Statistical Distributions in Mobile Communications Using Multiple Scattering

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ABSTRACT

A new model utilizing multiple scattering replaces the traditional subdivision of the total fading into a slow lognormal and a fast Rayleigh component. The distribution agrees well with experimental results from a forest and an urban environment. It is shown that part of the slow fading in general may not be due to shadowing, but rather the slow variation of the coupling between scatterers, when the mobile is moving. This means that this slow fading is just as unpredictable as the fast fading, since it originates from the same scatterers. Shadowing will still exist behind major changes in the environment, like street corners.

INTRODUCTION

Due to a multitude of physical phenomena like reflection, scattering, diffraction and guiding effects the field strength will vary with position. Traditionally ([1], [2]) the narrowband fading has been split into two parts, the fast fading and the slow or shadow fading. The slow fading is supposed to give the local mean of the fast Rayleigh fading, and experimentally the division has been made by choosing a proper averaging length for the fast fading. The amplitude $r(x)$ is usually separated into two parts

$$r(x) = m(x) r_0(x) \tag{1}$$

the long-term (or slow) fading $m(x)$ and the short-term (or fast) fading $r_0(x)$. Typically m is described by a lognormal distribution and r_0 by a Rayleigh distribution. This mixed distribution is also called a Suzuki distribution [3]

From a practical point of view the division into the two types of fading has been successful, but theoretically it is unsatisfactory and arbitrary. There is in the general case no good explanation of the slow fading distribution (the lognormal distribution), and it is of interest to see if there are other distributions, which satisfy experimental results and give better explanations. The lognormal distribution has also been used for satellite propagation, especially through foliage [4], and as a combination with the Rice-distribution where the 'constant' part is assumed to obey a lognormal distribution [5].

The main intention of this paper is to give a physically justified distribution **without** averaging over the fast fading. By comparison with experimental results from various environments these distributions will be used to explain the propagation mechanisms of fast and slow fading.

The justification of the classical distributions

The lognormal distribution is hard to justify. Since it is a normal distribution of dB values it is obvious that a *product* of a large number of amplitudes lead to a lognormal distribution by using the central limit theorem. It is difficult to see, however, how such a product would appear in practice for urban propagation, since if it was a multiple forward scattering phenomenon where a new scattering cross-section is multiplied by the previous one as a factor, this would essentially lead to exponential decay of the power in contrast to the well established d^{-n} power law with distance. If the entire scattering occurred locally at street level, it would be strange only to observe the multiply reflected parts, and not the single, dual et cetera reflections. Seen from a statistical point of view the variance of the sum will equal the sum of variances, so for a large number of sums (in dB) the variance will rapidly

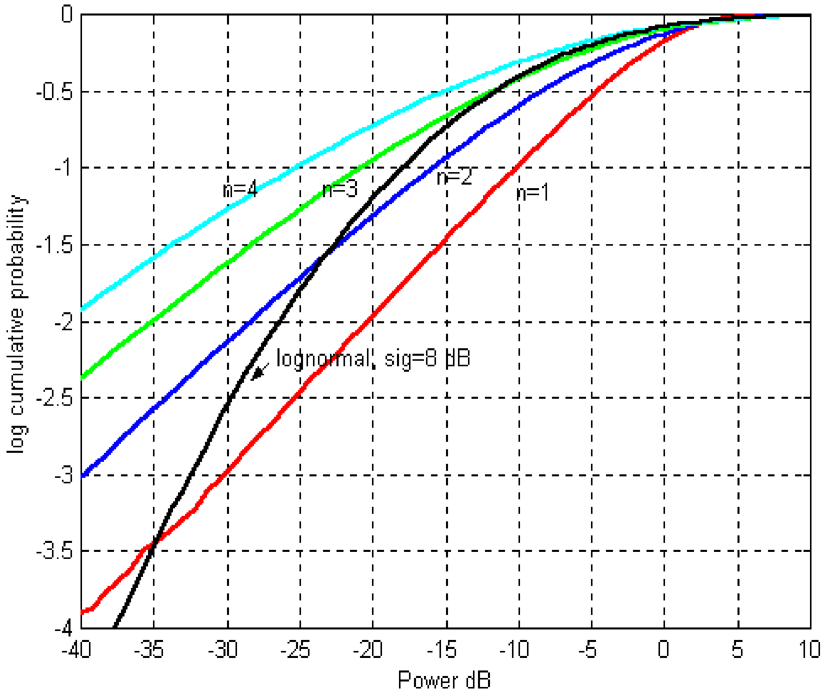

Fig. 1. The log cumulative probability curve for a product of n Rayleigh distributions and a lognormal distribution for comparison.

grow. This is illustrated in Fig. 1, which shows a set of distributions of *n* Rayleigh fading signals multiplied together as in eq. 2, i.e.

$$V_n = \prod_1^n V_i \tag{2}$$

They are normalized to their own mean power, and it is noted that they rapidly deviate from the Rayleigh case (*n*=1). The ordinate is the logarithm of the cumulative probability. The distributions are simulated by using 100.000 realizations. Also shown is the lognormal case with a standard deviation of 8 dB, typical in urban areas, and it shows no similarity to the other curves. Even if they are normalized to have the same standard deviation the case of *n=4* is far from the lognormal distribution, in fact simulations show that a large number of factors in the product are needed to get close to the lognormal distribution at the 10^{-3} cumulative probability. This agrees with the observations by Coulson et al [7].

