

# SEMIBLIND SEPARATION AND EQUALIZATION OF COCHANNEL SIGNALS IN AD-HOC NETWORKS

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Wireless ad-hoc networks are limited by a wide range of interfering signals, partially due to their ease of setup and growing popularity. We propose to use multiple antennas to enable separation of partially overlapping packets in multipath fading. In such an environment, a correct identification of the user of interest requires the use of known information, *e.g.*, a code or a short training segment. In bursty and asynchronous communications, instead of a single block of training, it is more robust to disperse the known symbols over the packet. A semiblind algorithm for the separation and equalization of the packet of the user of interest is proposed. The algorithm is based on a direct equalizer approach, combined with ideas from blind “mutually referenced equalizers” (MRE), and uses the available training-based information.

## 1. INTRODUCTION

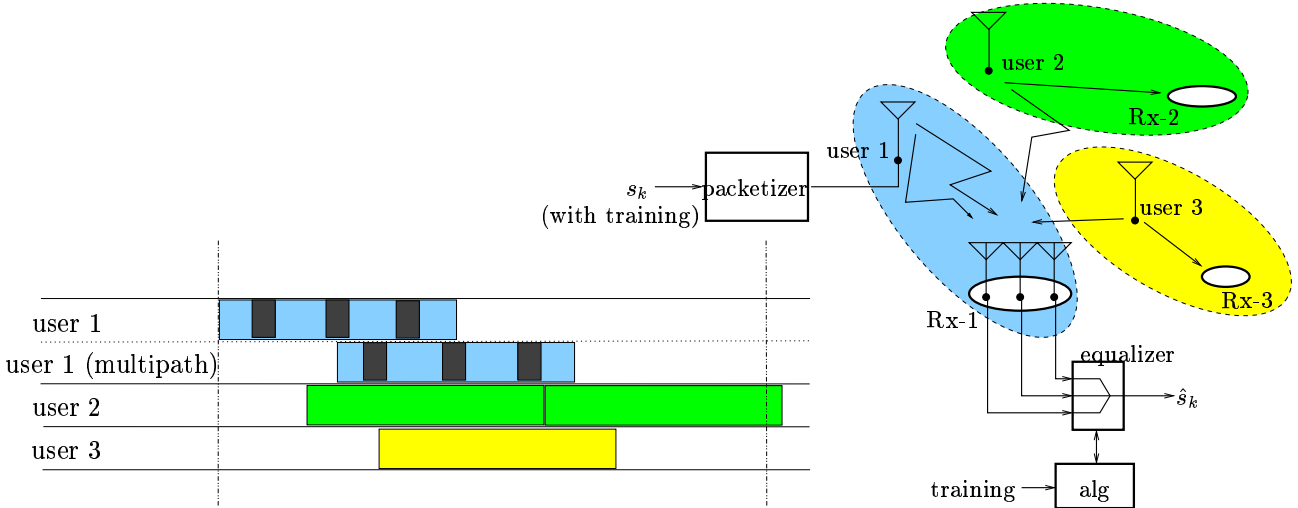
The increasing adoption of WLAN and all types of wireless ad-hoc networks operating in the unlicensed band already causes quality degradation and service unavailability. Sources of interfering signals are not limited to wireless network transceivers, but also Bluetooth devices, cellular phones, satellite links and even microwave ovens generate interference.

