NUMERICAL SCATTERING SIMULATIONS FROM TIME-EVOLVING OCEAN-LIKE SURFACES AT L- AND X-BAND: DOPPLER ANALYSIS AND COMPARISONS WITH A COMPOSITE SURFACE ANALYTICAL MODEL

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ABSTRACT

Direct numerical simulations of sea surface scattering are used to evaluate the accuracy of a recently proposed analytical model for ocean-generated Doppler spectra. Comparisons are performed for backscattering at incidence angles varying from near-nadir to low grazing at L- and X-bands. Doppler centroids predicted by the model show, as expected, large discrepancies close to nadir and grazing directions. However, even for moderate incidence angles, a disagreement of 10-25% is observed, especially at X-band. At present, the reason for this deviation is not well understood. The Doppler widths, however, show good agreement across the broad range of incidence angles.

INTRODUCTION

Due to advances in numerical scattering methods and computing technology, a growing number of practical problems in ocean-related radar applications and remote sensing can now be addressed using a computationally-based approach. Doppler analysis of signals scattered from the ocean surface is one such enhanced capability. A particular remote sensing application that relies heavily on *a priori* knowledge of the Doppler spectrum is the determination of ocean currents using the along-track interferometric synthetic aperture radar (AT-InSAR). By comparing the InSAR Doppler measurements to the expected Doppler spectra for stationary surfaces and by interpreting any discrepancy as being due to the effects of an underlying current, one can produce a map of the ocean currents in the area [1].

In practice, the anticipated Doppler spectra are usually derived analytically, using approximate scattering models and, perhaps, rather crude assumptions about ocean surface behavior [2, 3]. The obvious benefit of such an approach is the resulting analytical expressions that are relatively easy to evaluate and that readily lend themselves to further analysis and incorporation in remote sensing retrieval algorithms. In fact, until recently, this has been the only available approach, but there are legitimate concerns about its accuracy, and even its applicability in certain regimes such as low grazing angle (LGA) incidence. The maturing direct numerical simulation methods could eventually provide an alternative tactic for the retrieval of environmental parameters, and they definitely constitute an excellent tool to gauge the accuracy and validity of the approximate models. In this paper, we focus on utilizing the direct numerical simulations to validate one of the recent Doppler models put forward by Romeiser and Thompson in [3] specifically for AT-InSAR applications.

SCATTERING CALCULATIONS

Direct numerical simulations

Scattering simulations for the ocean-like surfaces were performed as described in [4]. The method assumes a two-dimensional geometry shown in Fig. 1, with only the backscattering being considered, and the scattering surface is

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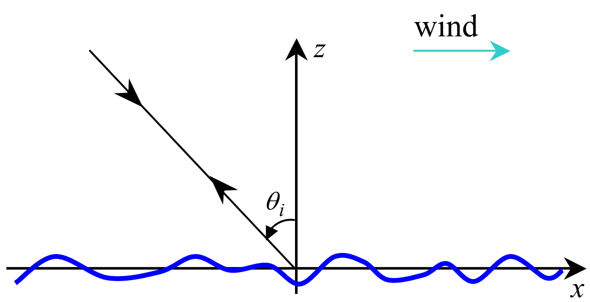

Fig. 1. Problem geometry.

treated as a perfect electric conductor (PEC). A random realization of the surface profile is generated according to the one-dimensional Pierson-Moskowitz spectrum:

$$\Psi_1(K_x) = \begin{cases} \dfrac{0.0081}{2K_x^3} \exp\left\{-\dfrac{0.74g^2}{K_x^2 U^4}\right\}, & K_x \geq 0 \\ 0, & K_x < 0 \end{cases} \tag{1}$$

