PERFORMANCE EVALUATION OF DOA ESTIMATION ALGORITHMS IN DIGITAL IMPLEMENTATION

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ABSTRUCT

In this paper, we study properties and drawbacks of representative DOA (direction of arrival) estimation algorithms such as MUSIC, Root-MUSIC, and Unitary-ESPRIT by investigating the influence of quantization errors in digital operations for various cases. We also verify which algorithm is the most suitable for the implementation by finite word-length digital processors. First we examine the characteristics of DOA algorithms through some computer simulation. Then we study the internal quantization error of DOA algorithms, and derive the bit length required for the accurate DOA estimation.

INTRODUCTION

Recently eigendecomposition based algorithms such as MUSIC, Root-MUSIC [1][2], and Unitary-ESPRIT have been often used in DOA estimation. There have been many studies on examining the characteristics of the algorithms, but few quantitative investigations assuming digital implementation. Nowadays, FPGA has attracted attention for the property of low power and high-speed device which is based on parallel processing and fixed-point operation. Considering that DOA estimation algorithms will be implemented such digital devices in the near future, the algorithms should be quantitatively investigated.
In this paper, we study properties and drawbacks of representative DOA estimation algorithms by investigating the influence of quantization errors in digital operations for various cases. We also verify which algorithm is the most suitable for the implementation by finite word-length digital processors. First we examine the characteristics of DOA algorithms through some computer simulation. Here we change DOA angle, the number of snapshots, SNR, array elements, and the correlation of incident waves. Then we study the internal quantization error of DOA algorithms, and derive the bit length required for the accurate DOA estimation. We change a bit length of the internal operation of each algorithm in fixed-point operation, and examine the relation between estimation accuracy and bit length.

SPECIFICATIONS OF SIMULATION

Simulation on Characteristics of DOA Algorithms

In this simulation, as it is given in Fig.1, we used uniform linear array antenna. The algorithm used in the simulation is as follows: Spectral (original) MUSIC, Root-MUSIC, Unitary Spectral MUSIC, Unitary Root-MUSIC, ESPRIT, and Unitary ESPRIT, hereafter we use the abbreviations in Table 1. Here we change arrival angles, the number of snapshots, SNR, array elements, and the correlation of incident waves. We used the FB (forward-backward) space averaging technique [3] as a correlation suppressing method. Let θ_{DOA} and θ_{EST} denote the DOA angle and its estimation, respectively. The estimation error used for this evaluation is given by $\left|\theta_{DOA} - \theta_{EST}\right|$.

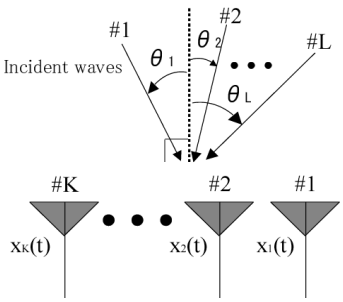

Fig.1. K-elements uniform linear array antenna

Table 1. Meanings of Abbreviation

Abbr.	Meanings
S	Spectral MUSIC
US	Unitary Spectral MUSIC
R	Root-MUSIC
UR	Unitary Root-MUSIC
E	ESPRIT
UE	Unitary ESPRIT

Simulation on The Internal Quantization Error of DOA Algorithms

We studied on the required bit length for the internal operations of each DOA algorithm. We examined the relation between estimation accuracy and bit length, when we change the bit length of the decimal part and the integer part respectively in a fixed-point operation from 4bits to 32bits. We calculated the value of MUSIC spectrum in a floating-point operation, because it sometimes becomes very large. Moreover, we used the values of $\sqrt{\ }$ and $\sin\theta$, $\cos\theta$, $\tan\theta$ which were transformed from the calculation results in PC into the fixed-point values.