Wireless packet transmission is by nature burst-like, and subject to burst-like interference. By using a unique and short training sequence, each user is identifiable, in a multiuser communications scenario. Multiple antennas are used to separate users, and avoid packet loss and transmission repetitions. Additionally, the training is not placed mid-amble, but dispersed over the complete packet, as bursts from interfering users can appear and disappear at any point (Figure 1). It has been shown that dispersed training can lead to more accurate channel estimates in similar scenarios [1], as based on an analysis of the Cramer-Rao Bound (CRB). To maximize payload and thus spectral efficiency, training is used sparingly, and the known symbol information is combined with structural properties of the other received symbols, leading to a semiblind algorithm. There are many such algorithms (*e.g.* [2] for a comprehensive study), but their usage is limited to channel estimation, and requiring knowledge of all user’s training sequences in a joint estimation approach. The algorithm described in this paper performs source separation with equalization based on dispersed training symbols, only estimating the desired user, and only requiring knowledge of that user’s training. The algorithm is based on a “direct equalizer approach”, in which not the channel is estimated, but the signal at the output of the beamformer/equalizer. In this framework, it is straightforward to pose conditions on the resulting sequence (the training symbols), and to exploit the structure in the signal matrix due to the convolutive FIR channel model. The latter is based on ideas from “mutually referenced equalizers” (MRE) [3], in which a bank of equalizers is defined, such that the output of one is a shift of the next equalizer.

**Notation:**  $^T$  denotes a matrix transpose,  $^H$  the matrix complex conjugate transpose,  $^\dagger$  the pseudo inverse,  $\mathbf{0}$  an appropriately sized vector or matrix of all 0s, and  $\mathbf{1}$  an appropriately sized vector or matrix of all 1s.

## 2. DATA MODEL

For the user of interest, we consider a single-transmit / multiple-receive antenna (SIMO) model with a convolutive channel. The user transmits a digital symbol sequence  $[s_i]$  through a medium, which is received by an array of  $M \geq 1$  sensors. The received signals are sampled  $P \geq 1$  times faster than the symbol rate, normalized



**Figure 1.** Packet structure, and wireless ad-hoc communication scenario

to  $T = 1$ . During each symbol period, a total of  $MP$  measurements are available, which can be stacked into  $MP$ -dimensional vectors  $\mathbf{x}_i = [x_i^1, \dots, x_i^{MP}]^T$ . Assuming an FIR channel, we can model  $\mathbf{x}_i$  as the output of an  $MP$ -dimensional vector channel with impulse response  $[\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{L-1}]$ , where  $L$  denotes the channel length. In the interference- and noise-free case,  $\mathbf{x}_i$  is then given by  $\mathbf{x}_i = \sum_{k=0}^{L-1} \mathbf{h}_k s_{i-k}$ . After  $N$  observations, we can define the corresponding matrices  $\mathbf{X} = [\mathbf{x}_0 \ \mathbf{x}_1 \ \dots \ \mathbf{x}_N]$ ,  $\mathbf{H}_1 = [\mathbf{h}_0 \ \mathbf{h}_1 \ \dots \ \mathbf{h}_{L-1}]$ , and

$$\mathbf{S}_1 = \begin{bmatrix} s_0 & s_1 & \dots & s_N \\ s_{-1} & s_0 & \dots & s_{N-1} \\ \dots & \dots & \dots & \dots \\ s_{-L+1} & s_0 & \dots & s_{N-L+1} \end{bmatrix}, \quad (1)$$

such that we can write  $\mathbf{X}$  as a factorization  $\mathbf{X} = \mathbf{H}_1 \mathbf{S}_1$ . Denoting  $\mathbf{N}$  the appropriately sized AWGN matrix, and defining  $\mathbf{H}_q$  and  $\mathbf{S}_q$  similarly to  $\mathbf{H}_1$  and to  $\mathbf{S}_1$ , the extension to a multiuser system with  $Q$  users in the analysis window, synchronized to the user of interest, is straightforward and leads to

$$\mathbf{X} = \sum_{q=1}^Q \mathbf{H}_q \mathbf{S}_q + \mathbf{N} = \mathbf{H} \mathbf{S} + \mathbf{N}. \quad (2)$$