with g=9.81 m/s^2 being the gravity acceleration and U being the wind speed. The surface is then deterministically evolved in time (with a certain time step) in accordance with a non-linear hydrodynamic model, as will be explained shortly. At each discrete time, the complex amplitude of the field scattered by a "time-frozen" surface profile is calculated exactly using a robust integral equation-based numerical technique called the Method of Ordered Multiple Interactions (MOMI) [5]. This technique is further augmented by the Novel Spectral Acceleration (NSA) algorithm [6], which improves its speed dramatically. Repetition of the calculations for consecutive profiles of evolving surface yields a series of complex field amplitudes that represents a time-varying scattered field and contains all the Doppler information. To obtain statistical averages, the Monte Carlo approach is used and the procedure outlined above is repeated a number of times using different randomly generated starting surface profiles. In this study, the number of statistical realizations is 100. Once the average Doppler spectrum $S_a(f)$ is calculated, the important parameters such as Doppler centroid f_c and the Doppler width γ can be evaluated as

$$f_c = \frac{\int f\, S_a(f)\, df}{\int S_a(f)\, df} \quad \text{and} \quad \gamma^2 = \frac{\int (f - f_c)^2\, S_a(f)\, df}{\int S_a(f)\, df} \tag{2}$$

To describe the surface evolution, a non-linear hydrodynamic model by Creamer *et al* [7] is used. This model was derived from "first principles" and captures important non-linear features such as interactions of short waves with the orbital currents of longer waves. The procedure involves generating and propagating an underlying "linear surface" (each harmonic propagates independently of others according to the water wave dispersion relation, cf. [4]), and at each time of interest, the Creamer integral transformation (cf. [4, 7]) is applied to the linear surface profile.

Romeiser-Thompson analytical model

The model presented in [3] follows the composite surface approach and uses the modulation transfer function (MTF) method and analytical averaging to derive the Doppler centroid f_c, and the spectral width γ as follows:

$$f_c = f_{bragg} + \mathrm{Re}\left\{ \int D^*(\vec{K})\, M(\vec{K}) k^2 \Psi(\vec{K})\, d^2K \right\} \quad \text{and} \quad \gamma^2 = \int D^*(\vec{K})\, D(\vec{K}) k^2 \Psi(\vec{K})\, d^2K \tag{3}$$

In (3), D is a "Doppler modulation transfer function" associated with the orbital motion of the longer waves, and M is the linear modulation transfer function that relates first-order slope variations to cross section changes. These quantities depend on the problem geometry (incidence angle), electromagnetic wavelength, and surface properties such as the behavior of the surface spectrum in the vicinity of the Bragg wavenumber. In addition, M depends on polarization, while D does not. Explicit expressions can be found in [3]. f_{bragg} is the Doppler frequency associated with freely propagating Bragg waves [4], and integrations in (3) include only the "long-wave" part of the of the surface directional spectrum Ψ (the cutoff wavenumber could range from $k_{bragg}/6$ to $k_{bragg}/3$; in our study we used the latter value). Once f_c and γ^2 are known, the Doppler spectrum can be approximated by a Gaussian curve with those two parameters.

For meaningful comparisons with the 2D numerical simulation results, the geometry in Fig. 1 with no variation in y-coordinate (axis points into the paper) was assumed. Consequently the surface spectrum Ψ in (3) was related to Ψ_1 in (1) as

$$\Psi(K_x, K_y) = \Psi_1(K_x)\,\delta(K_y) \tag{4}$$

As with numerical simulations, perfect surface conductivity was used when evaluating MTFs.

The model explicitly accounts for non-linear surface hydrodynamics by including in M a "hydrodynamic" term that describes the amplitude enhancement of short-scale waves near the long-wave crests. Other, more implicit (and perhaps rather heuristic) assumptions about the long-short wave interactions are incorporated in the Doppler MTF D. Namely, (5) in [3] implies that the facets of Bragg waves already propagating at their "free" phase velocity, will acquire additional speed equal to the orbital velocity of the longer wave.