RESULTS OF SIMULATION

Simulation on Characteristics of DOA Algorithms

First, we investigated the accuracy of DOA estimation when we changed arrival angles, the number of snapshots, and SNR to see how the above algorithms work. Fig.2 shows the characteristics when changing arrival angle. We verified that the estimation accuracy becomes worse in nearly ±90 degrees. Fig.3 shows the characteristics when changing the number of snapshots. Fig.4 shows the characteristics when changing SNR. We verified that the estimation accuracy could be improved by increasing the number of snapshots or SNR. These simulation conditions are given in Table 2.
The obtained characteristics by the algorithms using MUSIC spectrum are different from those by the algorithms estimating DOA numerically. In other words, the estimation accuracy of the algorithms using MUSIC spectrum depends on how precise we measure the steering vectors. We used the steering vectors whose minimum interval of value was 0.5 degrees that can be considered sufficiently precise.
Fig.5 shows the characteristics when changing SNR in the case where uncorrelated 3 waves come. Comparing with Fig.4, we verified that the estimation accuracy deteriorates by increasing the number of incident waves. Fig.6 shows the characteristic when changing SNR in the case where full-correlated 3 waves come. Compared with Fig.5, we verified that the estimation accuracy becomes worse when incident waves were correlated.

Table 2. Settings of Parameters in Simulation

	Fig.2.	Fig.3.	Fig.4.	Fig.5.	Fig.6.	Fig.7.	Fig.8.	Fig.9.
Array Elements	4	4	4	4	6	Change	Change	10
Sub-Array Elements					4		4	Change
Incident waves	1	1	1	3	3	2	2	2
Arrival Angle [deg] (Power)	Change (1.0)	−35.2 (1.0)	−35.2 (1.0)	−35.2(1.0) 10.2(1.0) 25.2(1.0)	−35.2(1.0) 10.2(1.0) 25.2(1.0)	15.2 (1.0) Change	15.2 (1.0) Change	15.2 (1.0) Change
Snapshots	300	Change	300	300	300	300	300	300
SNR [dB]	7	7	Change	Change	Change	7	7	7