Furthermore, we will assume that  $\mathbf{H}_1$  is tall and full column rank, and  $\mathbf{S}_1$  wide and full row rank  $L$ , so that  $\mathbf{H}_1 \mathbf{S}_1$  is rank  $L$ . We assume that  $r = \text{rank}(\mathbf{H} \mathbf{S}) < MP$ , so that  $\text{row}(\mathbf{X})$  contains  $\text{row}(\mathbf{S}_1)$  ([4] provides further insight on oversampling and augmenting the rank of  $\mathbf{X}$ .) These conditions are essential to the existence of (zero-forcing) equalizers  $\mathbf{w}$  that can reconstruct rows of  $\mathbf{S}_1$  via  $\mathbf{w}^H \mathbf{X}$ , at least in the noise free case. To avoid equalizers in the null space of  $\mathbf{X}$ , in all algorithms to follow a preprocessing is necessary, consisting of a prewhitening and dimension reduction to the rank of  $\mathbf{X}$ . The processing consists of computing a singular value decomposition of  $\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}$ , and replacing  $\mathbf{X}$  by the first  $r$  rows of  $\mathbf{V}$ . Refer to [4, 5] for further details.

### 3. ALGORITHM

The proposed semiblind algorithm for direct equalization is presented in detail in Section 3.3, after introducing the usage of training information in Section 3.1 and "mutually referenced equalizers" (MRE) in Section 3.2. Although not hard to generalize, we assume from now on a simplified case where the channel has length  $L = 2$  symbols and no noise.

#### 3.1. Training and sequence shifts

Consider a finite block of data and define the  $MP \times N$  submatrices of  $\mathbf{X}$ ,  $\mathbf{X}^{(i)} = [\mathbf{x}_i \ \mathbf{x}_{i+1} \ \dots \ \mathbf{x}_{i+N-1}]$ , where the superscript  $(i)$  denotes an offset (we will consider  $i = 0$  and  $i = 1$ ). From (2),  $\mathbf{X}^{(i)}$  has a factorization

as  $\mathbf{X}^{(i)} = \mathbf{H}\mathbf{S}^{(i)}$ , where  $\mathbf{H}$  is the  $MP \times 2$  channel matrix defined before, and  $\mathbf{S}^{(i)}$  is a  $2 \times N$  submatrix of  $\mathbf{S}$ ,

$$\mathbf{H} = [\mathbf{h}_0 \quad \mathbf{h}_1] \quad , \quad \mathbf{S}^{(i)} = \begin{bmatrix} s_i & s_{i+1} & \cdots & s_{i+N-1} \\ s_{i-1} & s_i & \cdots & s_{i+N-2} \end{bmatrix}. \quad (3)$$

Let us denote the transmitted sequence by  $\mathbf{s} = [s_0 \quad s_1 \quad \cdots \quad s_N]$ , and the shifted and truncated version by  $\mathbf{s}^{(i)} = [s_i \quad s_{i+1} \quad \cdots \quad s_{i+N-1}]$ . The sequence  $\mathbf{s}$  contains  $L_T$  training symbols  $\bar{s}_i$ ,  $i = 0 \cdots L_T$ , in arbitrary positions, unique for each user. We define a selection matrix  $\mathbf{J}$  such that  $\mathbf{s}\mathbf{J}$  selects exclusively the training symbols of the sequence  $\mathbf{s}$ .  $\mathbf{J}$  is an  $N + 1 \times L_T$  augmented identity matrix, where all-zeros rows were inserted in the positions corresponding to unknown symbols and

$$\mathbf{s}_T = [\bar{s}_0 \cdots \bar{s}_{L_T}] = \mathbf{s}\mathbf{J}. \quad (4)$$

Similarly to (3,4) we define  $\mathbf{J}^{(i)}$  such that  $\mathbf{s}^{(i)}\mathbf{J}^{(i)} = \mathbf{s}_T$ , assuming that all training symbols are present in  $\mathbf{s}^{(i)}$ .