RESULTS

The Doppler centroids at L band (λ=23 cm) and X band (λ=3 cm) are shown if Figs. 2a and 2b, respectively. The vertical bars indicate $\pm 3\sigma$ statistical errors in evaluating centroids from Monte Carlo trials, which corresponds to a 0.9985 confidence interval. In both cases the model gives adequate qualitative prediction of the Doppler centroid behavior for $\theta_i \geq 20^{\circ}$. Namely, it captures HH-VV centroid separation, as well as the decreasing trend around 20^{o}-30^{o}.

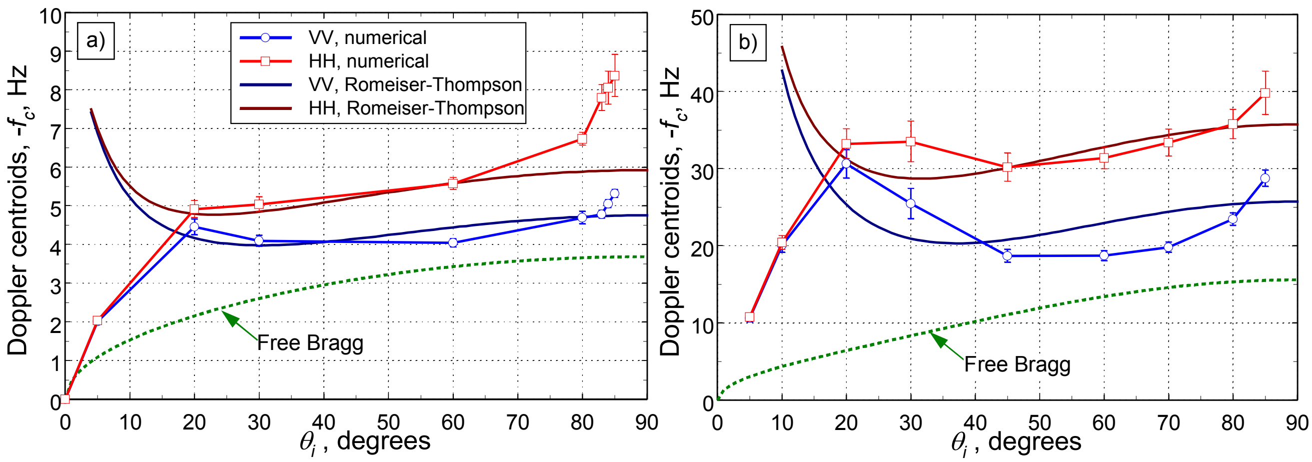

Fig. 2. Doppler centroids for 5 m/s Pierson-Moskowitz surface. a) L-band; b) X-band.

Within the model validity range of 30^{o} to 60^{o} specified in [3], the agreement with the numerical simulation results at L-band seems to be quite good (note though, that in Fig. 2a the numerical data points for intermediate angles are rather sparse). Even outside the stated validity range, the L-band data occasionally continues to show good match. For X-band, however, the agreement is not as good. Even within the 30^{o} - 60^{o} validity range, the differences of up to 25% are encountered for both polarizations. We observed a similar increase in this discrepancy at L-band when the wind speed was increased to 7 m/s.

Figs. 3a and 3b illustrate the behavior of the second parameter, Doppler width. The prediction of Romeiser-Thompson