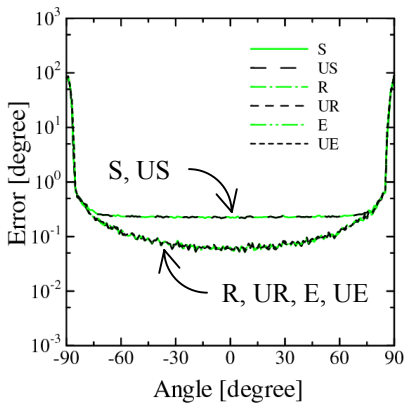

Fig.2. Estimation Error when changing arrival angle

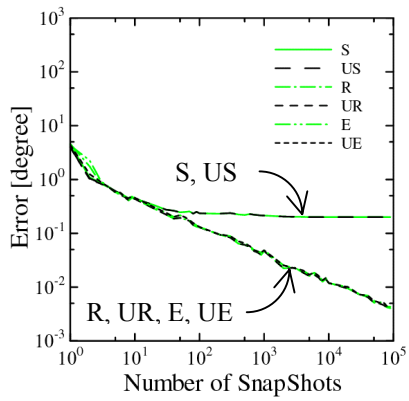

Fig.3. Estimation Error when changing the number of snapshots

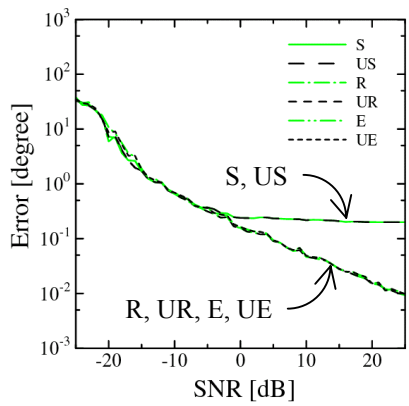

Fig.4. Estimation Error when changing SNR

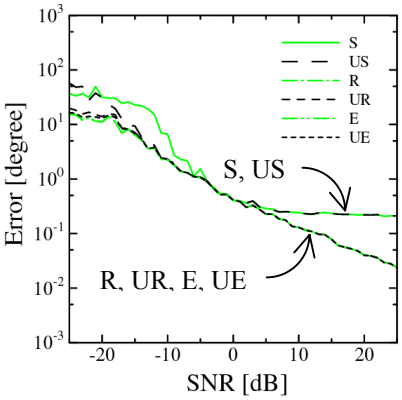

Fig.5. Estimation Error when changing SNR

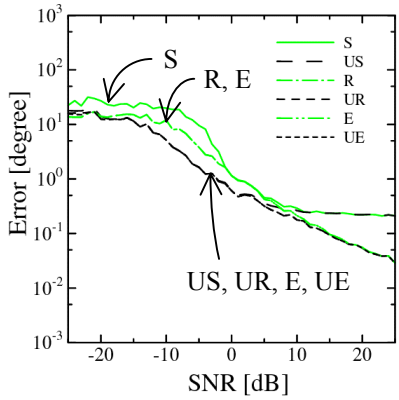

Fig.6. Estimation Error when changing SNR

Next, for the cases that two incident waves are uncorrelated or full-correlated, we investigate the relation between the number of array elements and the range of angles which can achieve enough accuracy (the range of angles which can be estimated within the error less than 1 degree). Fig.7 shows the characteristics in the case where uncorrelated 2 waves come. As shown in the previous simulation result, we confirmed the difference of the characteristics between the algorithms using MUSIC spectrum and those not using it (means, those estimating DOAs numerically). Note that the estimation accuracy can be improved in any algorithm by increasing the number of elements.

Especially in the case where full-correlated 2 waves come, we simulated two cases. One is that changing the number of array elements (the number of sub-array elements are fixed to 4 elements), and the other is that changing the number of sub-array elements (the number of array elements are fixed to 10 elements). Simulation results are given in Fig.8 and Fig.9, respectively. Consequently, we observed the followings: when increasing the number of array elements, the algorithms not employing unitary transform (non-unitary algorithms) cannot improve the DOA estimation accuracy at all, they just keep the accuracy at the same level, but the algorithms employing unitary transform (unitary algorithms) can improve the estimation accuracy. On the other hand, when increasing the number of sub-array elements, the non-unitary algorithms can improve the DOA estimation accuracy, while the unitary algorithms cannot improve the estimation accuracy. From this fact, we can remark that the estimation accuracy for the non-unitary algorithms and that for the unitary ones are dependent on the number of sub-array elements and the number of sub-arrays, respectively, in the case that full-correlated waves come.

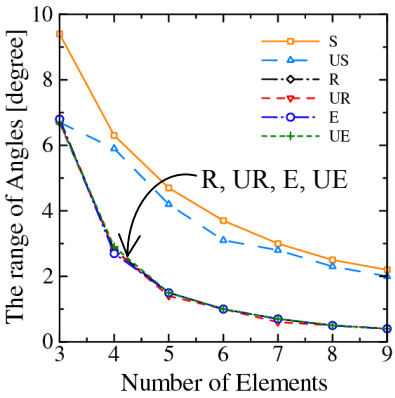

Fig.7. The range of angles when changing the number of array elements

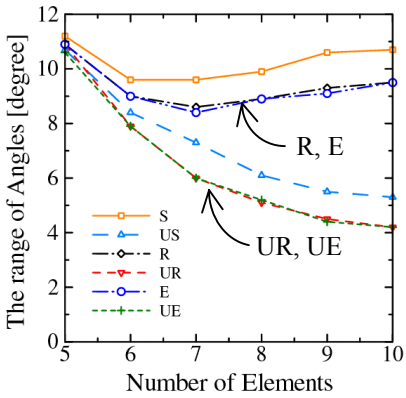

Fig.8. The range of angles when changing the number of array elements

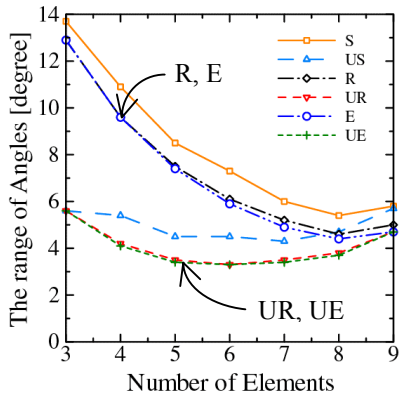

Fig.9. The range of angles when changing the number of sub-array

Simulation on The Internal Quantization Error of DOA Algorithms

We derive the bit length required for the accurate DOA estimation for implementation in fixed-point operation. Simulation condition is as follows: full-correlated 3 waves come because the estimation accuracy when 3 waves come is lower than that when 1 wave comes, and the estimation accuracy when correlated waves come is lower than that when uncorrelated waves come. Arrival angles are –5.2 degrees, 15.2 degrees and 35.2 degrees (intervals of 20 degrees). SNR is 7 dB because we assume actual (e.g. open air) environment. The number of snapshot is 300 because we consider that

