when the scattering is weak and the scattered field magnitude is linearly related to the Fourier spectrum of the rough surface profile (Born approximation). In practice this condition can be formulated as the condition under which the coherent component of the reflected field dominates the non-coherent component. As far as the correlation function of the reflected field phase is concerned, in all simulated scenarios it remains the same as the correlation function of the surface roughness.
CONCLUSION

A theoretical model for the surface clutter near a rough air-ground interface has been developed. Novelty of the model is the use of a band-limited power-law spectrum for the surface roughness and the use of the fast forward solver based on a small-slope approximation. Using this model properties of the surface clutter have been investigated. It has been found that the magnitude of the surface clutter increases with increase of the surface RMS height and exceeds the mean value of the ground reflection if the surface RMS height is larger than 0.065\(\lambda\) (that is, if the real part of dielectric permittivity is about 4). With the increase of the ground dielectric permittivity, the magnitude of the surface clutter increases relatively to the mean value of the ground reflection. For relatively smooth surfaces the correlation function of the clutter coincides with the correlation function of the surface roughness.

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REFERENCES