### 3.2. Mutually referenced equalizers

An equalizer can be viewed as a vector  $\mathbf{w}$  acting on  $\mathbf{X}^{(i)}$  to produce an output sequence  $\mathbf{z} = \mathbf{w}^H \mathbf{X}^{(i)}$ . Since  $\mathbf{S}^{(i)}$  has two rows, there are two different equalizers,  $\mathbf{w}_0$  and  $\mathbf{w}_1$ , to recover the source symbols at the specified delays,

$$\mathbf{w}_0^H \mathbf{x}_k = \mathbf{w}_1^H \mathbf{x}_{k+1} \Leftrightarrow \begin{cases} \mathbf{w}_0^H \mathbf{X}^{(i)} & = [s_i \quad s_{i+1} \quad \cdots \quad s_{i+N-1}] \\ \mathbf{w}_1^H \mathbf{X}^{(i)} & = [s_{i-1} \quad s_i \quad \cdots \quad s_{i+N-2}] \end{cases} \quad (5)$$

For the considered case of a channel length  $L = 2$ , the two delays of the inputs can be paired, which is the idea behind the MRE technique [3], and  $\mathbf{w}_1^H \mathbf{X}^{(1)} = [s_0 \quad s_1 \quad \cdots \quad s_{N-1}] = \mathbf{w}_0^H \mathbf{X}^{(0)}$ . The equalizers can be found in various ways, adaptively or using subspace intersections, *cf.* [5], essentially by solving

$$\min_{\mathbf{w}_0, \mathbf{w}_1} \left\| \begin{bmatrix} \mathbf{w}_0^H & \mathbf{w}_1^H \end{bmatrix} \begin{bmatrix} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{bmatrix} \right\|^2$$

with a suitable norm constraint on  $[\mathbf{w}_0^H \quad \mathbf{w}_1^H]$ . The solution is given by the left singular vector corresponding to the smallest singular value of  $\begin{bmatrix} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{bmatrix}$ . The corresponding right singular vector is the source sequence  $\beta[s_0 \quad s_1 \quad \cdots \quad s_{N-1}]$ , where  $\beta$  is an indetermined scaling. In a multiuser scenario, the MRE condition is not sufficient for the identification of the user of interest. For this, a known and user-specific segment is included in the transmitted signal, and equalizer outputs forced to match it.

### 3.3. Semiblind MRE implementation

The proposed MRE semiblind equalization algorithm is designed to work on the first  $L$  rows of the signal row space  $\mathbf{V}$ , obtained from an initial svd of the received signal matrix,  $\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}$ . In a multiuser scenario,  $r$  rows of  $\mathbf{V}$  corresponding to the signal subspace are selected. Let us define  $\mathbf{V}^{(i)}$  in a similar manner as  $\mathbf{X}^{(i)}$ . For the particular case of  $L = 2$  shifts,  $\mathbf{V}^{(0)}$  and  $\mathbf{V}^{(1)}$  are the  $L \times N$  shifted and truncated copies of  $\mathbf{V}$ .

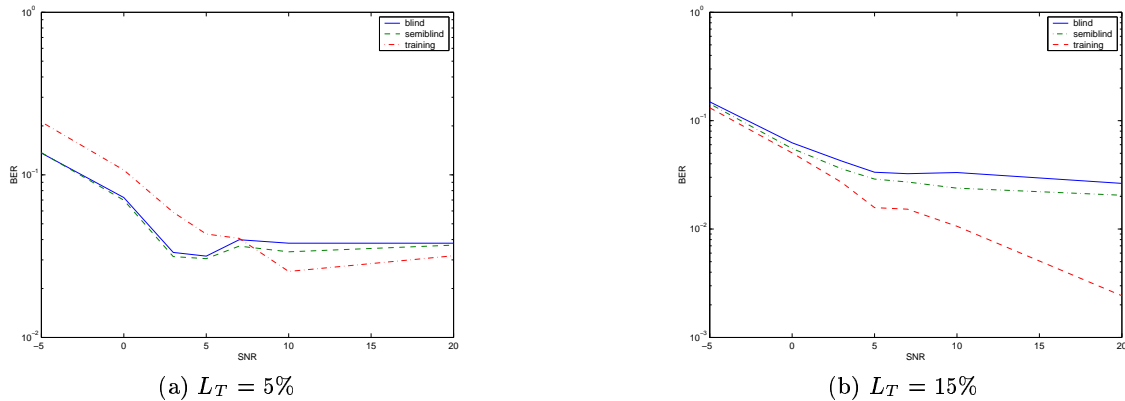
The semiblind MRE implementation must include both blind and training-based equalizer conditions, corresponding to the pairing of both equalizer outputs for the several shifts of the transmitted sequence. As in Section 3.2, the blind conditions are expressed by  $\mathbf{w}_0^H \mathbf{V}^{(0)} = \mathbf{w}_1^H \mathbf{V}^{(1)}$ . The training-based equalization conditions are expressed by  $\mathbf{w}_0^H \mathbf{V}_T^{(0)} = \mathbf{w}_1^H \mathbf{V}_T^{(1)} = [\bar{s}_0 \cdots \bar{s}_{L_T}]$ , where  $\mathbf{V}_T^{(0)} = \mathbf{V}\mathbf{J}^{(0)}$ , and  $\mathbf{V}_T^{(1)} = \mathbf{V}\mathbf{J}^{(1)}$ . The two conditions can be combined as a single condition