300 snapshots are proper for the estimation accuracy and the amount of calculation.
Figure 10 shows the characteristics when changing a bit length of the decimal part in a fixed-point operation at 6 elements array and 4 elements sub-array (a bit length of integer part is fixed to 64 bits). We found that only unitary algorithms could estimate within the error of 1 degree with 16 bits, while non-unitary algorithms could not. So only for unitary algorithms, we simulated the characteristics when changing a bit length of the integer part in a fixed-point operation in the case where a bit length of decimal part was fixed to 16 bits. The result is given in Fig.11. We confirmed that we needed 12 bits to estimate within the error of 1 degree.
Figures 12 shows the characteristics when changing a bit length of the integer part, at 10 elements array and 6 elements sub-array (in Fig.12, a bit length of decimal part is fixed to 12 bits). In Fig.12, we confirmed, by increasing the number of array elements, the required bit length of Root MUSIC and Unitary Root-MUSIC were very large because the calculation amount of Root-MUSIC polynomial depended on the number of array element: 2(K−1).
As far as the result of our simulation, in the case where full-correlated 3 waves come at 6 elements array and 4 elements sub-array, at least 28 bits (16bits + 12bits) is necessary for the DOA estimation with sufficient accuracy.

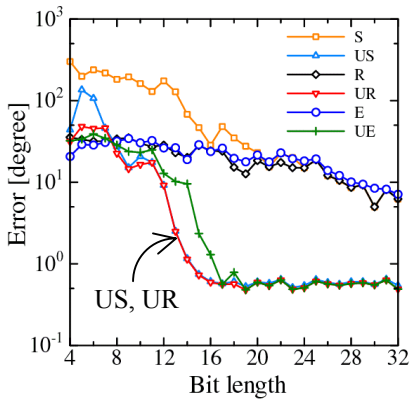

Fig.10. Estimation Error when changing a bit length of the decimal part at 6 elements array and 4 elements sub-array

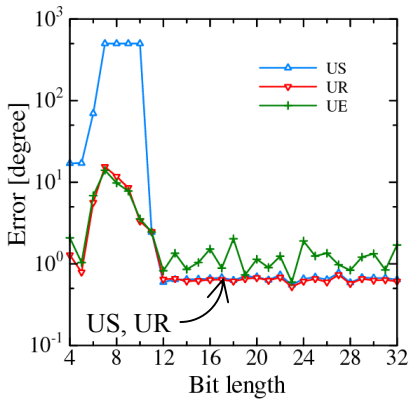

Fig.11. Estimation Error when changing a bit length of the integer part at 6 elements array and 4 elements sub-array

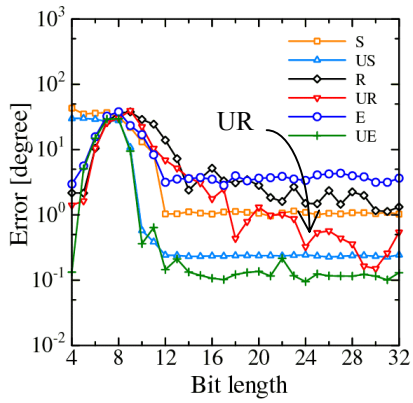

Fig.12. Estimation Error when changing a bit length of the integer part at 10 elements array and 6 elements sub-array

CONCLUSION

In this work, we investigated the characteristics of various DOA estimation algorithms. Throughout the simulations, we confirmed that unitary algorithms were superior to the others, and unitary algorithms were more effective for finite bit-length operation (although the required bit length of Unitary Root-MUSIC becomes larger as the number of array elements increases). Consequently, we conclude that Unitary ESPRIT is the best algorithm in 6 algorithms we examined for digital implementation. In the case where full-correlated 3 waves come at 6 elements array and 4 elements sub-array, at least 28 bits (16bits + 12bits) is necessary for the DOA estimation with sufficient accuracy.

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