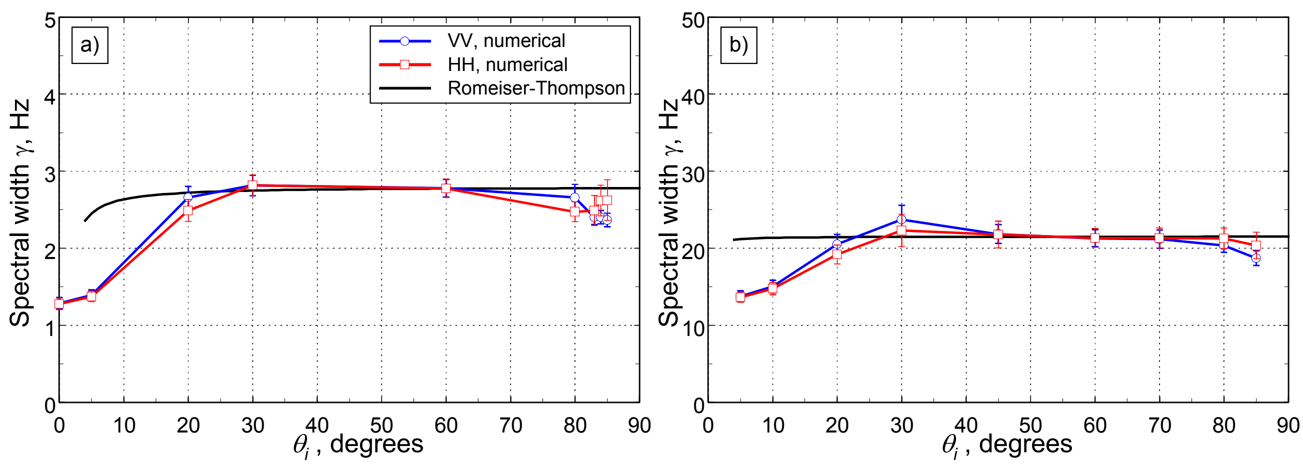

Fig. 3. Doppler widths for 5 m/s Pierson-Moskowitz surface. a) L-band; b) X-band.

model in (3) does not depend on polarization and is represented by a single line. At both bands, the numerical simulations and the model show good agreement across the wide range of incidence angles.

Finally, Fig. 4 compares Doppler spectra obtained form direct numerical simulations with the Gaussian-shape approximations from the model (spectra are normalized to unit area). As could be expected from the comparisons in Fig. 2b, at 30° the model Doppler spectrum is shifted with respect to its numerical counterpart. Also note that the numerical Doppler spectrum in Fig. 4a is too skewed to be accurately represented by a Gaussian curve (this skewness is

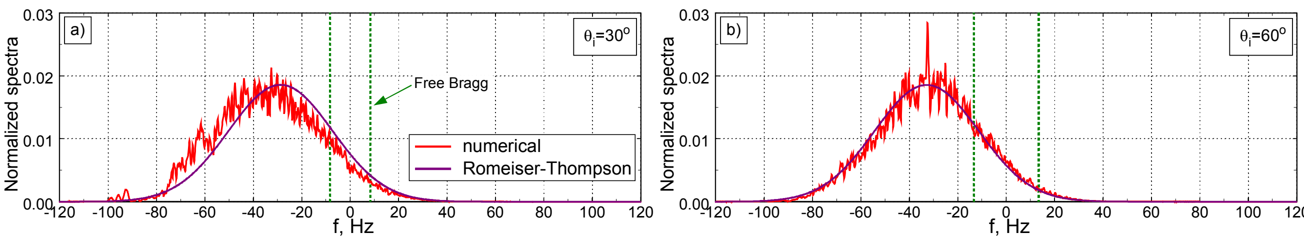

Fig. 4. Doppler spectra for 5 m/s Pierson-Moskowitz surface. X-band, HH.

even more pronounced in the simulation spectra at a smaller angle of 20°). However, at 60°, the positions and shapes of both numerical and the model spectra match very well.

CONCLUSION

Doppler centroids predicted by the Romeiser-Thompson model show good qualitative agreement with the results of direct numerical simulations for moderate and even large incidence angles. The quantitative agreement is good for the L-band example, but at X-band the differences of up to 25% are observed even at intermediate angles within the expected model validity range. The reason for this discrepancy is not quite clear. While the two-scale scattering model at the core of the Romeiser-Thompson approach is supposed to work well at intermediate angles, the hydrodynamic assumptions used there might not be quite adequate. On the other hand, the spectral width predictions are based on the same assumptions and do show good agreement for a broad range of incidence angles. Since the electromagnetic and hydrodynamic parts of the model are intricately bundled together, more detailed analysis of the sources of the observed discrepancies is problematic.

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