$$\mathcal{V} = \begin{bmatrix} \mathbf{V}^{(0)} & \left| \begin{array}{cc} \alpha \mathbf{V}_T^{(0)} & \mathbf{0} \\ \mathbf{0} & \alpha \mathbf{V}_T^{(1)} \end{array} \right. \\ -\mathbf{V}^{(1)} & \end{bmatrix},$$

where  $\alpha$  is a scaling for the training condition. With  $\mathbf{t} = [\mathbf{0} \quad \mathbf{s}_T \quad \mathbf{s}_T]$ , the problem can be posed as

$$[\mathbf{w}_0^H \quad \mathbf{w}_1^H] \mathcal{V} = \mathbf{t}. \quad (6)$$

If  $\mathcal{V}$  is full rank, the equalizer pair  $[\mathbf{w}_0 \quad \mathbf{w}_1]$  is the unique solution to Eq. (6). The design and placement of the training symbols such that the full rank condition is always observed is an interesting and open issue, in



**Figure 2.** Bit Error Rate results for  $Q = 2$

particular for the asynchronous multiuser case. In the presence of noise, the solution is  $[\mathbf{w}_0^H \quad \mathbf{w}_1^H] = \mathbf{t}\mathcal{V}^\dagger$ , and Eq. (6) is replaced by the minimisation of the cost function

$$\min_{\mathbf{w}_0, \mathbf{w}_1} \left\| [\mathbf{w}_0^H \quad \mathbf{w}_1^H] \begin{bmatrix} \mathbf{V}^{(0)} & \alpha \mathbf{V}_T^{(0)} \\ -\mathbf{V}^{(1)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \alpha \mathbf{V}_T^{(0)} & \mathbf{0} \\ \mathbf{0} & \alpha \mathbf{V}_T^{(1)} \end{bmatrix} - \mathbf{t} \right\|^2. \quad (7)$$

#### 4. SIMULATION RESULTS

The simulated wireless ad-hoc communications scenario included two interfering asynchronous users, transmitting with equal power, as shown in Figure 1. The results presented in Figure 2 are for  $N = 100$  samples, two antennas, and 200 Monte Carlo runs in a two-path equal power Rayleigh fading channel. In (a), 5% of the message payload was used for training, while in (b), it was increased to 15%. For short amounts of training, the semiblind equalizer outperforms the training-based equalizer and the blind MRE, while as training length and SNR increase, the training-based equalizer shows better performance. This is because  $\alpha$  was kept fixed to  $\alpha = 1$  in all simulations.

#### 5. CONCLUSIONS

A novel semiblind algorithm for packet separation in wireless ad-hoc networks, using dispersed training and mutually referenced equalizers was introduced. It presents the advantage of relying on less known information than the training-only solutions, and always outperforms the pure blind MRE. Nevertheless, the derivation of an optimal weighting for the combination of training and blind equalizers is a relevant open issue, as the semiblind equalizers do not always find to the best solution for a fixed value of  $\alpha$ .

#### References

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