It should be noted that in [6] it is shown that a random distribution of building heights in it self does not lead to a lognormal distribution, but that additional features may be added from the construction of the buildings to give a lognormal distribution over the 5%-95% range. It is interesting that Lin [8] in a study of rain attenuation has introduced the loglognormal distribution, where the dBs are lognormally distributed and found good agreement with observations. It seems unlikely as an explanation that many dBs should be multiplied to give the total attenuation. Thus, the lognormal distribution cannot be easily justified from a propagation point of view, and it is merely a practical solution to the distribution of local means used for planning purposes

3. Physically motivated distributions

The model assumed in this paper is essentially one consisting of two (or more) layers. Each layer consists of a number of scatterers. It is clear that the model may be extended to a multiplicity of layers. Only forward scattering is assumed. The model may be easily generalized to a multiplicity of layers leading to triple Rayleighs et cetera.
In general, the following model results, where a constant term (a Ricean term *K*) has been added

$$H = K + H_1 + \alpha H_2 H_3 + \beta H_4 H_5 H_6 + ... \tag{11}$$

This is the new model to be applied in the following where it is assumed that H_i are complex, independent Gaussian fading signals. The normalization is such that each product of complex Gaussians has a mean of one. The cumulative distributions of single, double, triple, and quadruple product of Rayleigh fading paths are shown in Figure 1, and it is seen that the deep fading increases significantly. The double fading has been described earlier by Erceg et al [9] as cascaded Rayleigh fading. The basic Rayleigh term H_1 has a mean power of 1, H_2H_3 has also a mean power of 1, and so on.

5. Comparison with experimental results

A forest is an environment where multiple scattering should be present. Figure 2 shows some examples at 1800 MHz for a mobile-to-mobile link. The best fit multiple-Rayleigh model gives the parameters in Table 1.

d	α	β	K
150 m	1.05	0	0
330 m	14.5	0	0

Table 1 Best fit parameters for forest propagation (Fig. 2)

The trend of increasing α with distance is in good agreement with the model. We can assume that a group of trees near the antennas form the effective set of scatterers. At close distance they tend to overlap and single Rayleigh is dominating. For large distances the two groups separate, and the double Rayleigh dominates, even if there are trees in the intervening area. There are also cases, not shown, where β is larger than zero.

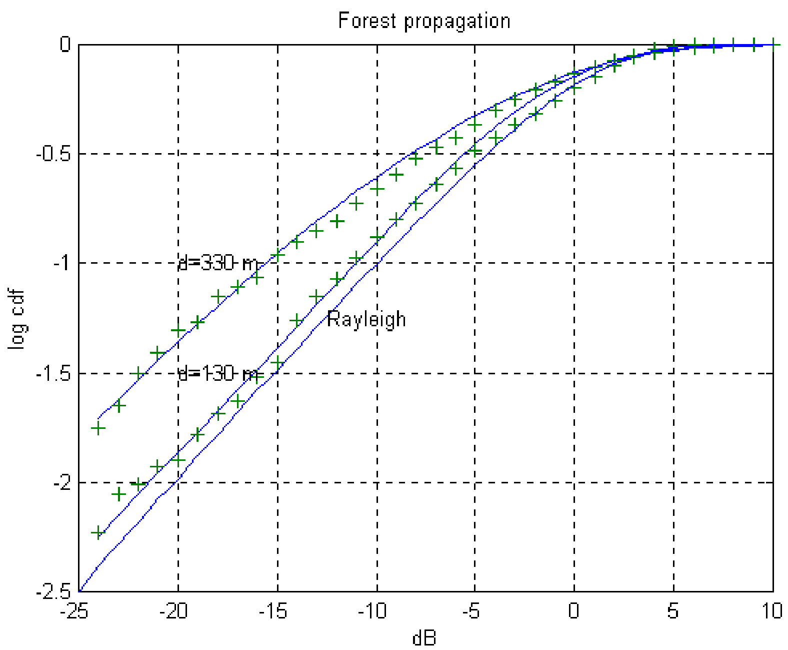

Figure 2 Measurements (+) in a forest at two different distances compared with multiple Rayleigh model (-). Single Rayleigh shown for reference.

CONCLUSION

The traditional way of separating the spatial power variations in a short term and a long-term fading is satisfactory for many purposes. However, it is not related to the physical propagation mechanisms, and the standard lognormal distribution used for the slow fading does not have a simple interpretation. In stead we have chosen not to separate the fading in several parts, but rather study the total fading. The physical basis is a model of forward scattering between scatterers introducing multiply scattered waves defining a new transfer function. This function consists of a sum of a small number of terms, where each term is a multiple product of complex Gaussians. Under certain conditions multi-Rayleigh distributions, like the double Rayleigh, will dominate, but in general there will be a mix of single, double, triple Rayleighs, which form the complete picture. The advantage of the new distribution is the insight it gives into the origin of the slow fading, its disadvantage is the lack of a simple analytical function except in special cases.

The new model is similar in shape to the single reflected times lognormal (Suzuki), but it has a different interpretation. The lognormal is usually interpreted as a shadowing function, which influences the local mean value. The shadowing is supposed to be dependent on the local environment. The multi-Rayleigh distribution has a constant mean power for the single scattering for the whole environment, and the variation of the mean of the total power stems from the slowly varying scattering between the scatterers as the antenna moves. Thus there is no need for a shadowing argument to explain the slow fading. The resulting parameters from the fitting of the distributions may be interpreted as revealing the propagation mechanisms. One important conclusion from this study is that the slow fading is unpredictable, since it originates from the same random elements as the fast fading.

Two environments have been used for comparison, a forest and an urban environment. In both cases the agreement with the model has been excellent, in fact even better than the Suzuki model, which involves the lognormal distribution. Thus it seems that the lognormal distribution is just a practical tool, without any explaining power.

Shadowing will still occur and may give major predictable changes in the power, e.g. at street crossings and general terrain changes, but this will be additional mechanisms on top of those discussed here.

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